# INTENSIVE SCIENCE & VIRTUAL PHILOSOPHY

manuel delanda

*Intensive Science and Virtual Philosophy*

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# INTENSIVE SCIENCE AND VIRTUAL PHILOSOPHY

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continuum
LONDON • NEW YORK

**Continuum**
The Tower Building, 11 York Road, London SE1 7NX
370 Lexington Avenue, New York, NY 10017–6503
*www.continuumbooks.com*

First published in 2002



**British Library Cataloguing-in-Publication Data**
A catalogue record for this book is available from the British Library.

ISBN 0–8264–5622–7 (hardback)
0–8264–5623–5 (paperback)

Typeset by CentraServe, Saffron Walden, Essex
Printed and bound in Great Britain by
MPG Books Ltd, Bodmin, Cornwall

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To Julieta Pereda Pineda,
who taught so much about the world

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# *Introduction: Deleuze's World*

There are always dangers in writing a book with a specific audience in mind. The most obvious one is the danger of missing the target audience completely, either because the subject matter fails to grab its attention or because the style of presentation does not meet its standards or expectations. Then there is the associated danger of losing readers who, had not that particular target been chosen, would have formed the real audience of the book. A book may end up this way without any readership at all. In the world of Western philosophy, for example, history and geography have conspired to divide this world into two almost mutually exclusive camps, the Anglo-American and the Continental camps, each with its own style, research priorities and long traditions to defend. A philosophical book which refuses to take sides, attempting, for example, to present the work of a philosopher of one camp in the terms and style of the other, may end up being a book without an audience: too Anglo-American for the Continentals, and too Continental for the Anglo-Americans.

Such a danger is evident in a book like this, which attempts to present the work of the philosopher Gilles Deleuze to an audience of analytical philosophers of science, and of scientists interested in philosophical questions. When confronted with Deleuze's original texts this audience is bound to be puzzled, and may even be repelled by the superficial similarity of these texts with books belonging to what has come to be known as the 'post-modern' tradition. Although as I argue in these pages Deleuze has absolutely nothing in common with that tradition, his experimental style is bound to create that impression. Another source of difficulty is the philosophical *resources* which Deleuze brings to his project. Despite the fact that authors like Spinoza and Leibniz, Nietzsche and Bergson, have much to offer to philosophy today, they are not generally perceived by scientists or analytical philosophers of science as a legitimate resource. For this reason what I offer here is not a direct interpretation of Deleuze texts but a *reconstruction* of his philosophy, using entirely different theoretical resources and lines of argument. The point of this reconstruction is not just to make his ideas seem legitimate to my intended audience, but also to show that his conclusions do not depend on his particular choice of resources, or the particular lines of argument he uses, but that they are *robust to changes* in theoretical assumptions and strategies. Clearly, if the same conclusions can be reached from entirely different points of departure and following entirely different paths, the validity of those conclusions is thereby strengthened.

I must qualify this statement, however, because what I attempt here is far from a comprehensive reconstruction of all of Deleuze's philosophical ideas. Instead, I focus on a particular yet fundamental aspect of his work: *his ontology*. A philosopher's ontology is the set of entities he or she assumes to exist in reality, the types of entities he or she is committed to assert actually exist. Although in the history of philosophy there are a great variety of ontological commitments, we can very roughly classify these into three main groups. For some philosophers reality has no existence independently from the human mind that perceives it, so their ontology consists mostly of mental entities, whether these are thought as transcendent objects or, on the contrary, as linguistic representations or social conventions. Other philosophers grant to the objects of everyday experience a mind-independent existence, but remain unconvinced that theoretical entities, whether unobservable relations such as physical causes, or unobservable entities such as electrons, possess such an ontological autonomy. Finally, there are philosophers who grant reality full autonomy from the human mind, disregarding the difference between the observable and the unobservable, and the anthropocentrism this distinction implies. These philosophers are said to have a *realist ontology*. Deleuze is such a realist philosopher, a fact that by itself should distinguish him from most post-modern philosophies which remain basically non-realist.

Realist philosophers, on the other hand, need not agree about the contents of this mind-independent reality. In particular, Deleuze rejects several of the entities taken for granted in ordinary forms of realism. To take the most obvious example, in some realist approaches the world is thought to be composed of fully formed objects whose identity is guaranteed by their possession of an *essence*, a core set of properties that defines what these objects are. Deleuze is *not* a realist about essences, or any other *transcendent* entity, so in his philosophy something else is needed to explain what gives objects their identity and what preserves this identity through time. Briefly, this something else is *dynamical processes*. Some of these processes are material and energetic, some are not, but even the latter remain *immanent* to the world of matter and energy. Thus, Deleuze's process ontology breaks with the essentialism that characterizes naive realism and, simultaneously, removes one of the main objections which non-realists make against the postulation of an autonomous reality. The extent to which he indeed deprives non-realists from this easy way out depends, on the other hand, on the details of his account of how the entities that populate reality are produced without the need for anything transcendent. For this reason I will not be concerned in this reconstruction with the textual source of Deleuze's ideas, nor with his style of argumentation or his use of language. In short, I will not be concerned with Deleuze's *words* only with Deleuze's *world*.

The basic plan of the book is as follows. Chapter 1 introduces the formal ideas needed to think about the abstract (or rather *virtual*) structure of dynamical processes. I draw on the same mathematical resources as Deleuze (differential geometry, group theory) but, unlike him, I do not assume the reader is already familiar with these fields. Deleuze's grasp of the technical details involved is, I hope to show, completely adequate (by analytical philosophy standards), but his discussion of technical details is so compressed, and assumes so much on the part of the reader, that it is bound to be misinterpreted. Chapter 1 is written as an alternative to his own presentation of the subject, guiding the reader step by step though the different mathematical ideas involved (manifolds, transformation groups, vector fields) and giving examples of the application of these abstract ideas to the task of modelling concrete physical processes. Despite my efforts at unpacking as much as possible the contents of Deleuze's highly compressed descriptions, however, the subject matter remains technical and some readers may still find it hard to follow. I recommend that such readers skip this first chapter and, if need be, come back to it once the point of the formal resources becomes clear in its applications to less abstract matters in the following chapters.

Chapters 2 and 3 deal with the production of the different entities that populate Deleuze's world. The basic theme is that, within a realist perspective, one does not get rid of essences until one replaces them with something else. This is a burden which affects only the realist philosopher given that a non-realist can simply declare essences mental entities or reduce them to social conventions. One way to think about essentialism is as a theory of the genesis of form, that is, as a theory of *morphogenesis*, in which physical entities are viewed as more or less faithful realizations of ideal forms. The details of the process of realization are typically never given. Essences are thought to act as *models*, eternally maintaining their identity, while particular entities are conceived as mere *copies* of these models, resembling them with a higher or lower degree of perfection. Deleuze replaces the false genesis implied by these pre-existing forms which remain *the same* for all time, with a theory of morphogenesis based on the notion of *the different*. He conceives difference not negatively, as lack of resemblance, but positively or productively, as that which drives a dynamical process. The best examples are *intensive differences*, the differences in temperature, pressure, speed, chemical concentration, which are key to the scientific explanation of the genesis of the form of inorganic crystals, or of the forms of organic plants and animals. Chapter 2 is concerned with the *spatial* aspects of this intensive genesis while Chapter 3 deals with its *temporal* aspects.

After reconstructing Deleuze's ontology I move on in Chapter 4 to give a brief account of his *epistemology*. For any realist philosopher these two areas must be, in fact, intimately related. This may be most clearly seen in the case of naive realism, where truth is conceived as a relation of *correspondence* between, on one hand, a series of facts about the classes of entities populating reality and, on the other, a series of sentences expressing those facts. If one assumes that a class of entities is defined by the essence which its members share in common, it becomes relatively simple to conclude that these classes are basically given, and that they exhaust all there is to know about the world. The ontological assumption that the world is basically closed, that entirely novel classes of entities cannot emerge spontaneously, may now be coupled with the epistemological one, and the correspondence between true sentences and real facts can be made absolute. It is unclear to what extent any realist philosopher actually subscribes to this extremely naive view, but it is clear that a reconstruction of Deleuze's realism must reject each one of these assumptions and replace them with different ones.

While in the first three chapters I attempt to eliminate the erroneous assumption of a closed world, in Chapter 4 I try to replace not only the idea of a simple correspondence but, beyond that, *to devalue the very idea of truth*. In other words, I will argue that even if one accepts that there are true sentences expressing real facts it can still be maintained that most of these factual sentences are *trivial*. The role of the thinker is not so much to utter truths or establish facts, but to distinguish among the large population of true facts those that are important and relevant from those that are not. *Importance and relevance*, not truth, are the key concepts in Deleuze's epistemology, the task of realism being to ground these concepts preventing them from being reduced to subjective evaluations or social conventions. This point can be made clearer if we contrast Deleuze's position not with the linguistic version of correspondence theory but with the mathematical one. In this case a relation of correspondence is postulated to exist between the *states* of a physical object and the *solutions* to mathematical models capturing the essence of that object. By contrast, Deleuze stresses the role of correctly posed *problems*, rather than their true solutions, a problem being well posed if it captures an objective distribution of the important and the unimportant, or more mathematically, of *the singular and the ordinary*.

Chapter 4 explores this *problematic epistemology* and compares it with the more familiar axiomatic or theorematic versions which predominate in the physical sciences. To anticipate the main conclusion of the chapter, while in an axiomatic epistemology one stresses the role of *general laws*, in a problematic one laws as such disappear but without sacrificing the objectivity of physical knowledge, an objectivity now captured by distributions of the singular and the ordinary. If such a conclusion can indeed be made plausible, it follows that despite the fact that I reconstruct Deleuze to cater to an audience of scientists and analytical philosophers of science, nothing is yielded to the orthodox positions held by these two groups of thinkers. On the contrary, both physical science and analytical philosophy emerge transformed from this encounter with Deleuze, the former retaining its objectivity but losing the laws it holds so dear, the latter maintaining its rigour and clarity but losing its exclusive focus on facts and solutions. And more importantly, *the world itself* emerges transformed: the very idea that there can be a set of true sentences which give us the facts once and for all, an idea presupposing a closed and finished world, gives way to an open world full of divergent processes yielding novel and unexpected entities, the kind of world that would not sit still long enough for us to take a snapshot of it and present it as the final truth.

To conclude this introduction I must say a few words concerning that other audience which my reconstruction may seem to overlook: Deleuzian philosophers, as well as thinkers and artists of different kinds who are interested in the philosophy of Deleuze. First of all, there is much more to Deleuze's books than just an ontology of processes and an epistemology of problems. He made contributions to such diverse subjects as the nature of cinema, painting and literature, and he held very specific views on the nature and genesis of subjectivity and language. For better or for worse, these are the subjects that have captured the attention of most readers of Deleuze, so it will come as a surprise that I will have nothing to say about them. Nevertheless, if I manage to reconstruct Deleuze's world these other subjects should be illuminated as well, at least indirectly: once we understand Deleuze's world we will be in a better position to understand what could cinema, language or subjectivity be in that world.

On the other hand, if this reconstruction is to be faithful to Deleuze's world it is clear that I must rely on an adequate interpretation of his words. There is a certain violence which Deleuze's texts must endure in order to be reconstructed for an audience they were not intended for, so whenever I break with his own way of presenting an idea I explain in detail the degree of rupture and the reason for it in a footnote. A different kind of violence is involved in wrenching his ideas from his collaboration with Félix Guattari. In this reconstruction I use Deleuze's ontology and epistemology as exposed in his early texts, and use only those parts of his collaborative work which can be directly traced to those early texts. For this reason I always ascribe the source of those ideas to him, using the pronoun 'he' instead of 'they' even when quoting from their joint texts. Finally, there is the violence done to Deleuze's fluid style, to the way he fights the premature solidification of a terminology by always keeping it in a state of flux. Fixing his terminology will seem to some akin to pinning down a live butterfly. As an antidote I offer an appendix where I relate the terms used in my reconstruction to all the different terminologies he uses in his own texts and in his collaborative work, setting his words free once again after they have served their purpose of giving us his world. The hope is that this world will retain all its openness and divergence, so that the intense expressivity and even madness so often attributed to Deleuze's words may be seen as integral properties of the world itself.

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## CHAPTER 1

# *The Mathematics of the Virtual: Manifolds, Vector Fields and Transformation Groups*

Of all the concepts which populate the work of Gilles Deleuze there is one that stands out for its longevity: the concept of *multiplicity*. This concept makes its appearance in his early books and remains one of central importance, with almost unchanged meaning and function, until his final work.[1] Its formal definition is highly technical, including elements from several different branches of mathematics: differential geometry, group theory and dynamical systems theory. In this chapter I will discuss the technical background needed to define this important concept but some preliminary informal remarks will prove helpful in setting the stage for the formal discussion. In the first place, one may ask what role the concept of a multiplicity is supposed to play and the answer would be a replacement for the much older philosophical concept of an *essence*. The essence of a thing is that which explains its *identity*, that is, those fundamental traits without which an object would not be what it is. If such an essence is shared by many objects, then possession of a common essence would also explain the fact that these objects *resemble* each other and, indeed, that they form a distinct *natural kind* of things.

Let's take one of the most traditional illustrations of an essence. When one asks what makes someone a member of the human species the answer may be, for example, being a 'rational animal'. The exact definition of the human essence is not what is at issue here (if rationality and animality are not considered to be essential human properties some other set will do). The important point is that there be some set of defining characteristics, and that this set explain both the identity of the human species and the fact that particular members of the species resemble each other. In a Deleuzian ontology, on the other hand, a species (or any other natural kind) is not defined by its essential traits but rather by the *morphogenetic process* that gave rise to it. Rather than representing timeless categories, species are historically constituted entities, the resemblance of their members explained by having undergone common processes of natural selection, and the enduring identity of the species itself guaranteed by the fact that it has become reproductively isolated from other species. In short, while an essentialist account of species is basically static, a morphogenetic account is inherently dynamic. And while an essentialist account may rely on factors that transcend the realm of matter and energy (eternal archetypes, for instance), a morphogenetic account gets rid of all *transcendent* factors using exclusively form-generating resources which are *immanent* to the material world.

Animal and plant species are not, of course, the only natural kinds traditionally defined by essences. Many other natural kinds, the chemical elements or the set of elementary particles, for example, are also typically so defined. In each of these cases we would need to replace timeless categories by historical processes. Yet, even if successful this replacement would take us only half-way towards our goal. The reason is that even if the details of a given process account for the resemblance among its products, the similarities which make us classify them as members of the same kind, there may be *similarities of process* which still demand an explanation. And when accounting for these common features we may be tempted to reintroduce essences through the back door. These would not be essences of objects or kinds of objects, but essences of processes, yet essences nevertheless. It is in order to break this vicious circle that multiplicities are introduced. And it is because of the tenacity of this circle that the concept of multiplicity must be so carefully constructed, justifying each step in the construction by the way it avoids the pitfalls of essentialism. To anticipate the conclusion I will reach after a long and technical definitional journey: multiplicities specify *the structure of spaces of possibilities*, spaces which, in turn, explain the regularities exhibited by morphogenetic processes. I will begin by defining an appropriate notion of 'space', a notion which must not be purely geometrical but also capable of being linked to questions of process.

The term 'multiplicity' is closely related to that of 'manifold', a term which designates a geometrical space with certain characteristic properties. To grasp what is special about manifolds (and what resources this concept can offer to avoid essentialism) it will be useful to give a brief account of its historical origins. Although the use of geometrical procedures for the solution of problems is an ancient practice inherited from the Greeks, the extensive use of curves and trajectories in the formulation of a variety of physical problems from the sixteenth century on made it necessary to develop new problem-solving resources. With this in mind, René Descartes and Pierre de Fermat invented the now familiar method of embedding curves into a two-dimensional space on which arbitrary axes could be fixed. Once so embedded, the fixed axes allowed the assignment of a pair of numbers, or coordinates, to every point of the curve, so that the geometric relations between points could now be expressed as relations between numbers, a task for which the newly developed algebra was perfectly suited. This translation scheme, in short, allowed the combinatorial resources of algebra to be brought to bear on the solution of geometrical problems.

The term 'manifold' does not belong to the analytical geometry of Descartes and Fermat, but to the *differential geometry* of Friedrich Gauss and Bernhard Riemann, but the basic idea was the same: tapping into a new reservoir of problem-solving resources, the reservoir in this case being the differential and integral calculus. In its original application the calculus was used to solve problems involving relations between the changes of two or more quantities. In particular, if these relations were expressed as a *rate of change* of one quantity relative to another, the calculus allowed finding the instantaneous value for that rate. For example, if the changing quantities were spatial position and time, one could find instantaneous values for the rate of change of one relative to the other, that is, for velocity. Using this idea as a resource in geometry involved the realization that a geometrical object, a curved line or surface, for instance, could also be characterized by the rate at which some of its properties changed, for example, the rate at which *its curvature* changed between different points. Using the tools of the calculus mathematicians could now find 'instantaneous' values for this rate of change, that is, the value of the curvature at a given infinitesimally small point.

In the early nineteenth century, when Gauss began to tap into these differential resources, a curved two-dimensional surface was studied using the old Cartesian method: the surface was embedded in a three-dimensional space complete with its own fixed set of axes; then, using those axes, coordinates would be assigned to every point of the surface; finally, the geometric links between points determining the form of the surface would be expressed as algebraic relations between the numbers. But Gauss realized that the calculus, focusing as it does on infinitesimal points on the surface itself (that is, operating entirely with local information), allowed the study of the surface *without any reference to a global embedding space*. Basically, Gauss developed a method to implant the coordinate axes on the surface itself (that is, a method of 'coordinatizing' the surface) and, once points had been so translated into numbers, to use differential (not algebraic) equations to characterize their relations. As the mathematician and historian Morris Kline observes, by getting rid of the global embedding space and dealing with the surface through its own local properties 'Gauss advanced the totally new concept that *a surface is a space in itself*'.[2]

The idea of studying a surface as a space in itself was further developed by Riemann. Gauss had tackled the two-dimensional case, so one would have expected his disciple to treat the next case, three-dimensional curved surfaces. Instead, Riemann went on to successfully attack a much more general problem: that of N-dimensional surfaces or spaces. It is these N-dimensional curved structures, defined exclusively through their intrinsic features, that were originally referred to by the term 'manifold'. Riemann's was a very bold move, one that took him into a realm of abstract spaces with a variable number of dimensions, spaces which could be studied without the need to embed them into a higher-dimensional (N+1) space. As Morris Kline puts it: 'The geometry of space offered by Riemann was not just an extension of Gauss's differential geometry. It reconsidered the whole approach to the study of space.'[3] And we could add that this new way of *posing spatial problems* would, a few decades later in the hands of Einstein and others, completely alter the way physicists approached the question of space (or more exactly, of spacetime).

A Deleuzian multiplicity takes as its first defining feature these two traits of a manifold: its variable number of dimensions and, more importantly, the absence of a supplementary (higher) dimension imposing an extrinsic coordinatization, and hence, *an extrinsically defined unity*. As Deleuze writes: 'Multiplicity must not designate a combination of the many and the one, but rather an organization belonging to the many as such, which has no need whatsoever of unity in order to form a system.'[4] Essences, on the other hand, do possess a defining unity (e.g. the unity of rationality and animality defining the human essence) and, moreover, are taken to exist in a transcendent space which serves as a container for them or in which they are embedded. A multiplicity, on the other hand, 'however many dimensions it may have, . . . never has a supplementary dimension to that which transpires upon it. This alone makes it natural and immanent.'[5] It may be objected that these are purely *formal* differences between concepts, and that as such, they do not necessarily point to a deeper ontological difference. If we are to replace essences as the explanation of the identity of material objects and natural kinds we need to specify the way in which multiplicities relate to the physical processes which generate those material objects and kinds.

Achieving this goal implies establishing a more intimate relation between the geometric properties of manifolds and the properties which define morphogenetic processes. The resources in this case come from the theory of dynamical systems where the dimensions of a manifold are used to represent properties of a particular physical process or system, while the manifold itself becomes *the space of possible states* which the physical system can have.[6] In other words, in this theory manifolds are connected to material reality by their use as *models* of physical processes. When one attempts to model the dynamical behaviour of a particular physical object (say, the dynamical behaviour of a pendulum or a bicycle, to stick to relatively simple cases) the first step is to determine the number of relevant ways in which such an object can *change* (these are known as an object's *degrees of freedom*), and then to relate those changes to one another using the differential calculus. A pendulum, for instance, can change only in its position and momentum, so it has two degrees of freedom. (A pendulum can, of course, be melted at high temperatures, or be exploded by dynamite. These are, indeed, other ways in which this object can change, they simply are not relevant ways from the point of view of dynamics.) A bicycle, if we consider all its moving parts (handlebars, front wheels, crank-chain-rear-wheel assembly and the two pedals) has ten degrees of freedom (each of the five parts can change in both position and momentum).[7]

Next, one maps each degree of freedom into one of the dimensions of a manifold. A pendulum's space of possibilities will need a two-dimensional plane, but the bicycle will involve a ten-dimensional space. After this mapping operation, the state of the object at any given instant of time becomes a *single point* in the manifold, which is now called a *state space*. In addition, we can capture in this model an object's *changes of state* if we allow the representative point to move in this abstract space, one tick of the clock at a time, describing a curve or trajectory. A physicist can then study the changing behaviour of an object by studying the behaviour of these representative trajectories. It is important to notice that even though my example involves two objects, what their state space captures is not their static properties but the way these properties change, that is, *it captures a process*. As with any model, there is a trade-off here: we exchange the complexity of the object's changes of state for the complexity of the modelling space. In other words, an object's instantaneous state, no matter how complex, becomes a single point, a great simplification, but the space in which the object's state is embedded becomes more complex (e.g. the three-dimensional space of the bicycle becomes a ten-dimensional state space).

Besides the great simplification achieved by modelling complex dynamical processes as trajectories in a space of possible states, there is the added advantage that mathematicians can bring new resources to bear to the study and solution of the physical problems involved. In particular, *topological resources* may be used to analyse certain features of these spaces, features which determine *recurrent or typical behaviour* common to many different models, and by extension, common to many physical processes. The main pioneer of this approach was another great nineteenth-century mathematician, Henri Poincaré. Poincaré began his study not with a differential equation modelling a real physical system, but with a very simple equation, so simple it had no physical application, but which nevertheless allowed him to explore the recurrent traits of *any model with two degrees of freedom*. He discovered and classified certain special topological features of two-dimensional manifolds (called *singularities*) which have a large influence in the behaviour of the trajectories, and since the latter represent actual series of states of a physical system, a large influence in the behaviour of the physical system itself.[8]

Singularities may influence behaviour by acting as *attractors* for the trajectories. What this means is that a large number of different trajectories, starting their evolution at very different places in the manifold, may end up in exactly the same final state (the attractor), as long as all of them begin somewhere within the 'sphere of influence' of the attractor (the *basin of attraction*). Given that, in this sense, different trajectories may be attracted to the same final state, singularities are said to represent the inherent or intrinsic *long-term tendencies* of a system, the states which the system will spontaneously tend to adopt in the long run as long as it is not constrained by other forces. Some singularities are topological points, so the final state they define as a destiny for the trajectories is a *steady state*. Beside these, Poincaré also found that certain closed loops acted as attractors and called them 'limit cycles'. The final state which trajectories attracted to a limit cycle (or periodic attractor) are bound to adopt is an oscillatory state. But whether we are dealing with steady-state, periodic or other attractors what matters is that they are *recurrent topological features*, which means that different sets of equations, representing quite different physical systems, may possess a similar distribution of attractors and hence, similar long-term behaviour.

Let me give a simple example of how singularities (as part of what defines a multiplicity) lead to an entirely different way of viewing the genesis of physical forms. There are a large number of different physical structures which form spontaneously as their components try to meet certain energetic requirements. These components may be constrained, for example, to seek a point of minimal free energy, like a soap bubble, which acquires its spherical form by minimizing surface tension, or a common salt crystal, which adopts the form of a cube by minimizing bonding energy. We can imagine the state space of the process which leads to these forms as structured by a single point attractor (representing a point of minimal energy). One way of describing the situation would be to say that *a topological form* (a singular point in a manifold) guides a process which results in many different physical forms, including spheres and cubes, each one with different *geometric* properties. This is what Deleuze means when he says that singularities are like 'implicit forms that are topological rather than geometric'.[9] This may be contrasted to the essentialist approach in which the explanation for the spherical form of soap bubbles, for instance, would be framed in terms of the essence of sphericity, that is, of geometrically characterized essences acting as ideal forms.

I will discuss in a moment the meaning and relevance of the topological nature of singularities. What matters at this point is that singularities, by determining long-term tendencies, structure the possibilities which make up state space, and by extension, structure the possibilities open to the physical process modelled by a state space. In addition, singularities tend to be recurrent, that is, they tend to characterize processes independently of their particular physical mechanisms. In the example above, the mechanism which leads to the production of a soap bubble is quite different from the one leading to a salt crystal, yet both are minimizing processes. This *mechanism-independence* is what makes singularities (or rather the multiplicities they define) perfect candidates to replace essences.[10] As I said before, however, we must be careful at this stage not to make singularities the equivalent of the essence of a process. To avoid this error I will discuss some additional formal properties of multiplicities distinguishing them from essences and then, as above, I will discuss the way in which these purely conceptual differences connect with questions of physical process.

The formal difference in question has to do with the way essences and multiplicities are specified as entities. While essences are traditionally regarded as possessing a *clear and distinct* nature (a clarity and distinctiveness also characterizing the ideas which appear in the mind of a philosopher who grasps one of these essences), multiplicities are, by design, *obscure and distinct*: the singularities which define a multiplicity come in sets, and these sets are not given all at once but are structured in such a way that they *progressively specify the nature of a multiplicity* as they unfold following recurrent sequences.[11] What this means may be illustrated first by a metaphor and then given a precise technical definition. The metaphor is that of a fertilized egg prior to its unfolding into a fully developed organism with differentiated tissues and organs. (A process known as *embryogenesis*.) While in essentialist interpretations of embryogenesis tissues and organs are supposed to be already given in the egg (*preformed*, as it were, and hence having a clear and distinct nature) most biologists today have given up preformism and accepted the idea that differentiated structures emerge progressively as the egg develops. The egg is not, of course, an undifferentiated mass: it possesses an obscure yet distinct structure defined by zones of biochemical concentration and by polarities established by the asymmetrical position of the yolk (or nucleus). But even though it does possess the necessary biochemical materials and genetic information, these materials and information do not contain a clear and distinct blueprint of the final organism.[12]

Although the egg metaphor does provide a vivid illustration of the distinction I am trying to draw here, it is nevertheless just a useful analogy. Fortunately, there are technical ways of defining the idea of *progressive differentiation* which do not rely on metaphors. The technical resources in this case come from another crucial nineteenth-century innovation, the theory of groups, a field of mathematics which, like the differential geometry I discussed before, eventually became an integral part of the basic mathematical technology of twentieth-century physics. The term 'group' refers to a set of entities (with special properties) and a rule of combination for those entities. The most important of the properties is the one named 'closure', which means that, when we use the rule to combine any two entities in the set, the result is an entity also belonging to the set. For example, the set of positive integers displays closure if we use addition as a combination rule: adding together any two positive integers yields another positive integer, that is, another element in the original set.[13]

Although sets of numbers (or many other mathematical objects) may be used as illustrations of groups, for the purpose of defining progressive differentiation we need to consider groups whose members are not objects but *transformations* (and the combination rule, a consecutive application of those transformations). For example, the set consisting of rotations by ninety degrees (that is, a set containing rotations by 0, 90, 180, 270 degrees) forms a group, since any two consecutive rotations produce a rotation also in the group, provided 360 degrees is taken as zero. The importance of groups of transformations is that they can be used to classify geometric figures by their *invariants*: if we performed one of this group's rotations on a cube, an observer who did not witness the transformation would not be able to notice that any change had actually occurred (that is, the visual appearance of the cube would remain invariant relative to this observer). On the other hand, the cube would not remain invariant under rotations by, say, 45 degrees, but a sphere would. Indeed, a sphere remains visually unchanged under rotations by *any amount* of degrees. Mathematically this is expressed by saying that the sphere has *more symmetry* than the cube relative to the rotation transformation. That is, degree of symmetry is measured by the number of transformations in a group that leave a property invariant, and relations between figures may be established if the group of one is included in (or is a subgroup of) the group of the other.

Classifying geometrical objects by their degrees of symmetry represents a sharp departure from the traditional classification of geometrical figures by their essences. While in the latter approach we look for a set of properties common to all cubes, or to all spheres, groups do not classify these figures on the basis of their static properties but in terms of how these figures are affected (or not affected) by active transformations, that is, figures are classified by *their response to events that occur to them*.[14] Another way of putting this is that even though in this new approach we are still classifying entities by a property (their degree of symmetry), this property is never an intrinsic property of the entity being classified but always a property relative to a specific transformation (or group of transformations). Additionally, the symmetry approach allows dynamic relations to enter into the classification in a different way. When two or more entities are related as the cube and the sphere above, that is, when the group of transformations of one is a subgroup of the other, it becomes possible to envision *a process which converts one of the entities into the other* by losing or gaining symmetry. For example, a sphere can 'become a cube' by loosing invariance to some transformations, or to use the technical term, by undergoing a *symmetry-breaking transition*. While in the realm of pure geometry this transmutation may seem somewhat abstract, and irrelevant to what goes on in the worlds of physics or biology, there are many illustrations of symmetry-breaking transitions in these more concrete domains.

In a physical process transmutations through broken symmetry may occur, for example, in the form of *phase transitions*. Phase transitions are events which take place at critical values of some parameter (temperature, for example) switching a physical system from one state to another, like the critical points of temperature at which water changes from ice to liquid, or from liquid to steam. The broken symmetry aspect here can be clearly seen if we compare the gas and solid states of a material, and if for simplicity we assume perfectly uniform gases and perfect crystal arrangements. In these ideal conditions, the gas would display invariant properties under all translations, rotations and reflections, while the solid would be invariant to only a subset of these transformations. For example, while the gas could be displaced by any amount and remain basically the same (that is, an observer would be unable to tell whether a displacement occurred at all) the solid would remain visually unchanged only under displacements which moved it one unit crystal at a time (or multiples of that unit). In other words, the gas has more symmetry than the solid, and can become the solid by undergoing a symmetry-breaking phase transition.[15] The metaphorical example I gave above, that of a fertilized egg which differentiates into a fully formed organism, can now be made quite literal: the progressive differentiation of the spherical egg is achieved through a complex cascade of symmetry-breaking phase transitions.[16]

Let me now incorporate the idea of progressive differentiation into the concept of multiplicity by showing how it can be translated into state-space terms. I said before that for the purpose of defining an entity to replace essences the aspect of state space that mattered was its singularities. One singularity (or set of singularities) may undergo a symmetry-breaking transition and be converted into another one. These transitions are called *bifurcations* and may be studied by adding to a particular state space one or more 'control knobs' (technically, control parameters) which determine the strength of external shocks or perturbations to which the system being modelled may be subject. These control parameters tend to display *critical values*, thresholds of intensity at which a particular bifurcation takes place breaking the prior symmetry of the system. A state space structured by one point attractor, for example, may bifurcate into another with two such attractors, or a point attractor may bifurcate into a periodic one, losing some of its original symmetry.[17] Much as attractors come in recurrent forms, so bifurcations may define *recurrent sequences* of such forms. There is a sequence, for instance, that begins with a point attractor which, at a critical value of a control parameter, becomes unstable and bifurcates into a periodic attractor. This cyclic singularity, in turn, can become unstable at another critical value and undergo a sequence of instabilities (several period-doubling bifurcations) which transform it into a chaotic attractor.

This symmetry-breaking cascade of bifurcations can, in turn, be related to actual recurring sequences in physical processes. There is, for example, a realization of the above cascade occurring in a well-studied series of distinct hydrodynamic flow patterns (steady-state, cyclic and turbulent flow). Each of these recurrent flow patterns appears one after the other at well-defined critical thresholds of temperature or speed. The sequence of phase transitions may be initiated by heating a water container from below. At low temperatures the flow of heat from top to bottom, referred to as thermal *conduction*, is simple and steady, displaying only a bland, featureless overall pattern, having the degree of symmetry of a gas. At a critical point of temperature, however, this steady flow suddenly disappears and another one takes its place, thermal *convection*, in which coherent rolls of water form, rotating either clockwise or anti-clockwise. The water container now has structure and, for the same reason, has lost some symmetry. As the temperature continues to intensify another threshold is reached, the flow loses its orderly periodic form and a new pattern takes over: *turbulence*. The cascade that yields the sequence conduction–convection–turbulence is, indeed, more complicated and may be studied in detail through the use of a special machine called the Couette–Taylor apparatus, which speeds up (rather than heats up) the liquid material. At least seven different flow patterns are revealed by this machine, each appearing at a specific critical point in speed, and thanks to the simple cylindrical shape of the apparatus, each phase transition may be directly related to a broken symmetry in the group of transformations of the cylinder.[18]

As can be seen from this example, a cascade of bifurcations may be faithfully realized in a physical system. This realization, however, bears no resemblance to the mathematical cascade. In particular, unlike the latter which is *mechanism-independent*, the physical realization involves specific mechanisms. To begin with there are causal interactions and their effects. To return to our example, the flow of heat into the container causes a graded density difference to form, given that water expands when heated (that is, becomes less dense). This density gradient, in turn, interacts with other forces like the viscosity of the water, their balance of power determining whether a system switches from one flow pattern to the next. For example, the density gradient will tend to amplify small differences in movement (fluctuations) which could add some detail to the bland steady-state flow, but which are damped by the viscosity of the fluid. As the flow of heat is intensified, however, the system reaches a critical point at which the density gradient is strong enough to overcome viscosity, leading to the amplification of fluctuations and allowing the formation of coherent rolls. Thus, a very specific sequence of events underlies the transition to convection. On the other hand, as the biologist Brian Goodwin has pointed out, portions of this hydrodynamic sequence may be observed in a completely different process, the complex morphogenetic sequence which turns a fertilized egg into a fully developed organism. After describing another instance of a sequence of flow patterns in hydrodynamics Goodwin says:

> The point of the description is not to suggest that morphogenetic patterns originate from the hydrodynamic properties of living organisms . . . What I want to emphasize is simply that many pattern-generating processes share with developing organisms the characteristic that spatial detail unfolds progressively simply as a result of the laws of the process. In the hydrodynamic example we see how an initially smooth fluid flow past a barrier goes through a symmetry-breaking event to give a spatially periodic pattern, followed by the elaboration of local nonlinear detail which develops

> out of the periodicity. Embryonic development follows a similar qualitative course: initially smooth primary axes, themselves the result of spatial bifurcation from a uniform state, bifurcate to spatially periodic patterns such as segments [in an insect body], within which finer detail develops . . . through a progressive expression of nonlinearities and successive bifurcations . . . The role of gene products in such an unfolding is to stabilize a particular morphogenetic pathway by facilitating a sequence of pattern transitions, resulting in a particular morphology.[19]

From a Deleuzian point of view, it is this *universality* (or mechanism-independence) of multiplicities which is highly significant. Unlike essences which are always abstract and general entities, multiplicities are *concrete universals*. That is, concrete sets of attractors (realized as tendencies in physical processes) linked together by bifurcations (realized as abrupt transitions in the tendencies of physical processes). Unlike the generality of essences, and the resemblance with which this generality endows instantiations of an essence, the universality of a multiplicity is typically *divergent*: the different realizations of a multiplicity bear no resemblance whatsoever to it and there is in principle no end to the set of potential divergent forms it may adopt. This lack of resemblance is amplified by the fact that multiplicities give form to processes, not to the final product, so that the end results of processes realizing the same multiplicity may be highly dissimilar from each other, like the spherical soap bubble and the cubic salt crystal which not only do not resemble one another, but bear no similarity to the topological point guiding their production.

The concept of progressive differentiation which I have just defined was meant, as I said, to distinguish the obscure yet distinct nature of multiplicities from the clear and distinct identity of essences, as well as from the clarity afforded by the light of reason to essences grasped by the mind. A final distinction must now be made: unlike essences, which as abstract general entities coexist side by side sharply distinguished from one another, concrete universals must be thought as *meshed together into a continuum*. This further blurs the identity of multiplicities, creating zones of indiscernibility where they blend into each other, forming a continuous immanent space very different from a reservoir of eternal archetypes. Multiplicities, as Deleuze writes, coexist

> but they do so at points, on the edges, and under glimmerings which never have the uniformity of a natural light. On each occasion, obscurities and zones of shadow correspond to their distinction. [Multiplicities] are distinguished from one another, but not at all in the same manner as forms and the terms in which these are incarnated. They are objectively made and unmade according to the conditions that determine their fluent synthesis. This is because they combine the greatest power of being differen*t*iated with an inability to be differen*c*iated.[20]

Although I will not stick to this subtle typographical distinction, Deleuze distinguishes the progressive unfolding of a multiplicity through broken symmetries (differen*t*iation), from the progressive specification of the continuous space formed by multiplicities as it gives rise to our world of discontinuous spatial structures (differen*c*iation). Unlike a transcendent heaven which exists as a *separate dimension* from reality, Deleuze asks us to imagine a continuum of multiplicities which *differenciates itself* into our familiar three-dimensional space as well as its spatially structured contents.

Let me explain in what sense a continuous space may be said to become progressively defined giving rise to discontinuous spaces. First of all, a space is not just a set of points, but a set together with a way of binding these points together into *neighbourhoods* through well-defined relations of *proximity or contiguity*. In our familiar Euclidean geometry these relations are specified by fixed lengths or distances which determine how close points are to each other. The concept of 'length' (as well as related ones, like 'area' or 'volume') is what is called a *metric* concept, so the spaces of Euclidean geometry are known as *metric spaces*.[21] There exist other spaces, however, where fixed distances cannot define proximities since distances do not remain fixed. A topological space, for example, may be stretched without the neighbourhoods which define it changing in nature. To cope with such exotic spaces, mathematicians have devised ways of defining the property of 'being nearby' in a way that does not presuppose any metric concept, but only nonmetric concepts like 'infinitesimal closeness'. However one characterizes it, the distinction between *metric and nonmetric spaces* is fundamental in a Deleuzian ontology.[22] Moreover, and this is the crucial point, there are well-defined technical ways of linking metric and nonmetric spaces in such a way that the former become the product of the progressive differentiation of the latter. To explain how such a symmetry-breaking cascade would work in this case, I will need to take a brief detour through the history of nineteenth-century geometry.

Although in that century most physicists and mathematicians thought the structure of physical space was captured by Euclidean geometry, many other geometries, with very different properties, had come into existence. Some of them (such as the non-Euclidean geometry developed by Lobatchevsky) shared with the geometry of Euclid the property of being metric. There were, however, other geometries where metric concepts were not in fact fundamental. The differential geometry of Gauss and Riemann which gave us the concept of a manifold is one example, but there were several others (projective geometry, affine geometry, topology). Moreover, and despite the fact that Euclidean geometry reigned supreme, some mathematicians realized that its basic concepts could in fact be *derived* from the nonmetric concepts which formed the foundation of the newcomers. In particular, another influential nineteenth-century mathematician, Felix Klein, realized that all the geometries known to him could be categorized by their invariants under groups of transformations, and that the different groups were embedded one into the other.[23] In modern terminology this is equivalent to saying that the different geometries were related to each other by relations of broken symmetry.

In Euclidean geometry, for example, lengths, angles and shapes remain unaltered by a group containing rotations, translations and reflections. This is called the group of *rigid transformations*. These metric properties, however, do not remain invariant under the groups of transformations characterizing other geometries. There is one geometry, called *affine geometry*, which adds to the group characterizing Euclidean geometry new transformations, called *linear transformations*, under which properties like the parallelism or the straightness of lines remain invariant, but not their lengths. Then there is *projective geometry*, which adds to rigid and linear transformations those of projection, corresponding to shining light on a piece of film, and section, the equivalent of intercepting those light rays on a screen. (More technically, this geometry adds transformations called 'projectivities'.) These transformations do not necessarily leave Euclidean or affine properties unchanged, as can be easily pictured if we imagine a film projector (which typically increases the magnitude of lengths) and a projection screen at an angle to it (which distorts parallel lines).

If we picture these three geometries as forming the levels of a hierarchy (projective–affine–Euclidean) it is easy to see that the transformation group of each level includes the transformations of the level below it and adds new ones. In other words, each level possesses more symmetry than the level below it. This suggests that, as we move down the hierarchy, a symmetry-breaking cascade should produce progressively more differentiated geometric spaces, and, vice versa, that as we move up we should lose differentiation. For example, as we ascend from Euclidean geometry more and more figures become equivalent to one another, forming *a lesser number of distinct classes*. Thus, while in Euclidean geometry two triangles are equivalent only if their sides have the same length, in affine geometry all triangles are the same (regardless of lengths). In other words, as we move up the class of equivalent triangles becomes less differentiated. Or to take a different example, while in Euclidean geometry two conic sections (the family of curves containing circles, ellipses, parabolas and hyperbolas) are equivalent if they are both of the same type (both circles or both parabolas) and have the same size, in affine geometry they only need to be of the same type (regardless of size) to be equivalent, while in projective geometry all conic sections, without further qualification, are the same.[24] In short, as we move up the hierarchy figures which used to be fully differentiated from one another become progressively less distinct eventually blending into a single one, and vice versa, as we move down, what used to be one and the same shape progressively differentiates into a variety of shapes.

This hierarchy can be expanded to include other geometries, such as differential geometry and topology. The latter, for example, may be roughly said to concern the properties of geometric figures which remain invariant under bending, stretching, or deforming transformations, that is, transformations which do not create new points or fuse existing ones. (More exactly, topology involves transformations, called 'homeomorphisms', which *convert nearby points into nearby points* and which can be reversed or be continuously undone.) Under these transformations many figures which are completely distinct in Euclidean geometry (a triangle, a square and a circle, for example) become one and the same figure, since they can be deformed into one another. In this sense, topology may be said to be the *least differentiated* geometry, the one with the least number of distinct equivalence classes, the one in which many discontinuous forms have blended into one continuous one.[25] Metaphorically, the hierarchy 'topological–differential–projective–affine–Euclidean' may be seen as representing an abstract scenario for the birth of real space. As if the metric space which we inhabit and that physicists study and measure *was born* from a nonmetric, topological continuum as the latter differentiated and acquired structure following a series of symmetry-breaking transitions.

This *morphogenetic* view of the relation between the different geometries is a metaphor in the sense that to mathematicians these relations are purely *logical*, useful because theorems which are valid at one level are automatically valid at the levels below it.[26] But this cascade of broken symmetries may be also given *an ontological dimension*. One way in which this scenario for the birth of metric space can be made less metaphorical and more directly ontological, is through a comparison between metric and nonmetric geometrical properties, on one hand, and *extensive and intensive physical properties*, on the other. Extensive properties include not only such metric properties as length, area and volume, but also quantities such as amount of energy or entropy. They are defined as properties which are *intrinsically divisible*: if we divide a volume of matter into two equal halves we end up with two volumes, each half the extent of the original one. Intensive properties, on the other hand, are properties such as temperature or pressure, which cannot be so divided. If we take a volume of water at 90 degrees of temperature, for instance, and break it up into two equal parts, we do not end up with two volumes at 45 degrees each, but with two volumes at the original temperature.[27]

Deleuze argues, however, that an intensive property is not so much one that is indivisible but one which *cannot be divided without involving a change in kind*.[28] The temperature of a given volume of liquid water, for example, can indeed be 'divided' by heating the container from underneath creating a temperature difference between the top and bottom portions of the water. Yet, while prior to the heating the system is at equilibrium, once the temperature difference is created the system will be away from equilibrium, that is, we can divide its temperature but in so doing we change the system qualitatively. Indeed, as we just saw, if the temperature difference is made intense enough the system will undergo a phase transition, losing symmetry and changing its dynamics, developing the periodic pattern of fluid motion which I referred to above as 'convection'. Thus, in a very real sense, phase transitions do divide the temperature scale but in so doing they mark sudden changes in the spatial symmetry of a material.

Using these new concepts we can define the sense in which the metric space we inhabit emerges from a nonmetric continuum through a cascade of broken symmetries. The idea would be to view this genesis not as an abstract mathematical process but as a concrete physical process in which an undifferentiated *intensive space* (that is, a space defined by continuous intensive properties) progressively differentiates, eventually giving rise to *extensive structures* (discontinuous structures with definite metric properties). We can take as an illustration of this point some recent developments in quantum field theories. Although the concept of spontaneous symmetry breaking, and its connection with phase transitions, developed in rather humble branches of physics, like the fields of hydrodynamics and condensed matter physics, it was eventually incorporated into the main stream.[29] Today, this concept is helping unify the four basic forces of physics (gravitational, electromagnetic, strong and weak nuclear forces) as physicists realize that, at extremely high temperatures (the extreme conditions probably prevailing at the birth of the universe), these forces lose their individuality and blend into one, highly symmetric, force. The hypothesis is that as the universe expanded and cooled, a series of phase transitions broke the original symmetry and allowed the four forces to differentiate from one another.[30] If we consider that, in relativity theory, gravity is what gives space its metric properties (more exactly, a gravitational field constitutes the metric structure of a four-dimensional manifold), and if we add to this that gravity itself emerges as a distinct force at a specific critical point of an intensive property (temperature), the idea of an intensive space giving birth to extensive ones through progressive differentiation becomes more than a suggestive metaphor.[31]

Let me pause for a moment to summarize the argument so far. I began by establishing some purely formal differences between the concepts of 'essence' and of 'multiplicity': while the former concept implies a unified and timeless identity, the latter lacks unity and implies an identity which is not given all at once but is defined progressively; and while essences bear to their instantiations the same relation which a model has to its copies, that is, a relation of greater or lesser resemblance, multiplicities imply divergent realizations which bear no similarity to them. These formal differences, I said, are insufficient to characterize the distinction between essences and multiplicities as *immaterial entities* whose job is to account for the genesis of form: replacing eternal archetypes involves supplying an alternative explanation of morphogenesis in the world. Unlike essences which assume that matter is a passive receptacle for external forms, multiplicities are immanent to material processes, defining their spontaneous capacity to generate pattern without external intervention. I used certain features of mathematical models (state spaces) to define the nature of multiplicities: a multiplicity is defined by distributions of singularities, defining tendencies in a process; and by a series of critical transitions which can take several such distributions embedded within one another and unfold them. Finally, I said that a population of such concrete universals forms a real dimension of the world, a nonmetric continuous space which progressively specifies itself giving rise to our familiar metric space as well as the discontinuous spatial structures that inhabit it.

No doubt, despite my efforts these remarks remain highly metaphorical. First of all, I have defined multiplicities in terms of attractors and bifurcations but these are features of mathematical models. Given that I want the term 'multiplicity' to refer to a concrete universal (to replace abstract general essences) the question may arise as to the legitimacy of taking features of a model and reifying them into the defining traits of a real entity. Second, the relation between a continuum of multiplicities and the discontinuous and divisible space of our everyday world was specified entirely by analogy with a purely mathematical construction, the hierarchy of geometries first dreamt by Felix Klein. Eliminating the metaphorical content will involve not only a thorough ontological analysis of state space so that its *topological invariants* can be separated from its variable mathematical content, but in addition, a detailed discussion of how these topological invariants may be woven together *to construct* a continuous, yet heterogeneous, space. In the following chapter I will show in technical detail how this construction can be carried out and how the resulting continuum may replace the top or least metric level in the hierarchy of geometries. I will also discuss how the intermediate levels may be replaced by intensive processes of individuation which yield as their final product the fully differentiated metric structures that populate the bottom level. At the end of chapter two the metaphor of a genesis of metric space through a cascade of broken symmetries should have been mostly eliminated, and a literal account taken its place.

Meanwhile, in what remains of this chapter I would like to make a more detailed analysis of the nature of multiplicities. The first set of issues to be discussed will involve the technical details of Deleuze's ontological interpretation of the contents of state space. His approach is very unorthodox as will be shown by a comparison with the state space ontologies proposed by analytical philosophers. Then I will move on to a second set of issues concerning the *modal status* of multiplicities. Modal logic is the branch of philosophy which deals with the relations between *the possible and the actual*. Here the question to be answered is if state space is a space of possible states what is the status of attractors and bifurcations in relation to these possibilities? Can multiplicities be interpreted in terms of the traditional modal categories, the possible and the necessary, or do we need to postulate an original form of physical modality to characterize them? Finally, a third set of issues that needs to be dealt with is related to the speculative dimension of Deleuze's project. Replacing essences with social conventions or subjective beliefs is a relatively safe move, but putting in their place a new set of objective entities inevitably involves philosophical speculation. What guides this speculation? One way of looking at this question is to see Deleuze as engaged in a constructive project guided by certain *proscriptive constraints*, that is, constraints which tell him not what to do but what to avoid doing. One such constraint is, of course, to avoid the trap of essentialism, but there are others and these need to be discussed.

Let me begin with Deleuze's ontological analysis of state space. Many philosophers are today looking at these abstract spaces as objects of study and reflection. A recent shift in the analytical philosophy of science, for example, moving away from logic (and set theory) and towards an analysis of the actual mathematics used by scientists in their everyday practice, has brought the importance of state spaces to the foreground.[32] Yet none of the philosophers involved in this new movement has attempted such an original analysis of state space as Deleuze has. In particular, analytical philosophers seem unaware of (or at least unconcerned with) Poincaré's topological studies and of the ontological difference that may be posited between the recurrent features of state space and the trajectories these features determine. Given that this ontological difference is key to the idea of a Deleuzian multiplicity, I will need to explain how state spaces are constructed. First of all, it is important to distinguish the different operators involved in this construction. As I said above, given a relation between the changes in two (or more) degrees of freedom expressed as a rate of change, one operator, differentiation, gives us the instantaneous value for such a rate, such as an instantaneous velocity (also known as a *velocity vector*). The other operator, integration, performs the opposite but complementary task: from the instantaneous values it reconstructs a full trajectory or series of states.

These two operators are used in a particular order to generate the structure of state space. The modelling process begins with a choice of manifold to use as a state space. Then from experimental observations of a system's changes in time, that is, from actual series of states as observed in the laboratory, we create some trajectories to begin populating this manifold. These trajectories, in turn, serve as the raw material for the next step: we repeatedly apply the differentiation operator to the trajectories, each application generating one velocity vector and in this way we generate a *velocity vector field*. Finally, using the integration operator, we generate from the vector field further trajectories which can function as predictions about future observations of the system's states. The state space filled with trajectories is called the 'phase portrait' of the state space.[33] Deleuze makes a *sharp ontological distinction between the trajectories* as they appear in the phase portrait of a system, on one hand, *and the vector field*, on the other. While a particular trajectory (or integral curve) models a succession of actual states of a system in the physical world, the vector field captures the inherent tendencies of many such trajectories, and hence of many actual systems, to behave in certain ways. As mentioned above, these tendencies are represented by singularities in the vector field, and as Deleuze notes, despite the fact that the *precise nature* of each singular point is well defined only in the phase portrait (by the form the trajectories take in its vicinity) *the existence and distribution* of these singularities is already completely given in the vector (or direction) field. In one mathematician's words:

> The geometrical interpretation of the theory of differential equations clearly places in evidence two absolutely distinct realities: there is the field of directions and the *topological accidents* which may suddenly crop up in it, as for example the existence of . . . singular points to which no direction has been attached; and there are the integral curves with the form they take on in the vicinity of the singularities of the field of directions . . . The existence and distribution of singularities are notions relative to the field of vectors defined by the differential equation. The form of the integral curves is relative to the solution of this equation. The two problems are assuredly complementary, since the nature of the singularities of the field is defined by the form of the curves in their vicinity. But it is no less true that the field of vectors on one hand and the integral curves on the other are *two essentially distinct mathematical realities*.[34]

There are several other features of singularities, or more specifically, of attractors, which are crucial in an ontological analysis of state space, and which further differentiate its two 'distinct mathematical realities'. As is well known, the trajectories in this space always approach an attractor *asymptotically*, that is, they approach it *indefinitely close but never reach it*.[35] This means that unlike trajectories, which represent the actual states of objects in the world, attractors are *never actualized*, since no point of a trajectory ever reaches the attractor itself. It is in this sense that singularities represent only the long-term tendencies of a system, never its actual states. Despite their lack of actuality, attractors are nevertheless real and have definite effects on actual entities. In particular, they confer on trajectories a certain degree of stability, called *asymptotic stability*.[36] Small shocks may dislodge a trajectory from its attractor but as long as the shock is not too large to push it out of the basin of attraction, the trajectory will naturally return to the stable state defined by the attractor (a steady state in the case of point attractors, a stable cycle in the case of periodic attractors, and so on). Another important feature involves not the stability of the trajectories but that of the distribution of attractors itself (its *structural stability*). Much as the stability of trajectories is measured by their resistance to small shocks, so the stability of a particular distribution of attractors is checked by submitting the vector field to perturbations, an effect achieved by adding a small vector field to the main one, and checking whether the resulting distribution of attractors is *topologically equivalent* to the original one.[37] Typically, distributions of attractors are structurally stable and this, in part, is what accounts for their recurrence among different physical systems. On the other hand, if the perturbation is large enough a distribution of attractors may cease to be structurally stable and change or bifurcate into a different one. Such a bifurcation event is defined as a continuous deformation of one vector field into another topologically *inequivalent* one through a structural *instability*.[38]

Using the technical terms just introduced I can give now a final definition of a multiplicity. A multiplicity *is a nested set of vector fields related to each other by symmetry-breaking bifurcations, together with the distributions of attractors which define each of its embedded levels*. This definition separates out the part of the model which carries information about the actual world (trajectories as series of possible states) from that part which is, in principle, *never actualized*. This definition presupposes only the two concepts of 'differential relation' and 'singularity'. I will return in the next chapter to a discussion of what further *philosophical transformation* these two concepts need to undergo in order to be truly detached from their mathematical realization. At this point, granting that the definition I just gave could specify a concrete entity, we may ask what ontological status such an entity would have? To speak as I did of patterns of hydrodynamic flow and of patterns of embryological development as divergent *realizations* of a universal multiplicity is misleading since it suggests that these patterns are real, while the multiplicity itself is not. So Deleuze speaks not of 'realization' but of *actualization*, and introduces a novel ontological category to refer to the status of multiplicities themselves: *virtuality*. This term does not refer, of course, to the virtual reality which digital simulations have made so familiar, but to a *real virtuality* forming a vital component of the objective world. As he writes:

> The virtual is not opposed to the real but to the actual. *The virtual is fully real in so far as it is virtual* . . . Indeed, the virtual must be defined as strictly a part of the real object – as though the object had one part of itself in the virtual into which it plunged as though into an objective dimension . . . The reality of the virtual consists of the differential elements and relations along with the singular points which correspond to them. The reality of the virtual is structure. We must avoid giving the elements and relations that form a structure an actuality which they do not have, and withdrawing from them a reality which they have.[39]

What is *the modal status* of the virtual? If state space trajectories have the status of possibilities (possible series of states) what modality do virtual multiplicities represent? This is not an easy question to answer given that the ontological status of even the familiar modal categories is a thorny issue. So before dealing with virtuality let me discuss the question of possibility. Traditionally, ontological discussion of possibilities has been very controversial due to their elusive nature, and in particular, to the difficulty of giving a clear criterion for *individuating* them, that is, for telling when we have one instead of another possibility. As a famous critic of modal logic, the philosopher Willard Van Orman Quine, jokes:

> Take, for instance, the possible fat man in the doorway; and again, the possible bald man in the doorway. Are they the same possible man, or two possible men? How do we decide? How many possible

> men there are in that doorway? Are there more possible thin ones than fat ones? How many of them are alike? Or would their being alike make them one? Are not two possible things alike? Is this the same as saying that it is impossible for two things to be alike? Or, finally, is the concept of identity simply inapplicable to unactualized possibles? But what sense can be found in talking of entities which cannot be meaningfully said to be identical with themselves and distinct from one another?[40]

Most approaches to modal logic concentrate on language, or more specifically, on an analysis of sentences which express *what could have been*, sentences such as 'If J.F.K. had not been assassinated then the Vietnam War would have ended sooner.' Given that human beings seem capable of routinely using and making sense of these counterfactual sentences, the modal logician's task is to explain this ordinary capability.[41] However, the fact that linguistically specified *possible worlds* (like the possible world where J.F.K. survived) are so devoid of structure, and allow so much ambiguity as to what distinguishes one possible world from another, is what has prompted criticisms such as Quine's. But as some philosophers have suggested, the problem here would seem to be with linguistic representations and their lack of resources to structure possible worlds, and not with possibilities as such. The philosopher of science Ronald Giere, for instance, thinks the extra constraints which structure state space can overcome the limitations of other modal approaches:

> As Quine delights in pointing out, it is often difficult to individuate possibilities . . . [But] many models in which the system laws are expressed as differential equations provide an unambiguous criterion to individuate the possible histories of the model. They are the trajectories in state space corresponding to all possible initial conditions. Threatened ambiguities in the set of possible initial conditions can be eliminated by explicitly restricting the set in the definition of the theoretical model.[42]

Giere argues that state spaces may be viewed as a way of specifying possible worlds for a given physical system, or at least, possible histories for it, each trajectory in the phase portrait representing one possible historical sequence of states for a system or process. The individuality of the different possible histories within state space is defined by *laws*, expressed by the differential equations that functionally relate the system's degrees of freedom, as well as by *initial conditions*, the specific state, or point in the manifold, where a system begins its evolution. Given a specific initial condition and a deterministic law (such as those of classical physics) one and only one trajectory is individuated, a fact that may be used to challenge Quine's sceptical stance. The phase portrait of any particular state space will be typically filled with many such individual trajectories, one for each possible initial condition. One may reduce this number by adding other laws which forbid certain combinations of values for the degrees of freedom, that is, which make some initial conditions not available for a given system, but still, one ends up with many possible histories.[43]

The problem for the philosopher becomes what *ontological status* to assign to these well-defined possibilities. One ontological stance, which Giere calls 'actualism', denies any reality to the possible trajectories, however well individuated they may be. A mathematical model, in this view, is simply a tool to help us in the control of particular physical systems (that is, the manipulation in the laboratory of the behaviour of real systems) as well as in the prediction of their future behaviour. For this limited purpose of prediction and control all we need to judge is the *empirical adequacy* of the model: we generate *one trajectory* for a given initial condition, then try to reproduce that particular combination of values for the degrees of freedom in the laboratory, and observe whether the sequence of *actual states* matches that predicted by the trajectory. Given the one trajectory we associate with the actual sequence in an experiment, the rest of the population of trajectories is merely a useful fiction, that is, ontologically unimportant.[44] As Giere argues, however, this ontological stance misses the fact that the population of trajectories as a whole *displays certain regularities* in the possible histories of a system, global regularities which play a role in shaping any one particular actual history.[45] To him, understanding a system is not knowing how it actually behaves in this or that specific situation, but knowing *how it would behave* in conditions which may in fact not occur. And to know that we need to use the global information embodied in the population of possible histories, information which is lost if we concentrate on the one trajectory which is compared with real sequences of states.[46]

As should be clear from the discussion in this chapter, Deleuze was not an 'actualist'. He held a realist position towards the modal structure of state space but would have disagreed with Giere in his interpretation of what constitutes that modal structure. In particular, in a Deleuzian ontology one must emphasize that the regularities displayed by the different possible trajectories are *a consequence* of the singularities that shape the vector field. The well-defined nature of the possible histories is not to be approached by a mere mention of laws expressed as differential equations, but by an understanding of how such equations in fact individuate trajectories. Each possible sequence of states, each possible history, is generated by following at each point of the trajectory the directions specified by the vector field, and any regularities or propensities exhibited by the trajectories should indeed be ascribed to the topological accidents or singularities of the field of directions. As Deleuze puts it, 'the singularities preside over the genesis' of the trajectories.[47] In other words, Giere is right in thinking that state space offers more resources than language to individuate possibilities (thus sidestepping Quine's criticisms) but wrong in his assessment of how the *process of individuation* takes place. To leave the vector field out of our ontological analysis (that is, to make it into an auxiliary construction or yet another useful fiction) hides the real source of the regularities or propensities in the population of possible histories.[48]

This point tends to be obscured in traditional philosophical analyses by the use of examples involving the simplest type of equation, *a linear equation*. Despite the fact that of all the types of equations available to physicists the linear type is *the least typical*, it happens to be the type that became dominant in classical physics. The vector fields of these differential equations are extremely simple, 'the only possible attractor of a linear dynamical system is a fixed point. Furthermore, this fixed point is unique – a linear dynamical system cannot have more than one basin of attraction.'[49] In other cases (in conservative systems which are quasi-isolated from their surroundings) there may be no attractors at all, only trajectories.Thus, in a linear conservative system (such as the harmonic oscillator used as an example by Giere) the vector field is so barely structured that it may, for most practical purposes, be ignored as a source of constraints in the individuation of trajectories. On the other hand, the more typical equations (nonlinear equations) have a more elaborate distribution of singularities, the state space being normally partitioned in a cellular fashion by many attractors and their basins, and these multiple attractors may be of different types. In these more common cases, the vector field has too much structure to be ignored.[50]

This argument, however, establishes only that there are in state space other constraints for the individuation of possible histories, but not that they should be given a separate modal status. We could, it would seem, take singularities to belong to the realm of the possible and save ourselves the trouble of introducing novel forms of physical modality, such as virtuality. One way of doing this would be to take a basin of attraction to be merely a subset of points of state space. Given that state space is a space of possible states, any subset of it will also be just a collection of possibilities.[51] Yet, as I mentioned before, despite the fact that the nature of singularities is well defined only in the phase portrait of a system, their *existence and distribution* is already given in the vector field, where they define overall flow tendencies for the vectors. It may seem plausible to think of point attractors, for example, as just one more point of state space, but this singular point is not an available possibility for the system since it is never occupied by a trajectory, only approached by it asymptotically. Trajectories will tend to approach it ever closer but never reach it, and even when one speaks of the end state of a trajectory, in reality the curve is fluctuating around its attractor, not occupying it. Strictly speaking, as I said above, attractors are *never actualized*.

Thus, it seems, a more complete analysis of state space does seem to demand a form of physical modality that goes beyond mere possibility. But could not that other traditional modal category, *necessity*, do the job? After all, in classical physics' models a general law relates all the successive points of a trajectory in a necessary or deterministic way, and which specific trajectory is generated is necessarily determined given a particular initial state.[52] This is, indeed, true, but the relative importance of general laws and particular initial conditions changes once we add singularities. On one hand, the role of any particular initial state is greatly diminished since many initial conditions (all those that are included within a particular basin) will be equivalent as far as the end state of the trajectory is concerned. The states a trajectory adopts on its way to the end state, what engineers call its transient states and which constitute the bulk of the trajectory, may be of interest sometimes, but clearly will not be as important as the stable end state, since the system will spend most of its time fluctuating around that state. On the other hand, the role of the general law will also be diminished because the behaviour of the trajectory at its end state, a steady-state or a cyclic behaviour, for example, will be determined not by its previous states (defined by the general law), but by the type of the attractor itself.

This argument, again, establishes the need to consider additional factors in the individuation of possible histories but not the need for additional modalities. After all, is not the end state of a trajectory necessary? In this case too, the complexity of the distribution of singularities makes a great difference in our interpretation of the modal structure of state space. A state space with a single attractor, and a single basin encompassing the entire space, has a unique end state for the evolution of the system. Concentrating on this atypical case, therefore, can mislead us into thinking that determinism implies a single necessary outcome. On the other hand, a space with multiple attractors *breaks the link between necessity and determinism*, giving a system a 'choice' between different destinies, and making the particular end state a system occupies a combination of determinism and chance. For instance, which attractor a system happens to be in at any one time is determined, in part, by its contingent history: a trajectory may be dislodged from an attractor by an *accident*, a strong-enough external shock pushing it out of one basin and into the sphere of influence of another attractor. Furthermore, which specific distribution of attractors a system has available at any one point in its history, may be changed by a bifurcation. When a bifurcation leads to two alternative distributions, only one of which can be realized, a deterministic system faces further 'choices'. Which alternative obtains, as nonlinear scientists Ilya Prigogine and Gregoire Nicolis have been arguing for decades, will be decided by chance fluctuations in the environment. Speaking of the emergence of convection cells at a phase transition, these authors write:

> As soon as [the critical value is reached] we know that the cells will appear: this phenomenon is therefore subject to strict determinism. In contrast, the *direction* of rotation of the cells [clock- or anti-clockwise] is unpredictable and uncontrollable. Only chance, in the form of the particular perturbation that may have prevailed at the moment of the experiment, will decide whether a given cell is right- or lefthanded. We thus arrive at a remarkable cooperation between chance and determinism . . . Stated more formally, *several solutions* are possible for the *same parameter value*. Chance alone will decide which of these solutions is realized.[53]

This line of argument for a different interpretation of the modal structure of state space is, in fact, not Deleuze's own, although it follows directly from his ontological analysis. Deleuze own arguments against the orthodox categories of the possible and the necessary are of a more general philosophical nature,[54] and are linked directly with the third set of issues I said needed to be discussed in the remainder of this chapter: the constraints that guide Deleuze's speculation about virtuality. I have already mentioned one such constraint, to avoid at all costs conceptualizing virtual multiplicities as eternal essences. Meeting this constraint requires rejecting much of what modal logic has to say about possibility and necessity. The reason is that the postulation of possible worlds existing alongside the actual world, as Quine and other critics have often remarked, almost always implies a commitment to one or another form of essentialism.[55] And, it should be emphasized, this criticism applies not only to modal philosophers but also to those physicists who seriously believe in the existence of alternate parallel universes.

When thinking about these parallel universes, both philosophers and physicists assume the existence of *fully formed individuals* populating the different possible worlds. This immediately raises a number of questions: Can the same individual exist, slightly altered, in other worlds? Can he or she maintain this identity across many worlds, after several slight alterations have accumulated? Could we identify him or her after all these changes? It is here that essences, either general or particular, are introduced to define the identity of these individuals and to guarantee its preservation across worlds. There are basically two different technical ways of achieving this effect. On one hand, one can claim that transworld identity is insured by the possession of a *particular essence*, that is, the property of being this particular individual. On the other hand, one can deny that there are, in fact, such transworld individuals, and speak simply of *counterparts*, that is, other possible individuals which closely resemble their real counterpart, but are not identical to it (in particular, they do not share the essence of being precisely this individual). These counterparts, however, would share a general essence. (Such as being 'rational animals', in the case of human beings.[56])

The alternative offered by Deleuze is to avoid taking as given fully formed individuals, or what amounts to the same thing, to always *account for the genesis of individuals* via a specific individuation process, such as the developmental process which turns an embryo into an organism. This emphasis on the objective production of the spatio-temporal structure and boundaries of individuals stands in stark contrast with the complete lack of process mediating between the possible and the real in orthodox modal thinking. The category of the possible assumes a set of predefined forms which retain their identity despite their non-existence, and which already resemble the forms they will adopt once they become realized. In other words, unlike the individuation process linking virtual multiplicities and actual structures, realizing a possibility does not add anything to the pre-existing form but mere reality. As Deleuze writes:

> What difference can there be between the existent and the non-existent if the non-existent is already possible, already included in the concept and having all the characteristics that the concept confers upon it as a possibility? . . . The possible and the virtual are . . . distinguished by the fact that one refers to the form of identity in the concept, whereas the other designates a pure multiplicity . . . which radically excludes the identical as a prior condition . . . To the extent that the possible is open to 'realization' it is understood as an image of the real, while the real is supposed to resemble the

> possible. That is why it is difficult to understand what existence adds to the concept when all it does is double like with like . . . Actualization breaks with resemblance as a process no less than it does with identity as a principle. In this sense, actualization or differenciation is always a genuine creation. Actual terms never resemble the singularities they incarnate . . . For a potential or virtual object to be actualized is to create divergent lines which correspond to – without resembling – a virtual multiplicity.[57]

Besides the avoidance of essentialist thinking, Deleuze's speculation about virtuality is guided by the closely related constraint of avoiding *typological* thinking, that style of thought in which individuation is achieved through the *creation of classifications and of formal criteria for membership in those classifications*. Although some classifications are essentialist, that is, use transcendent essences as the criterion for membership in a class, this is not always the case. For example, unlike Platonic essences which are transcendent entities, Aristotle's 'natural states', those states towards which an individual tends, and which would be achieved if there were not interfering forces, are not transcendent but *immanent* to those individuals. But while Aristotelian philosophy is indeed non-essentialist it is still completely typological, that is, concerned with defining the criteria which group individuals into species, and species into genera.[58]

For the purpose of discussing the constraints guiding Deleuze's constructive project, one historical example of typological thinking is particularly useful. This is the classificatory practices which were common in Europe in the seventeenth and eighteenth centuries, such as those that led to the botanical taxonomies of Linnaeus. Simplifying somewhat, we may say that these classifications took as a point of departure perceived *resemblances* among fully formed individuals, followed by precise comparisons aimed at an exhaustive listing of what differed and what stayed the same among those individuals. This amounted to a translation of their visible features into a linguistic representation, a tabulation of differences and *identities* which allowed the assignment of individuals to an exact place in an ordered table. Judgments of *analogy* between the classes included in the table were used to generate higher-order classes, and relations of *opposition* were established between those classes to yield dichotomies or more elaborate hierarchies of types. The resulting biological taxonomies were supposed to reconstruct a natural order which was *fixed and continuous*, regardless of the fact that historical accidents may have broken that continuity. In other words, given the fixity of the biological types, *time itself* did not play a constructive role in the generation of types, as it would later on in Darwin's theory of the evolution of species.[59]

Deleuze takes the four elements which inform these classificatory practices, *resemblance*, *identity*, *analogy* and *opposition* (or contradiction) as the four categories to be avoided in thinking about the virtual. Deleuze, of course, would not deny that there are objects in the world which resemble one another, or that there are entities which manage to maintain their identity through time. It is just that resemblances and identities must be treated as *mere results* of deeper physical processes, and not as fundamental categories on which to base an ontology.[60] Similarly, Deleuze would not deny the validity of making judgments of analogy or of establishing relations of opposition, but he demands that we give an account of that which allows making such judgments or establishing those relations. And this account is not to be a story about us, about categories inherent in our minds or conventions inherent in our societies, but a story about the world, that is, about the objective individuation processes which yield analogous groupings and opposed properties. Let me illustrate this important point.

I said before that a plant or animal species may be viewed as defined not by an essence but by the process which produced it. I characterize the process of *speciation* in more detail in the next chapter where I also discuss in what sense a species may be said to be *an individual*, differing from organisms only in spatio-temporal scale. The individuation of species consists basically of two separate operations: a sorting operation performed by natural selection, and a consolidation operation performed by reproductive isolation, that is, by the closing of the gene pool of a species to external genetic influences. If selection pressures happen to be uniform in space and constant in time, we will tend to find more resemblance among the members of a population than if those selection forces are weak or changing. Similarly, the degree to which a species possesses a clear-cut identity will depend on the degree to which a particular reproductive community is effectively isolated.

Many plant species, for example, retain their capacity to hybridize throughout their lives (they can exchange genetic materials with other plant species) and hence possess a less clear-cut genetic identity than perfectly reproductively isolated animals. In short, the degree of resemblance and identity depends on contingent historical details of the process of individuation, and is therefore not to be taken for granted. For the same reason, resemblance and identity should not be used as fundamental concepts in an ontology, but only as derivative notions.

In addition to showing, case by case, how similarity and identity are contingent on the details of an individuation process, the rejection of static categories and essences must be extended to all *natural kinds*, not just biological ones. We must show, also case by case, how terms which purport to refer to natural categories in fact refer to *historically constituted individuals*. In a way terms like 'human' are the easiest to de-essentialize given that Darwin long ago gave us the means to think about species as historical entities. But what of terms like 'gold' where the essentialist account seems more plausible? After all, all samples of gold must have certain atomic properties (such as having a specific atomic number) which, it can be plausibly argued, constitute the essence of gold. Part of the answer is that all atoms, not only gold atoms, need to be individuated in processes occurring within stars (nucleosynthesis), and that we can use these processes to specify what gold is instead of, say, giving its atomic number.[61] But a more compelling reason to reject essentialism here would be to deny that a given sample of gold large enough to be held in one's hand can be considered a mere sum of its atoms, hence reducible to its atomic properties.

In particular, much as between individual cells and the individual organisms which they compose there are several intermediate structures bridging the two scales (tissues, organs, organ systems) so between individual atoms of gold and an individual bulk piece of solid material there are intermediately scaled structures that bridge the micro and macro scales: individual atoms form crystals; individual crystals form small grains; individual small grains form larger grains, and so on. Both crystals and grains of different sizes are individuated following specific causal processes, and the properties of an individual bulk sample emerge from the causal interactions between these intermediate structures. There are some properties of gold, such as having a specific melting point, for example, which by definition do not belong to individual gold atoms since single atoms do not melt. Although individual gold crystals may be said to melt, in reality it takes a population of crystals with a minimum critical size (a so-called 'microcluster') for the melting point of the bulk sample to emerge. Moreover, the properties of a bulk sample do not emerge all at once at a given critical scale but appear one at a time at different scales.[62]

In conclusion, avoiding essentialist and typological thinking in all realms of reality are basic requirements in the construction of a Deleuzian ontology. But besides these *negative constraints* there must be some *positive resources* which we can use in this construction. I will develop these resources in the following chapter from a more detailed analysis of the intensive processes of individuation which actualize virtual multiplicities. The virtual, in a sense, leaves behind traces of itself in the intensive processes it animates, and the philosopher's task may be seen as that of a detective who follows these tracks or connects these clues and in the process, creates a reservoir of conceptual resources to be used in completing the project which this chapter has only started. This project needs to include, besides defining multiplicities as I did above, a description of how a population of multiplicities can form a virtual continuum, that is, it needs to include a theory of *virtual space*. Similarly, if the term 'virtual multiplicity' is not to be just a new label for old timeless essences, this project must include a theory of *virtual time*, and specify the relations which this non-actual temporality has with actual history. Finally, the relationship between virtuality and the *laws of physics* needs to be discussed, ideally in such a way that general laws are replaced by universal multiplicities while preserving the objective content of physical knowledge. Getting rid of laws, as well as of essences and reified categories, can then justify the introduction of the virtual as a novel dimension of reality. In other words, while introducing virtuality may seem like an inflationary ontological move, apparently burdening a realist philosophy with a complete new set of entities, when seen as a replacement for laws and essences it actually becomes deflationary, leading to an ultimately leaner ontology.

## CHAPTER 2

# *The Actualization of the Virtual in Space*

The picture of a relatively undifferentiated and continuous topological space undergoing discontinuous transitions and progressively acquiring detail until it condenses into the measurable and divisible metric space which we inhabit, is a powerful metaphor for the cosmic genesis of spatial structure. I attempted before to remove some of its metaphorical content by comparing the relation between topological and metric spaces to that between *intensive and extensive* properties: the latter are divisible in a simple way, like lengths or volumes are, while the former, exemplified by properties like temperature or pressure, are continuous and relatively indivisible. The cascade of symmetry-breaking events which progressively differentiates a topological space was, in turn, compared to phase transitions occurring at critical values of intensity. I gave an example from contemporary physics where such a scenario is becoming literally true but the fact is that, as a description of the genesis of space, this picture remains just that, a picture.

It is time now to give a less metaphorical account of how the intensive can engender the extensive, or more exactly, how processes of individuation characterized by intensive properties can yield as their final product individuals with specific spatial structures. In the first part of this chapter I will discuss two different aspects of the intensive, each illustrated with a specific individuation process. First I will describe the process which individuates biological species and from this description I will extract two of the main concepts which characterize intensive thinking: *populations and rates of change*. I will also show how these concepts can be used to replace the two main features of essentialist thinking: *fixed types and ideal norms*. Then I will move on to our second task, a discussion of how the extensive or metric features of individuals emerge from processes which are, at least in an approximate sense, nonmetric or topological, using as illustration the process which yields as its final product individual organisms. A more detailed discussion of embryogenesis will involve the first departure from our geometric metaphor given that its products are defined *not only by extensities but also by qualities*. In other words, an organism is defined both by its spatial architecture, as well as by the different materials (bone, muscle) which give that architecture its specific mechanical qualities. The intensive will then be revealed to be behind both the extensive and the qualitative.

Let's begin with the process of individuation of species. First of all, in what sense can we speak of 'individuation' here? For centuries biological species were one of the main examples of a *natural kind*. Whether one thought of natural kinds as defined by a transcendent essence, as Plato did, or by an immanent 'natural state' as did Aristotle, animal and plant species provided the exemplar of what an *abstract general entity* was supposed to be.[1] Charles Darwin, of course, broke with this tradition by showing that species, far from being eternal archetypes, are born at a particular historical time and die through extinction in an equally historical way, but the idea that *species are individuals*, not kinds, has only recently (and still controversially) gained ground. Much of the credit for the new view on species goes to the biologist Michael Ghiselin who has been arguing for decades that a species, formed through the double process of natural selection and reproductive isolation, does not represent a *higher ontological category* than the individual organisms that compose it.[2] Unlike the relation between a natural kind and its members, which is one of exemplification or instantiation, the relation of individual species to individual organisms is one of whole and parts, much as the relation between an organism and the individual cells that compose it. Moreover, unlike the relation between a particular instance and a general type, the relation of parts to whole is *causal*: the whole emerges from the causal interactions between the component parts.[3] A new species, for instance, may be said to be born when a portion of an old species becomes unable to mate with the rest. This reproductive isolation is a causal relation between the members of two sub-populations, and moreover, it is a relation which must be maintained through time. Anything that breaches the genetic, mechanical or geographical barriers maintaining this isolation will compromise the enduring genetic identity of a species.

Clearly, there are many differences between species and organisms, the most obvious ones being differences in scale. Spatially, a species has a much larger extension than an organism since it is typically comprised of several reproductive communities inhabiting geographically separated ecosystems. Temporally, a species also operates at much larger scales, its average life span being much greater than the lifecycles of organisms. But the fact that species are constructed through a historical process suggests that they are, in fact, just another individual entity, one which operates at larger spatio-temporal scales than organisms, but an individual entity nevertheless. One philosophical consequence of this new conception of species must be emphasized: while an ontology based on relations between general types and particular instances is *hierarchical*, each level representing a different ontological category (organism, species, genera), an approach in terms of interacting parts and emergent wholes leads to a *flat ontology*, one made exclusively of unique, singular individuals, differing in spatio-temporal scale but not in ontological status.[4] On the other hand, the new approach demands that we always specify a process through which a whole emerges, a process which in a Deleuzian ontology is characterized as intensive. The process of speciation may be said to be intensive, first of all, because its description involves the basic ideas of *population and heterogeneity*, two fundamental concepts which characterize a mode of biological explanation known as *population thinking*. What makes this form of thinking different from essentialist and typological thought is expressed in a famous quote by one of the creators of the modern synthesis of evolution and genetics, Ernst Mayr:

> [For the typologist there] are a limited number of fixed, unchangeable 'ideas' underlying the observed variability [in nature], with the *eidos* (idea) being the only thing that is fixed and real, while the observed variability has no more reality than the shadows of an object on a cave wall . . . [In contrast], the populationist stresses the uniqueness of everything in the organic world . . . All organisms and organic phenomena are composed of unique features and can be described collectively only in statistical terms. Individuals, or any kind of organic entities, form populations of which we can determine the arithmetic mean and the statistics of variation. Averages

> are merely statistical abstractions, only the individuals of which the populations are composed have reality. The ultimate conclusions of the population thinker and the typologist are precisely the opposite. For the typologist the type (eidos) is real and the variation an illusion, while for the populationist, the type (the average) is an abstraction and only the variation is real. No two ways of looking at nature could be more different.[5]

When one views species as natural kinds whose members share a common set of identical properties, the inevitable variation between the members of a class cannot be but an accident of history. From the point of view of determining the common set of properties which defines a fixed archetype, this variation is indeed quite unimportant. For population thinkers, on the other hand, variation, genetic variation that is, far from being unimportant is the fuel of evolution: without adaptive differences between organisms natural selection would be incapable of yielding any improvements in the population, let alone allow novel forms to emerge. Put differently, for population thinkers *heterogeneity* is the state we should expect to exist spontaneously under most circumstances, while *homogeneity* is a highly unlikely state which may be brought about only under very specific selection pressures, abnormally uniform in space and time.[6] Moreover, while the typologist thinks of the genesis of form in terms of the expression of *single* types, for the populationist the forms of organisms always evolve within *collectivities* (reproductive communities, for example) as selectively advantageous traits with different origins propagate through the population.

Population thinking eliminates one of the two undesirable aspects of essentialism, the existence of pre-existing archetypes defining the identity of species. The other aspect, the role which such archetypes play as *ideal norms* which their instantiations approximate to a more or less perfect degree, is eliminated by another key concept of Darwinism: the *norm of reaction*. To illustrate this concept let's imagine two different reproductive communities belonging to the same species but inhabiting different ecosystems. The norm of reaction refers to the fact that there is enough flexibility in the connection between genes and bodily traits that differences in the environment can yield different characteristics for the two communities, even though they are still the same species. For example, depending on the rate of availability of a particular resource (sunlight, for example, or a particular nutrient) the rates of growth of the organisms in the two communities may be different, with one consisting of smaller organisms than the other. In this case, there would be no point in saying that one community represents the normal, ideal, fixed phenotype, or that it approximates it to a greater degree of perfection. Since the phenotypes are flexible within certain limits, all realizations of the genotype are normal within those limits.[7] The concept of norm of reaction replaces the idea of *degrees of perfection* with that of *relations between rates of change* (in our example, rates of nutrient availability coupled to rates of growth). Deleuze credits Darwinism with this double blow to essentialism, challenging static classifications and the mode of thinking they imply with a dynamic form of thought which is at once *populational and differential*. As he writes:

> First . . . the forms do not preexist the population, they are more like statistical results. The more a population assumes divergent forms, the more its multiplicity divides into multiplicities of a different nature . . . the more efficiently it distributes itself in the milieu, or divides up the milieu . . . Second, simultaneously and under the same conditions . . . degrees are no longer measured in terms of increasing perfection . . . but in terms of differential relations and coefficients such as selection pressure, catalytic action, speed of propagation, rate of growth, evolution, mutation . . . Darwinism's two fundamental contributions move in the direction of a science of multiplicities: *the substitution of populations for types, and the substitution of rates or differential relations for degrees*.[8]

I said before that between organisms and the cells that are their working parts there are intermediately scaled individual structures, such as tissues or organs. Similarly, between these organisms and the species they compose there are halfway individuals called *demes*: concrete reproductive communities inhabiting a given ecosystem.[9] The intensive properties of these demes, such as how densely their component organisms are packed in their habitat, are characterized by rates. A key rate of change in this case will be the rate of growth of the deme, which is to be distinguished from the growth rate of individual organisms I just mentioned. The rate of growth of an individual deme depends on the birth, death and migration rates prevalent in the community, as well as on the rate of availability of resources (sometimes referred to as the carrying capacity of its environment.) A deme so defined is, indeed, a dynamical system, and as such may exhibit endogenously generated stable states (attractors) as well as abrupt transitions between stable states (bifurcations). In simple models, for instance, the system consisting of a deme coupled to its environment exhibits an unstable steady state (one with population at zero numbers, meaning extinction) as well as a stable steady state where population numbers match the carrying capacity.[10] More complex attractors, such as stable cycles, appear the moment we add nonlinearities to the model. This may be done, for example, by making the birth-rate term more realistic to reflect the fact that there are always nonlinear *delays* between the moment of birth and the moment of sexual maturity. When the growth dynamics of a deme are governed by a periodic attractor, the numbers characterizing its population will tend not to a fixed stable value but will oscillate between values.[11]

This simple example is meant only as an illustration of the sense in which a dynamical process occurring in populations and defined by coupled rates of change may be said to be intensive. How is such an intensive process related to the virtual multiplicities I discussed in the previous chapter? As I said, multiplicities consist of a structure defined by differential relations and by the singularities which characterize its unfolding levels. These two elements of the virtual find their counterpart in the intensive. The coupled rates of birth, death, migration and resource availability correspond *without resemblance* to the differential relations that characterize a multiplicity. The collectively stable states available to populations (steady-state or periodic, in my example) correspond, again without any similarity, to a distribution of singularities. This correspondence, in turn, is explained by the fact that a given intensive process of individuation embodies a multiplicity, and the lack of similarity between the virtual and the intensive is explained in terms of the divergent character of this embodiment, that is, by the fact that several different processes may embody the same multiplicity.[12] Finally, much as virtual multiplicities are meant to replace eternal essences, the intensive individuations that embody them, as well as the individuals that are their final product, are meant to replace general classes, a natural replacement given that general classes are often defined in terms of essences.

These would be, in a nutshell, the three ontological dimensions which constitute the Deleuzian world: the virtual, the intensive and the actual. Or to phrase this in terms of the metaphor that opened this chapter (and neglecting for a moment the temporal dimension) the individuals populating the actual world would be like the discontinuous spatial or metric structures which condense out of a nonmetric, virtual continuum. These metric individuals would exist at different spatial scales, since populations at one scale may form larger emergent individuals at another scale, but altogether (from the smaller individual particles to the largest cosmic individuals) they would constitute the familiar, measurable and divisible space of the actual world. At this point, however, we must make our first departure from the geometric metaphor: actual individuals differ from each other not only in their *extensity* (spatial structure and scale) but also in their *qualities*. A species, for example, possesses both an extensive aspect defining its *distribution in space* (its division into several reproductive communities inhabiting distinct ecosystems) as well as a qualitative aspect defined by *population-level qualities*, distinct from those of individual organisms, such as playing a particular role in a food chain or having a particular reproductive strategy.[13] This means that intensive individuation processes must be described in such a way that the origin of both extensities and qualities is accounted for.

To illustrate this important point I would like to move to a different level of scale, down from species to organisms, and discuss two examples of intensive processes in embryogenesis, one related to the production of extensities, the other to the production of qualities. Or more specifically, I would like to discuss two different embryological processes, one behind the *spatial structuration* of organisms through cellular migration, folding and invagination, and the other behind the *qualitative differentiation* of neutral cells into fully specialized muscle, bone, blood, nerve and other cell types.[14] Metaphorically, an egg may be compared to a topological space which undergoes a progressive spatial and qualitative differentiation to become the metric space represented by a fully formed organism. But in what sense can eggs and organisms be said to form spaces? As I said in the previous chapter, the distinction between metric and nonmetric spaces boils down to the way in which neighbourhoods (or the linkages between the points that form a space) are defined, either through exact lengths or through non-exact topological relations of proximity. In this sense, the fertilized egg, defined mostly by chemical gradients and polarities, as well as the early embryo defined by neighbourhoods with fuzzy borders and ill-defined qualities, may indeed be viewed as a topological space which acquires a rigidly metric *anatomical structure* as tissues, organs and organ systems become progressively better defined and relatively fixed in form.

Let's begin with the creation of distinct spatial structures, starting with the aggregation of individual cells into different neighbourhoods or collectives via a variety of *adhesion processes*. These neighbourhoods do not have a well-defined metric structure. Within any one neighbourhood, the *exact* location of a cell is immaterial as long as there are sufficiently many cells with a shared history located nearby. Similarly, the *exact* number of neighbours is not important and, at any rate, it is always subject to statistical fluctuations. What *is* important are the local, adhesive interactions between cells (or between cells and their extra-cellular matrix during migration), interactions which are typically both nonlinear (small changes may lead to large consequences) and statistical.[15] As the biologist Gerald Edelman has shown, these local interactions yield two stable states for collectives: cells may be tightly linked to each other by adhesion molecules into *sheets* (called epithelia) or be loosely associated via minimal interactions into *migratory groups* (referred to as mesenchyme). These two stable states are related to each other by a transformation which closely resembles a *phase transition*, and which leads to two different types of cellular motion: migration and folding.[16]

While cellular migrations move entire collectives into new places, where they can interact with different collectives, cellular folding and invagination create a large variety of three-dimensional structures which constitute the *external and internal spatial boundaries* of an organism. Just where a collective migrates and what extensive structures and borders will be formed is determined, in part, by intensive relations: not only the rates of synthesis and degradation of the different adhesion molecules (affecting the relative numbers of such molecules, which in turn mediate the phase transition between the two stable states), but also the birth and death rates of cells within a collective.[17] There is no detailed genetic control of the exact number of cell divisions, or of the exact number of cell deaths, but rather a nonlinear feedback relation between birth and death rates and the processes of migration and folding: these processes are affected by the rate at which new cells are born and die and, vice versa, the rates are strongly place-dependent and hence affected by migratory and folding motions.[18]

The intensive (populational and differential) aspects of this process may be said to be nonmetric in the following sense. Deleuze often speaks of the *anexact yet rigorous* style of thought which may be necessary whenever we need to think about nonmetric entities.[19] A good example would be the way Edelman approaches his cell collectives, where the exact number of members or their exact position is immaterial. This attitude towards *quantitative exactitude* is not a sign that biologists, unlike physicists, are less careful or disciplined. It indicates, on the contrary, the presence of a more sophisticated topological style of thought. To quote another biologist whose work will be discussed in the following chapter, Arthur Winfree:

> The sciences of life have never been admired for quantitative exactitude . . . But it cannot be said that living things are at heart sloppy, fuzzy, inexact, and unscientific. How does an oceanic salmon find its way home to spawn on the very rivulet it left in Oregon three years earlier? How is a meter-long sequence of billions of nucleotide base-pairs reversibly coiled without entanglement into a nucleus no more than a few thousand base-pairs in diameter? . . . Such miracles bespeak of reproducible precision. But that precision is not the kind we know how to write equations about, not the kind we can measure to eight decimal places. It is a more flexible exactitude which evades quantifying, like the exactitude of a cell's plasma membrane dividing the universe into an inside and an outside with not even a virus-sized hole lost somewhere in all that

> convoluted expanse: topological exactitude, *indifferent to quantitative details* of shape, force, and time.[20]

Thus, there is a well-defined sense in which the spatial relations characterizing an egg or the still developing parts of an embryo are, indeed, anexact yet rigorous. As migration and folding begin to yield finished anatomical structures, however, these nonmetric relations become progressively replaced by a less flexible set of metric ones. The finished product is a spatial structure adapted to specific functions. Like a building or a bridge, for example, an animal must be able to act under gravity as a *load-bearing structure*. On the other hand, the spatial architecture of an organism is not the only factor that determines its capacity to bear loads, the *qualities* of the materials making up that architecture also matter: the qualities of muscle that allow it to bear loads in tension, for instance, or the qualities of bone that allow it to bear them in compression. The intensive processes that create these materials are another example of a process of progressive differentiation, one which starts with a population of relatively undifferentiated cells and yields a structure characterized by qualitatively distinct *cell types*.

When cells begin their embryological development they are *pluripotent*, that is, they are capable of becoming any of the different types of cells which characterize the adult individual. This number varies from two in bacteria, to twenty or thirty for jellyfish, to about 254 for human beings.[21] Contact between different cells (or between different cellular collectives) leads to the important phenomenon of *induction*. This term refers to a complex process in which collectives exchange chemical signals which lead to the enhancement or suppression of cellular differentiation. However, as the biologist Stuart Kauffman has shown, these inductive signals act as *non-specific stimula* (or perturbations) which switch a cell among a variety of internally available stable states. The basic idea in Kauffman's model is that the regulatory genes within a cell form a complex network in which genes, interacting via their products, can turn one another on or off. Kauffman has found that there are certain recurrent patterns of gene activity within these networks, patterns which exhibit the kind of homeostatic stability associated with attractors. This has led him to believe that, in effect, each attractor may be considered to represent a recurrent cell type.[22]

Kauffman's model attempts to predict not only the number of different cell types in a given organism, but more importantly from our point of view, the number of cell types which a particular cell can *directly differentiate into*. Given a cell with a specific history, and a certain inductive signal which can change its fate, the outcome of their interaction will depend on how many other attractors exist *nearby* in the state space of the cell (or more exactly, in the state space of the network of genes within the cell). In other words, far from directly determining the qualities of a differentiated cell, inductive signals act as *triggers* causing cells to switch from one attractor to another nearby one, guiding a process of qualitative differentiation which follows attractors as so many stepping-stones. This property of *stimulus-independence* must be added to the mechanism-independence I discussed before as part of what defines the 'signature' of the virtual, or put differently, as part of what defines the traces which the virtual leaves in the intensive. But relative autonomy from specific stimula can be achieved only if the internal dynamics of a cell (or collectivities of cells) are rich enough in endogenously generated stable states. This condition is by no means guaranteed and depends on certain intensive properties of a network, those defining its *connectivity*: the number of genes directly or indirectly influenced by each single gene or the number of steps needed for the influence of one gene to be propagated to other genes. At critical values of connectivity a phase transition occurs leading to the crystallization of large circuits of genes, each displaying multiple attractors.[23]

Edelman's and Kauffman's models illustrate the sense in which the intensive may be said to be behind the genesis of both the extensive and the qualitative. Yet, neither one is a literal rendering of a simple cascade of broken symmetries. While the cellular neighbourhoods in Edelman's model do illustrate how non-rigidly metric spaces may be transformed into fixed spatial structures, the connection with topology is indirect. This is even more true in Kauffman's model where the connection with nonmetric questions is completely indirect, mediated by the topological invariants (such as connectivity) of abstract spaces of possibilities defining the available qualities.[24] Therefore, both examples should be seen not as directly illustrating but as *replacing* parts of the simple symmetry-breaking cascade. It is through such piece-meal replacements that literal content may be imparted to, and metaphorical content removed from, our guiding image for the actualization of the virtual in space. There is one more aspect of embryogenesis from which we can derive further resources to continue this process of progressive literalization. It involves looking at a developing embryo as a *process of assembly* of organisms, a process which must yield individuals with the *capacity to evolve*. As an illustration of this point I will contrast two different assembly processes, the process behind the creation of industrial products, as it takes place in an assembly-line factory, for example, and the process taking place within and among living cells which results in the assembly of tissues and organs.

The parts of an object put together in an assembly line are typically fully Euclidean, having rigid metric properties such as sizes, shapes and positions, a fact that limits the kind of procedures that may be followed for their assembly. These procedures must include a rigidly channelled transport system (using conveyor belts or pipes to transport raw materials, and wires to transport energy and information) as well as sequences of rigid motions to correctly position the parts relative to one another. By contrast, the component parts used in biological assembly are defined less by rigid metric properties than by their *topological connectivity*: the specific shape of a cell's membrane is less important than its continuity and closure, and the specific length of a muscle less important than its attachment points. This allows component parts to be *not inert but adaptive*, so that muscle lengths can change to fit longer bones, and skin can grow and fold adaptively to cover both. It also permits the transport processes not to be rigidly channelled, using simple diffusion through a fluid medium to bring the different parts together. Components may float around and randomly collide, using a lock-and-key mechanism to find matching patterns without the need for exact positioning.

All of this has consequences for the capacity to evolve through mutation and selection which each of these two assembly processes may have. If putting together organisms followed an assembly-line pattern, random mutations would have to occur simultaneously in matching parts, channels and procedures, in order to yield a viable entity on which natural selection could operate. The occurrence of such a large number of simultaneous mutations is, of course, a highly improbable event. In biological assembly, on the other hand, mutations do not have to be so coordinated and this greatly enhances the possibilities for evolutionary experimentation. As the scientist Eric Drexler writes:

> Because cells and organisms make widespread use of diffusive transport for energy, information and molecular parts, the evolution of new processing entities (enzymes, glands) is facilitated. A genetic change that introduces an enzyme with a new function can have immediate favorable effects because diffusion automatically links the enzyme to all other enzymes, energy sources and signal molecules in the same membrane compartment of the cell (and often beyond). No new channels need to be built . . . [and] no special space need be set aside for the enzyme, because device placement isn't geometric. Changes in the number of parts . . . become easy. There are no strong geometric or transport constraints; this often allows the number of molecular parts in a cell to be a variable, statistical quantity. With many copies of a part, a mutation that changes the instructions for some copies is less likely to be fatal . . . At the level of multicellular organisms, the striking adaptability of tissues and organs ensures that basic requirements for viability, such as continuity of skin and vascularization of tissues, continue to be met despite changes in size and structure. If skin and vascular systems were inert parts, they would require compensating adjustments for such changes.[25]

This example illustrates another indirect way in which the metric may be said to emerge from the nonmetric. Unlike a developing embryo, a finished organism has more specialized tubes and channels and some of its components lose adaptability and rigidify. This 'metrization' is, of course, never complete, even when an organism reaches maturity. But what is very significant is that, at least in the case of multi-cellular animals, if organisms were not individuated in an intensive environment which is not rigidly metric, their capacity to evolve would be greatly diminished. Thanks both to diffusive transport, lock-and-key matching assembly, topological and adaptive parts, on one hand, as well as stimulus-independence, on the other, evolution has an open space in which to carry out its blind search for new forms. Put differently, biological evolution can be divergent and lead to a proliferation of novelties thanks to the fact that the elements it uses to try out new combinations are neither rigidly connected (to specific stimula, to specific channels) nor intolerant to heterogeneity and variation.

Let me summarize what this discussion of embryogenesis has taught us about the actualization of the virtual in space. Intensive processes possess nonmetric properties in subtle and complex ways: sometimes they involve the spatial continuity and indivisibility of properties like temperature, pressure or density; other times the anexact yet rigorous way in which cellular spatial neighbourhoods are defined; sometimes what is involved is nothing specifically spatial, but rather that which remains topologically invariant in a spatial process; and other times specifically spatial capacities are concerned, such as the capability of adaptive components to fold, stretch or bend. Similarly, the final product of an intensive process is not just metric geometrically speaking: extensive properties include some geometric ones (like length or volume) but also several others that have nothing geometric about them, like entropy or amount of energy; then there are properties which are metric, such as channelled transport or rigidity of parts, but which expand the concept from structure to function; lastly, a finished product is characterized by qualities, which also result from intensities but which are metrically indivisible like intensities. Thus, the relation between the metric and the nonmetric in a process of individuation is not as simple and straightforward as the metaphor of a 'topological egg' progressively differentiating into a 'Euclidean organism' would suggest. But what this comparison has lost in simplicity it has, I believe, gained in literal adequacy.

Having clarified the relations between the intensive and the nonmetric, in the next part of this chapter I would like to probe more deeply into the nature of intensities. Although as I said in Chapter 1, the term 'intensive property' belongs to thermodynamics, it may be extended to cover other areas. Indeed, my use of the word 'intensive' in the descriptions of the individuation of species and organisms was already an extended usage. So the first task of this section will be to specify the connection between the standard definition and its several extensions. After this conceptual clarification is completed I will move on to discuss one of Deleuze's most important theses regarding the intensive. The basic idea is that once a process of individuation is completed, the intensive factors which defined this process disappear or become hidden underneath the extensive and qualitative properties of the final product. Or as Deleuze puts it, 'we know intensity only as already developed within extensity, and as covered over by qualities'.[26] This theme of the disguising of process under product is key to Deleuze's philosophy since his philosophical method is, at least in part, designed to overcome the objective illusion fostered by this concealment.

Let's begin this discussion with the textbook definition of the distinction between the intensive and the extensive: 'Thermodynamic properties can be divided into two general classes, namely intensive and extensive properties. If a quantity of matter in a given state is divided into two equal parts, each part will have the same value of intensive properties as the original, and half the value of the extensive properties. Pressure, temperature, and density are examples of intensive properties. Mass and total volume are examples of extensive properties.'[27] Although this definition does point to a basic difference between intensities and extensities, its emphasis on divisibility allows it to equally apply to qualities, such as colour or texture. But as we just saw, a crucial part of Deleuze's argument hinges precisely on the distinction between the intensive, on one hand, and the extensive *and* qualitative, on the other. Colours are, indeed, not divisible in extension: a certain patch of material of a given colour does not yield, when broken into equal halves, two smaller patches with half the value of its colour (half the hue and half the brightness). This lack of divisibility has misled some philosophers into failing to distinguish qualities, or even subjectively experienced intensities, such as pleasure, from objective intensive properties.[28] Thus, we need a characteristic other than indivisibility in extension to distinguish objective intensities from qualities.

There is, indeed, another way in which physicists state the distinction between the intensive and the extensive: while two extensive properties add up in a simple way (two areas add up to a proportionally larger area), intensive properties do not add up but rather *average*. This averaging operation is an objective operation, in the sense that placing into contact two bodies with different temperatures will trigger a spontaneous diffusion process which will equalize the two temperatures at some intermediate value.[29] This capacity to spontaneously reach an average value explains why temperatures or pressures cannot be divided in extension. A particular value of temperature or pressure, being an average, will remain the same when the body possessing these properties is broken into two or more parts. But beyond that, it points to a dynamical aspect of intensive properties not shared by qualities: differences in thermodynamic intensities are capable of driving a process of equilibration in a population of molecules, a process in which these differences will tend to average themselves out. The intensive would then be distinguished from the qualitative by the fact that *differences in intensity*, though not in quality, can *drive fluxes* of matter or energy.

Intensive differences may be sharp or gradual (in which case they are referred to as 'gradients') but in either case they are nothing like the external differences which distinguish one fully formed individual from another. In static typologies one confronts the *diversity* of objects in the world by a careful tabulation of that which stays the same and that which differs among them. The external differences between diverse objects are viewed simply as a lack of similarity so the concept of difference plays a purely *negative* role. Intensive or internal differences, such as a temperature or pressure gradient within one and the same body, are, on the contrary, *positive* or productive, forming the basis of simple processes of individuation. The soap bubbles and salt crystals I mentioned in the last chapter, for instance, are equilibrium structures which emerge from a process driven by intensive gradients, or more exactly, from the spontaneous tendency of the molecular components of bubbles or crystals to minimize a potential (or minimize an intensive difference). Given this morphogenetic role, it is not surprising that Deleuze makes intensive differences a key element in his ontology. As he writes:

> *Difference is not diversity*. Diversity is given, but difference is that by which the given is given . . . Difference is not phenomenon but the nuomenon closest to the phenomenon . . . Everything which happens and everything which appears is correlated with orders of differences: differences of level, temperature, pressure, tension, potential, difference of intensity.[30]

The first modification which must be made to the standard definition of intensive property is, then, that the intensities defining a particular physical system may indeed be 'divided' but the differences that result change the system in kind (from an equilibrium system, where differences are cancelled, to a non-equilibrium one). Moreover if these differences are made intense enough a critical threshold may be reached and the physical system in question will undergo a phase transition, its extensive properties suffering a radical change in nature. Thus, rather than indivisibility, the key concept in the definition of the intensive is *productive difference*, as well as the related concepts of endogenous stable state (such as a thermodynamic equilibrium state) and of critical transitions between states. How does this relate to the two concepts which I said defined the intensive in biology, populations and rates? The answer is relatively straightforward: intensive gradients are measured by rates of change, and the fluxes of matter and energy these differences drive are either the migratory movements of a molecular population, or movements of energy through such a population.[31] In this sense, the thermodynamic definition is directly related to the one I used in biology, but I also made several departures from it.

When I described population thinking in evolutionary biology a key issue was the role of genetic differences. While in essentialist or typological thinking uniformity is the natural state and difference what needs special explanation, for population thinkers it is difference that is unproblematic. This use of the concept of difference already constitutes an extension of the original notion of intensive gradient, but it is nevertheless related: a biological population where genetic differences have been eliminated is as unproductive as a thermodynamic system where differences in temperature or pressure have been cancelled through equilibration.[32] Yet, the biological examples I gave above involve a more radical departure from the original definition of the intensive. In particular, unlike the molecular populations studied in thermodynamics, the members of biological populations have a larger repertoire of ways to interact with each other. Like a thermodynamic system, a biological population may exhibit attractors (and thus be defined in part by the *tendencies* with which these singularities endow it) but in addition its members will typically display complex *capacities* for interaction which have no counterpart in the physics of heat.

An individual may be characterized by a fixed number of definite properties (extensive and qualitative) and yet possess an *indefinite* number of capacities *to affect and be affected* by other individuals. The degree of openness of this set of possible interactions will vary from individual to individual. In the realm of chemistry, for instance, different chemical elements have different capacities to form novel combinations with other elements, the capacities of carbon, for instance, vastly outperforming those of the inert gases. In biology, as we just saw, the flexible capabilities of adaptive parts or the capability to transport and match components without rigid channels or positioning procedures, lead to even more open combinatorial spaces. This openness is also related to the virtual as can be glimpsed from the fact that it demands from us the use of modal terms (such as 'unlimited possibilities'). Deleuze, in fact, always gives a two-fold definition of the virtual (and the intensive), using both singularities (unactualized tendencies) and what he calls *affects* (unactualized capacities to affect and be affected).[33]

Unlike singularities, which are relatively well studied thanks to the development of the topological approach to state space, the formal study of affects is relatively underdeveloped. Several scientists who had previously focused on the study of singularities, however, have recently switched to the study of a different type of formal system which allows the exploration of constructive capacities. Stuart Kauffman and Walter Fontana, among others, view the capacity to form novel assemblages when objects are put into functional relations with one another as a problem which is *complementary* to that of state space, a problem which may also lead to the discovery of universal features analogous to those revealed by classifications of attractors. Although the formal systems they have designed to study affects (Kauffman's random grammars, Fontana's algorithmic chemistry) are less well understood than the tools used to study singularities, they have already yielded valuable insights into questions of functional integration, including the discovery of a few *recurrent assembly patterns* (such as autocatalytic loops) which may turn out to be universal.[34]

While the relation between intensities and singularities does not involve any departure from the thermodynamic definition of 'intensive', adding capacities implies extending that definition. Let me first give a more detailed characterization of capacities and then show how the original definition may be naturally extended to include them. An individual organism will typically exhibit a variety of capabilities to form *assemblages* with other individuals, organic or inorganic. A good example is the assemblage which a walking animal forms with a piece of solid ground (which supplies it with a surface to walk) and with a gravitational field (which endows it with a given weight). Although the capacity to form an assemblage depends in part on the emergent properties of the interacting individuals (animal, ground, field) it is nevertheless not reducible to them. We may have exhaustive knowledge about an individual's properties and yet, not having observed it in interaction with other individuals, know nothing about its capacities.[35]

The term 'capacity' is closely related to the term 'affordance' introduced by James Gibson within the context of a theory of ecological interactions.[36] Gibson distinguishes between the intrinsic properties of things and their affordances. A piece of ground does have its own intrinsic properties determining, for example, how horizontal or slanted, how flat, concave or convex, and how rigid it is. But to be capable of affording support to a walking animal is not just another intrinsic property, it is a capacity which may not be exercised if there are no animals around. Given that capacities are *relational* in this sense, what an individual affords another may depend on factors like their relative *spatial scales*: the surface of a pond or lake may not afford a large animal a walking medium, but it does to a small insect which can walk on it because it is not heavy enough to break through the surface tension of the water. Affordances are also symmetric, that is, they involve both capacities to affect and be affected. For example, a hole in the ground affords a fleeing animal a place to hide, but such animal could also dig its own hole, thus affecting or changing the ground itself. Similarly, an animal may flee because a predator affords it danger but it itself affords nutrition to the predator.[37]

We may expand the meaning of the term 'intensive' to include the properties of assemblages, or more exactly, of the processes which give rise to them. An assembly process may be said to be characterized by intensive properties when it articulates *heterogeneous* elements as such.[38] In the assemblage formed by a walking animal, a piece of ground and a gravitational field, three heterogeneous individuals are joined together as such without the need for any homogenization. More generally, the interactions which organisms have with the organic and inorganic components of an ecosystem are typically of the intensive kind (in the enlarged sense), an ecosystem itself being a complex assemblage of a large number of heterogeneous components: diverse reproductive communities of animals, plants and micro-organisms, a geographical site characterized by diverse topographical and geological features, and the ever diverse and changing weather patterns. Similarly, the meaning of 'extensive' may be enlarged to refer to the properties of processes, such as the assembly-line process I mentioned before, where *homogeneous* components are linked together. The enlarged meaning of 'intensive' is related to the standard definition in the crucial role played by *difference*. Much as a thermodynamic intensive process is characterized by the productive role which differences play in the driving of fluxes, so in the enlarged sense a process is intensive if it relates *difference to difference*.[39] Moreover, as the example of assembly processes based on adaptive components showed, the flexible links which these components afford one another allow not only the meshing of differences, but also endow the process with the capacity of divergent evolution, that is, the capacity to further *differentiate differences*.

Armed with this more adequate definition of intensive process we can move on to the second set of issues I said needed to be discussed: the concealment of the intensive under the extensive, as well as the concealment of the concrete universals (singularities and affects) which animate intensive processes. To anticipate the conclusion I will reach in a moment, in the case of singularities the existence of the virtual is manifested in those situations where intensive differences *are not* *cancelled*. Similarly, in the case of affects it is the cases where an assemblage meshes differences as such, without cancelling them through homogenization, that exhibit the open set of possibilities calling for an explanation in terms of virtuality. Conversely, allowing differences in intensity to be cancelled or eliminating differences through uniformization, effectively hides the virtual and makes the disappearance of process under product seem less problematic. Although this concealment is partly the result of human intervention, of laboratory practices which focus on the final equilibrium state or which systematically homogenize materials, for example, it is also an objective phenomenon. Any area of the world which is in thermodynamic equilibrium, for instance, is an area where intensive differences have cancelled themselves out, and hence an area which conceals the virtual without the need for human intervention. These areas of the world, in short, would constitute an *objective illusion*.

Deleuze argues, for example, that despite the fact that classical thermodynamics yielded valuable insights into the importance of the intensive, this branch of physics did not provide the foundation needed for a theory of individuation given its exclusive focus on the final equilibrium state of a system. The problem with concentrating on the final state is that only during the difference-driven process can the equilibrium state be seen as a virtual attractor, a state which is not actualized yet but which is nevertheless real since it is actively attracting the successive states of the system towards itself. But while it is true that classical thermodynamics tends in this sense to underestimate the virtual and the intensive, 'this tendency would lead nowhere if intensity, for its own part, did not present a corresponding tendency within the extensity in which it develops and under the quality which covers it. Intensity is difference, but this difference tends to deny or to cancel itself out in extensity and underneath quality'.[40] In other words, while certain scientific practices tend to systematically down-grade the intensive and conceal the virtual, these practices only amplify an illusion which is objective and which is, therefore, much harder to overcome.

One way of allowing the virtual to manifest itself is to design experiments or to study phenomena in circumstances where intensive differences are not allowed to cancel themselves. This is what is done in the latest version of the science of heat, the field of *far-from-equilibrium* thermodynamics, where an intense flow of matter and energy continuously traverses the system under study acting as a constraint maintaining intensive differences alive.[41] I said in the previous chapter that the variety of attractors which a system may have depends on whether its dynamics are linear or nonlinear. While linear systems possess the simplest distribution of singularities, a single global optimum structuring the whole of state space, nonlinear ones typically have multiple attractors (or put more technically, nonlinear equations allow for multiple solutions). To the mathematical distinction between the linear and the nonlinear, therefore, we must now add a thermodynamic one, that between systems *near and far* from equilibrium. As Prigogine and Nicolis put it 'without the maintenance of an appropriate distance from equilibrium, nonlinearity cannot by itself give rise to multiple solutions. At equilibrium detailed balance introduces a further condition that restricts and even uniquely fixes' the solution.[42] In other words, to exhibit their full complexity nonlinear systems need to be driven away from equilibrium, or what amounts to the same thing, appropriately large differences in intensity need to be maintained by external constraints and not allowed to get cancelled or be made too small. In this sense, as these authors say, 'nonequilibrium reveals the potentialities hidden in the nonlinearities, potentialities that remain dormant at or near equilibrium'.[43]

This is important in the present context because it explains the physical source of the objective illusion Deleuze talks about. Take for example a linear system with a single attractor. As I just said, while the system is on its way to this attractor the unactualized end state is indeed there already, actively attracting the process towards itself. At this point its virtuality is relatively easy to grasp. But once the process is over it becomes easy to overlook the virtual nature of the end state, even though a system will never actually reach the attractor, only fluctuate in its vicinity. A nonlinear system with multiple attractors, on the other hand, continues to display its virtuality even once the system has settled into one of its alternative stable states, because the other alternatives are there all the time, *coexisting* with the one that happens to be actualized. All one has to do to reveal their virtual presence is to give a large enough shock to the system to push it out of one basin of attraction and into another. (Here we could, of course, refer to the alternative stable states as *possibilities*, not virtualities, but I have already argued for the need to replace the possible with a more adequate form of physical modality.)

A system with multiple attractors, in short, has a greater capacity to express or reveal the virtual. But this expressive capacity will depend, in turn, on the thermodynamic 'zone of intensity' in which the system operates: at low intensities (near equilibrium) a nonlinear system will in effect be *linearized*, that is, its potential complex behaviour will not be revealed. This procedure has, in fact, become routine in physics whenever troublesome nonlinear effects need to be eliminated: one simply studies the system in question at very low intensity values for the trouble-making variable.[44] However, by following procedures like this and systematically neglecting the high intensity values at which nonlinear effects are fully expressed, physicists promote an illusion which is originally objective but which now becomes subjectively amplified. On the other hand, studying systems which are both nonlinear and nonequilibrium, systems where the objective illusion is at it weakest, opens up windows into the virtual.

One of the tasks of a philosopher attempting to create a theory of virtuality is to locate those areas of the world where the virtual is still expressed, and use the unactualized tendencies and capacities one discovers there as sources of insight into the nature of virtual multiplicities. More exactly, Deleuze recommends following a very specific philosophical method in which, as he says, it is

> necessary to *return to the interior of scientific states of affairs or bodies in the process of being constituted*, in order to penetrate into consistency, that is to say, into the sphere of the virtual, a sphere that is only actualized in them. *It would be necessary to go back up the path that science descends*, and at the very end of which logic sets its camp.[45]

In other words, unlike the linear and equilibrium approach to science which concentrates on the final product, or at best on the process of actualization but always in the direction of the final product, philosophy should move in the opposite direction: from qualities and extensities, to the intensive processes which produce them, and from there to the virtual.

Let me give a concrete example of what it would mean to return to the interior of a body in the process of being constituted. Biological categories, particularly those above species, tend to be created by observing similarities (or technically, homologies) among the anatomical parts of fully formed organisms. To the extent that the process which generates these organisms is ignored these static classifications conceal the virtual. But the development of a nonlinear, nonequilibrium approach to embryology has revealed a different, more dynamic way of creating classifications. A good example is provided by a new approach to the study of the *tetrapod limb*, a structure which can take many divergent forms, ranging from the bird wing, to the single digit limb in the horse, to the human hand and its opposed thumb. It is very hard to define this structure in terms of the common properties of all the adult forms, that is, by concentrating on homologies at the level of the final product. But focusing instead on the embryological processes that produce this structure allows the creation of a more satisfactory classification. As one author puts it, this new classificatory approach 'sees limb homology as emerging from a common *process* (asymmetric branching and segmenting), rather than as a precisely repeated archetypal pattern'.[46]

Returning to the interior of the tetrapod limb as it is being constituted would mean to reveal how one and the same 'virtual limb' is unfolded through different intensive sequences, some blocking the occurrence of particular bifurcations (those leading to the branching out of digits, for example), some enabling a full series to occur, resulting in very different final products. This step in the method, however, can only constitute a beginning. The reason is that it still relies on the notion of similarity or homology, even if this now characterizes processes as opposed to products. A second step needs to be added to explain the source of these process homologies. Or to put this differently, once we have revealed the intensive process behind a product we still need to continue our ascent towards the virtual structures that can only be glimpsed in that process but which explain its regularities. Before engaging in a technical discussion of this second step I would like to sketch it in outline by returning to the metaphor which opened this chapter: a topological space which differentiates and divides its continuity as it becomes progressively more rigidly metric following a cascade of symmetry-breaking events.

Extensive structures would constitute the counterpart of the bottom level, while intensive processes would be the counterpart of the intermediate levels, each one representing a geometry which is not fully metric but which can, in fact, be metricized.[47] The top level, an ideally continuous and relatively undifferentiated space, would be the counterpart of the virtual. I use terms like 'top' and 'bottom' here informally, with no suggestion that these spaces actually form a hierarchical structure. A better image here would be a nested set of spaces, with the cascade acting to unfold spaces which are embedded into one another. Another important qualification is that each one of the spaces that comprises this nested set is classified not by its extensities or its qualities, but by its affects, that is, by its invariants under a transformation (or group of transformations). In other words, what matters about each space is its way of being affected (or not affected) by specific operations, themselves characterized by their capacity to affect (to translate, rotate, project, bend, fold, stretch). Without this caveat, we could run the danger of circularity, since the extensive properties of the bottom level would be used to define the other levels as well.

This metaphor supplies us with a target for a theory of the virtual: we need to conceive a *continuum* which yields, through progressive differentiation, all the discontinuous individuals that populate the actual world. Unlike the metaphor, however, this virtual continuum cannot be conceived as a single, homogeneous topological space, but rather as a heterogeneous space made out of a population of *multiplicities*, each of which is a topological space on its own. The virtual continuum would be, as it were, *a space of spaces*, with each of its component spaces having the capacity of progressive differentiation. Beside this multiplication of spaces, we need a way of meshing these together into a heterogeneous whole. Deleuze, in fact, refers to the virtual continuum as a *plane of consistency*, using the term 'consistency' in a unique sense, and in particular, in a sense having nothing to do with logical consistency, that is, with the absence of contradiction. Rather, consistency is defined as *the synthesis of heterogeneities as such*.[48]

There are two sets of issues that must be discussed before we can move beyond this metaphor. Both are issues relating to the *entities* that populate the virtual. First of all, Chapter 1's description of multiplicities left unresolved the question of their nature as concrete universal entities. In other words, I used certain features of mathematical models (the vector fields of state spaces) as a source for the notions that define a multiplicity but I did not discuss how the properties of an actual entity, a mathematical model, can be made into the properties of a virtual one. This is a task which will involve a specific *philosophical transformation* of the mathematical concepts involved, a means of detaching these concepts from their mathematical actualization, so to speak. In addition to this, the first part of this discussion needs to add to the last chapter's characterization a description of what makes multiplicities capable of being meshed together. I will argue that by extending each singularity into an *infinite series*, and defining these series without the use of metric or quantitative concepts, multiplicities can become capable of forming a heterogeneous continuum.

The second set of issues involves going beyond singularities and into a discussion of affects. I said before that there are two special cases of intensive processes that cry out for explanation in terms of virtuality (or at any rate, in terms of some kind of physical modality.) The first case was exemplified by physical systems with multiple attractors, systems which force on us the problem of accounting for the mode of existence of the available yet unactualized tendencies. The second case was flexible assembly processes which lead to an open set of potential combinations. When a process leads to a closed set of assemblages, this set may be given by exhaustive enumeration (that is, it may be defined extensionally) eliminating the need to bring in a modal explanation. But if the set is divergent (as in the case of biological evolution) then no exhaustive enumeration will do since there will always be novel assemblages not included in the list. The question now is, if multiplicities and their singularities correspond to multiple stable states, what corresponds to these unactualized capacities in the virtual continuum? Is there another virtual entity embodying the capacity to affect, and is the exercise of this capacity necessary for the assemblage of a heterogeneous continuum?

The first task is, then, to take the concepts which Deleuze adopts from mathematics (differential relation, singularity) and get rid of any trace of actuality that these concepts may still bear despite their already highly abstract nature. In particular, none of these concepts can *presuppose individuation*. They need to be transformed to become fully *pre-individual notions* so that they can form the logical and physical basis for the genesis of individuals. When physicists or mathematicians speak of 'differential relations', for example, they have in mind a particular mathematical object which embodies those relations: *a function*. Such an object may be viewed as a device which maps one domain of numbers (or other entities) into another, or to use a more technological metaphor, as a device which receives some inputs and maps them into an output.[49] As such, functions define *mathematical individuation processes*. For example, when a function is used to model a physical system, its inputs (or independent variables) become the dimensions of state space, while its output (dependent variable) *individuates* a particular state in that space. (A series of such states forms a trajectory.)

Although Deleuze does define virtual entities via differential relations (that is, as relations between changes or differences) it is clear that he cannot conceive of these relations as possessing the form of a function, since this would presuppose individuality. In other words, the differential relations defining multiplicities cannot involve the asymmetry between dependent and independent variables (or input and output). If anything, these relations must be like 'formless functions', where inputs and outputs are not yet distinguished, where the relation is not a rate of change of one quantity relative to another, but the rate at which two quantities change relative to each other. As Deleuze puts it, virtual relations must involve a purely *reciprocal determination* between their elements, a reciprocal synthesis between pure changes or differences which should not presuppose any prior individuation.[50] A philosophical transformation is also needed to lift the virtual content from the mathematical concept of singularity. Much as virtual differential relations must be distinguished from *individuating functions*, virtual singularities should be distinguished from *individuated states*.

Attractors, for example, may be defined as special subsets of state space, that is, as *limit states* (or limit sets of states). But viewing them as states would imply that they already possess a definite individuality. Hence Deleuze's idea that the pre-individual aspect of singularities can only be grasped before they acquire a well-defined identity in a state space full of trajectories, that is, when they are only vaguely defined by their *existence and distribution* in a vector field. Unlike trajectories, a vector field is not composed of individuated states, but of instantaneous values for rates of change. Individually, these *instantaneous rates* (or infinitesimals) have, in fact, no reality, but collectively they do exhibit topological invariants (singularities), and it is these invariants that should be given ontological significance. Ontologically, however, an invariant of a vector field is just a *topological accident*, a point in the field which *happens to be stationary* (more technically, a point at which the zero vector is attached). Deleuze proposes that these topological accidents should be given the ontological status of *an event*, but given their universality or recurrent nature, these events should be seen as ideal, not actual. A similar point applies to the bifurcations which unfold the embedded levels of a multiplicity: each one of these symmetry-breaking transitions should be seen as an ideal event, and not, of course, as an actual phase transition. As Deleuze writes:

> What is an ideal event? It is a singularity – or rather a set of singularities or of singular points characterizing a mathematical curve, a physical state of affairs, a psychological and moral person. Singularities are turning points and points of inflection; bottlenecks, knots, foyers, and centers; points of fusion, condensation and boiling; points of tears and joy, sickness and health, hope and anxiety, 'sensitive points' . . . [Yet, a singularity] is essentially pre-individual, non-personal, and a-conceptual. It is quite indifferent to the individual and the collective, the personal and the impersonal, the particular and the general – and to their oppositions. Singularity is *neutral*.[51]

To complete the characterization of multiplicities as entities we now need to discuss the capacities for interaction which these complex events may be expected to exhibit. Each of the singularities defining a multiplicity must be thought as possessing the *capacity to be extended or* *prolonged as an infinite series of ideal events*. Deleuze refers to this virtual process as a 'condensation of singularities'.[52] Let me first give a metaphorical description of this process and then give its technical definition. The metaphor is the occurrence of a phase transition in an actual material such as water. When steam is cooled down to a critical point (about 100°C at sea level) it will spontaneously change nature and *condense* into a liquid, but as we continue to decrease the temperature, the singular event which occurred at the critical point will be followed by a *series of ordinary events* (each additional lowering of temperature will have only a linear cooling effect on the liquid water), a series which extends up to the neighbourhood of another singularity (0°C, where the next critical event, freezing, occurs). A similar idea would apply to the virtual: the singularities defining a multiplicity would become the origin of series of ordinary ideal events extending up to the vicinity of other singularities belonging to other multiplicities. Unlike the metaphor, however, these series of ideal events would not form a sequence in time but rather a series of coexisting elements. (I will expand on this in the next chapter when I discuss the form of temporality of the virtual.[53])

To get rid of the metaphorical content and to show in what sense the series extending from singularities are nonmetric (thus capable of forming a virtual continuum) I will need to introduce one more technical term, that of an *infinite ordinal series*. Unlike an infinite series of cardinal numbers (one, two, three . . .) an ordinal series (first, second, third . . .) does not presuppose the existence of fully individuated numerical quantities. To be defined an ordinal series demands only certain asymmetrical relations between abstract elements, relations like that of *being in between* two other elements. In other words, it is only *the order* in a sequence that matters, and not the nature (numerical or otherwise) of the elements so ordered. Bertrand Russell, whose thought in these matters has influenced Deleuze, argues that much as nonmetric geometries eventually provided the foundation for the older metric ones, so ordinal series became the foundation for our very notion of numerical quantity.[54] There is, in fact, a direct relationship between metric spaces and cardinal numbers, on the one hand, and nonmetric spaces and ordinal numbers, on the other. Two metric entities, two lengths, for example, can be divided in a simple way into basic numerical units. This allows them to be exactly compared since we can establish unambiguously the *numerical identity* of the two lengths. Ordinal series, on the other hand, behave more like topological spaces, where we can rigorously establish that a point is nearby another, but not by exactly how much (given that their separation may be stretched or compressed).

Russell introduced the term *distance* (or intensity) to define relations of proximity between the elements of an ordinal series.[55] As a relation, an ordinal distance cannot be divided, and its lack of divisibility into identical units implies that two ordinal distances can never be *exactly compared* although we can rigorously establish that one is greater or less than another. The *difference* between two distances, in other words, *cannot be cancelled* through numerical identity, so the results of these comparisons are always anexact yet rigorous. In short, ordinal distances are a nonmetric or non-quantitative concept. Deleuze adopts these ideas from Russell but breaks with him at a crucial point: he does not conceive of the priority which the ordinal has over the cardinal as being purely logical or conceptual, but as being ontological. In other words, Deleuze establishes a *genetic relationship* between serial order and its defining nonmetric distances, on one hand, and numerical quantities, on the other. An ordinal series which is dense (that is, where between any two elements there is always another one) would form a *one-dimensional continuum* out of which cardinal numbers would emerge through a symmetry-breaking discontinuity.[56]

Let's return to the problem of assembling virtual multiplicities into a plane of consistency. As I said, each one of the singular ideal events defining a multiplicity needs to be imagined as being extended into a series of *ordinary events* which are still virtual or ideal but that, unlike singularities, already possess a minimal actualization.[57] Each of the series which emanates from a singularity should be imagined as being dense and defined exclusively by ordinal distances, thus constituting a one-dimensional continuum. A heterogeneous continuum could then be woven from the many serial continua springing from each member of the population of multiplicities. To ensure that multiplicities are meshed together by their differences, Deleuze argues that the relations among these series must be both *convergent and divergent*. In other words, the series must be made to come together and *communicate* but also to *ramify and proliferate*.[58] He shows how these relations of convergence and divergence do not presuppose any of the categories he wishes to avoid (identity, similarity, analogy and contradiction) and may be used to generate, as secondary consequences, the modal categories (possibility) he wishes to replace.[59]

At this point an important qualification should be made. Multiplicities should *not* be conceived as possessing the capacity to actively interact with one another through these series. Deleuze thinks about them as endowed with only a mere capacity to be affected, since they are, in his words, 'impassive entities – impassive results.'[60] The neutrality or sterility of multiplicities may be explained in the following way. Although their divergent universality makes them independent of any particular mechanism (the same multiplicity may be actualized by several causal mechanisms) *they do depend on the empirical fact that some causal mechanism or another actually exists*.[61] This is merely to say that they are not transcendent but immanent entities. But beyond this, unlike eternal and fixed archetypes which have no historical origin, Deleuze views multiplicities as *incorporeal effects of corporeal causes*, that is, as historical results of actual causes possessing no causal powers of their own. On the other hand, as he writes, 'to the extent that they differ in nature from these causes, they enter, with one another, into relations of *quasi-causality*. Together they enter into a relation with *a quasi-cause* which is itself incorporeal and assures them a very special independence . . .'[62]

I said before that the construction of a virtual continuum involves considering not only the role of singularities but also of affects. Unlike actual capacities, which are always capacities to affect and be affected, virtual affects are sharply divided into a pure capacity to be affected (displayed by impassible multiplicities) and a *pure capacity to affect*. This capacity, as I hinted above, is exhibited by another incorporeal entity which Deleuze refers to as a 'quasi-cause'. At this point, introducing more entities may strike us as artificial, or at least as inflationary, encumbering an already unfamiliar ontology with further unfamiliar features. But this introduction is far from being artificial. A key concept in the definition of a multiplicity is that of 'invariant', but invariances are always relative to some transformation (or group of transformations). In other words, whenever we speak of the invariant properties of an entity we also need to describe an operator, or group of operators, capable of performing rotations, translations, projections, foldings and a variety of other transformations on that entity. So the ontological content of the virtual must also be enriched with at least one operator. The quasi-cause is, indeed, this operator and it is defined not by its giving rise to multiplicities but by its capacity to affect them. 'The quasi-cause does not create, it operates', as Deleuze says.[63]

This new entity must be as carefully constructed as multiplicities were: every step in the construction must meet the constraint of avoiding essentialist and typological categories, and all the concepts involved in its definition must be shown to be pre-individual. Roughly, the task which the quasi-causal operator must accomplish is to create among the infinite series springing from each singularity 'resonances or echoes', that is, the most ethereal or least corporeal of relations.[64] The technical aspects of this task may be specified using concepts from abstract communication theory. In communication theory, the actual occurrence of an event is said to provide information in proportion to the *probabilities of the event's occurrence*: a rare event is said to provide more information on being actualized than a common one.[65] These events, each with its own probability of occurrence, may be arranged in a series. When two separate series of events are placed in communication, in such a way that a change in probabilities in one series affects the probability distribution of the other, we have an *information channel*. A telegraph, with its coupled series of events (electrical events defining letters in Morse code at both sending and receiving ends of the transmission line), is an example of an information channel. But in the abstract version of communication theory nothing whatsoever is said about the physical realization of a channel, such as the length of the transmission line, or the type of code used. Similarly, no mention is made of information flowing through a channel: an emission of a 'quantum' of information is associated with any change in probabilities in one series relative to the other series. (Technically, the two series are 'connected' only through a conditional probability matrix.)[66]

This definition of an information channel appeals to Deleuze precisely because of its highly abstract nature, presupposing nothing about details of physical implementation.[67] But mathematical models using differential relations are equally abstract and yet we saw they nevertheless imply some notions which are not pre-individual. So the concept of an abstract information channel creating communications among series of ideal events must be further transformed to become truly pre-individual. I will mention here only the most important requirement, although Deleuze discusses several more: the ideal events forming a virtual series must not be conceived as having *numerical* probabilities of occurrence associated with them; they must be arranged in series using only ordinal distances, and be distinguished from one another exclusively by the difference between the singular and the ordinary, the rare and the common, without further specification. In other words, the coupled changes in distributions which constitute an information transfer should not be conceived as changes in conditional probabilities, but simply *changes in the distribution of the singular and the ordinary* within a series.[68]

I will return in the next chapter to a more complete characterization of the relations between these three elements of the virtual (multiplicities, quasi-causal operator, plane of consistency). But to conclude the present chapter I would like to address a possible objection to this scheme. What motivates the postulation of a quasi-causal operator? After all, we feel confident postulating the existence of multiplicities to the extent that we can study in the laboratory certain phenomena (such as the series of flow patterns conduction–convection–turbulence) which embody such a progressively determinable entity. Moreover, we can also check empirically that a portion of the same symmetry-breaking cascade is exhibited by other processes (embryological processes, for example) which depend on such different causal mechanisms that they almost demand we postulate a mechanism-independent entity as part of their explanation. But what evidence do we have that there are intensive processes which can *spontaneously perform information transmission operations*? I will argue in a moment that the answer to this question is that there are in fact such processes, and that they provide the justification for thinking that such operations may indeed be performed virtually. But before doing that let me add that this reliance on 'evidence' from intensive processes (more exactly, a reliance on traces left by the virtual in the intensive) would constitute one of the main characteristics differentiating a theory of the virtual from a theory of eternal and immutable essences. Unlike the *a priori grasp* of essences in human thought postulated by those who believe in such entities, there would be *an empiricism of the virtual*. The concepts of virtual multiplicity, quasi-causal operator and plane of consistency would be, in this sense, *concrete empirico-ideal notions*, not abstract categories.[69]

Is there any evidence motivating the postulation of a quasi-causal operator? There is, in fact, a relatively new field of nonlinear science dedicated to the study of 'emergent computation', that is, to the study of physical processes in which the interactions among components can exhibit the capacity for non-trivial information processing.[70] The meaning of the term 'computation' in the context of natural phenomena is relatively easy to grasp if we think about DNA and the cellular machinery for its translation, since this involves the relatively unproblematic idea that biological mechanisms have been evolved for the purpose of storing, transferring and processing information. But I want to focus my discussion on a more general set of physical phenomena that do not involve any specialized hardware and yet can be said to transmit information. We need to keep in mind that information transfer need not involve any computer-like mechanism, but only the establishment (by whatever means) of a correlation between the probabilities of occurrence of two series of events. As the philosopher Kenneth Sayre puts it, we can conceive 'as an instance of information transmission any process in which the probability of one or more members of an ensemble of events or states is changed as the result of a change in probability of an event or state outside the ensemble. Thus conceived, information transmission occurs with every physical process.'[71]

The simplest non-biological instance of spontaneous correlation between the probabilities of events is the behaviour of materials *near phase transitions*. In this case the two series of events forming the information channel are, in a way, collapsed into one, since the correlations are established between the probabilities of occurrence of spatially separated events in one and the same system.[72] More exactly, material systems can be characterized thermodynamically by certain variables whose values are not fixed (even at equilibrium) but rather fluctuate (with definite probabilities) around a given state. It is these *fluctuations* that constitute the events among which correlations may be established. At equilibrium, the fluctuations are basically equiprobable, or put differently, they are mostly uncorrelated. No information transmission occurs. But as a system approaches a phase transition, these fluctuations begin to display correlations, the correlation length (the distance across which events influence each other's probabilities) increasing the closer the system gets to the critical point. In the *vicinity of the bifurcation the capacity to transmit information is maximized*. This phenomenon does not depend on the physical mechanisms underlying the phase transition: the same idea applies to a metallic material switching from the magnetized to the unmagnetized state, or to a material switching from the gas to the liquid state. In other words, the phenomenon of strong correlations between fluctuation events in the neighbourhood of a phase transition displays divergent universality.[73]

To scientists working in the field of emergent computation this universality is highly significant. Some even think that this universal capacity for information transmission is accompanied by complementary capacities to store and process information associated with other characteristics of phenomena near phase transitions.[74] This has led to the hypothesis that the specialized hardware which living organisms use to process information may have required that evolutionary forces kept early organisms *poised at the edge* of a phase transition, or what amounts to the same thing, away from any stable attractor. Christopher Langton, a pioneer in this field of research, puts it this way:

> Living systems are perhaps best characterized as systems that dynamically avoid attractors . . . Once such systems emerged near a critical transition, evolution seems to have discovered the natural-information processing capacity inherent in these near-critical dynamics, and to have taken advantage of it to further the ability of such systems to maintain themselves on essentially open-ended transients . . . There is ample evidence in living cells to support an intimate connection between phase transitions and life. Many of the processes and structures found in living cells are being maintained at or near phase transitions. Examples include the lipid membrane, which is kept in the vicinity of a sol-gel transition; the cytoskeleton,

in which the ends of microtubules are held at the point between growth and dissolution; and the naturation and de-naturation (zipping and unzipping) of the complementary strands of DNA.[75]

Kauffman's networks of regulatory genes which, as I discussed above, may form the basis of processes of differentiation in populations of cells, are also *poised systems* of this type. That is, in this case, too, the maximum information transferring capacity is achieved when the network is poised at the brink of a threshold, a threshold beyond which this capacity melts away. It is much too early in the development of this research programme to assess the full significance of these claims. Some of the early formal results (using cellular automata) have, in fact, been challenged.[76] But the basic claim that the vicinity of phase transitions is a special place when it comes to the emergence of spontaneous information *transmission* (as opposed to processing or storage) is still valid. And it is the existence of this emergent capacity in *systems which come very close to but do not actualize the phase transition*, which justifies us in postulating such an entity as a quasi-causal operator.

In conclusion I would like to add that, as unfamiliar and apparently complicated as Deleuze's scheme for the production of a virtual continuum may seem, he must at least be given credit for working out in detail (however speculatively) the requirements for the elimination of an immutable world of transcendent archetypes. Given that essences are typically postulated to explain the existence of individuals or of natural kinds, eliminating them involves giving an *alternative explanation*, not just reducing these individuals and kinds to social conventions. First, we must give a *detailed description of the intensive processes of individuation which generate actual forms*. Second, we must show in detail in what sense the resources involved in individuation processes are immanent to the world of matter and energy, that is, we must not simply deny transcendentality in general but *describe concrete mechanisms of immanence to explain how the virtual is produced* out of the actual. The two halves of this chapter are merely a sketch of how these two tasks are to be performed. The third and final requirement will involve discussing the temporal dimension of Deleuze's ontology. This will complete the elimination of essences we have begun here, ensuring that multiplicities possess their own historicity and preventing them from being confused with eternal archetypes. This is a complementary task to those performed in this chapter: developing a theory of time with actual and virtual parts, the two dissimilar halves linked through a properly intensive form of temporality. It is to this other task that I now turn.

CHAPTER 3

# *The Actualization of the Virtual in Time*

There is a conflict at the heart of physics, a conflict between two forms of scientific temporality. On one hand, there is the conception of time that developed in the most prestigious branches of physics, classical mechanics and later the special and general theories of relativity. On the other, the concept of time born in humble areas of applied physics, such as engineering and physical chemistry, a concept which eventually became the time of classical thermodynamics. The main difference between these two forms of time, beside their different degrees of intellectual prestige, is that while in classical and relativistic physics there is no arrow of time, the time of the science of heat contains a fundamental *asymmetry* between past and future. This asymmetry is exemplified by the fact that thermodynamic systems have a preferential direction always tending to approach thermal equilibrium as their final state. As long as these two conceptions of time simply coexisted side by side, as they did for most of the nineteenth century, their contradictory relations did not cause any major foundational conflicts in the scientific community. But when the physicist Ludwig Boltzmann attempted to unite classical physics and thermodynamics into one unified theory (statistical mechanics), the contradiction between *reversibility at the microscopic level*, at the level of the interactions between the molecules that make up a gas, for example, and *irreversibility at the macroscopic level*, at the level of collective quantities like temperature or entropy, could no longer be avoided.[1]

The term 'reversibility of time' has nothing to do with the idea of time flowing backwards, that is, with a flow of time going from the future towards the past. Rather it refers to the fact that if we took a certain process, seen as a series of events, and reversed their sequential order, the relevant properties of the process would not change.[2] A simple example from classical physics would be the motion of an object in a frictionless medium, such as a ball thrown upwards in a vacuum followed by its downward motion returning it to its initial position. A motion picture of this process would look exactly the same if projected in reverse. On the other hand, most processes in thermodynamics, such as diffusion or heat conduction, are not reversible in this sense. Diffusion, for example, tends to homogenize small differences or fluctuations, that is, tends to damp them. But if we reverse the sequence of events we get the opposite effect, a damping process turning into a process of amplification of fluctuations.[3] Mathematically, these ideas about processes are expressed in terms of the *invariance of the laws* governing a process: while the laws of classical and relativistic physics remain invariant under a time-reversal transformation, the laws of thermodynamics do not.[4]

I will argue in the following chapter that most of the objective content of classical physics can be recovered in an ontology *without laws*. But in the traditional ontology of physics, laws are clearly the single most important entity. Thus, given their ontological centrality and their invariance under time-reversal, it is not surprising that for most physicists the resolution of the conflict has taken the form of keeping the symmetry of the laws while explaining irreversibility away.[5] On the other hand, the emergence of new concepts in the nonlinear branches of classical physics, as well as the extension of thermodynamics to situations far from equilibrium, has added new models and new phenomena displaying irreversible temporal behaviour, forcing a re-evaluation of the conflict's resolution. Ilya Prigogine, a leading practitioner in both these fields, has been one of the most vocal critics of the attempts to eliminate irreversibility. As he argues, if reversing the sequence of events which makes up a process has no effect whatsoever on the nature of time, then time becomes a mere container for events happening in it:

> Consequently, as Henri Bergson and others emphasized, everything is given in classical physics: change is nothing but a denial of becoming and time is only a parameter unaffected by the transformation that it describes. The image of a stable world, a world that escapes the process of becoming, has remained until now the very ideal of theoretical physics . . . Today we know that Newtonian dynamics describes only part of our physical experience . . .

> [but relativity and quantum physics] inherited the idea of Newtonian physics: a static universe, a universe of *being* without *becoming*.[6]

The Deleuzian ontology I have described in these pages is, on the contrary, one characterizing a universe of *becoming without being*. Or more exactly, a universe where individual beings do exist but only as the outcome of becomings, that is, of irreversible processes of individuation. This is, of course, not a coincidence, since Deleuze was greatly influenced by those philosophers (such as Henri Bergson) who were the harshest critics of the reversible and uncreative temporality of classical science. Nevertheless, the theory of time created by Deleuze, a theory which I will attempt to reconstruct in this chapter, goes beyond the conflict between reversibility and irreversibility. The problem of time in a Deleuzian ontology needs to be approached in exactly the same terms as that of space: we need to conceive of a nonmetric time, a temporal continuum which through a symmetry-breaking process yields the familiar, divisible and measurable time of everyday experience. In particular, we cannot take for granted the existence of a linear flow of time already divided into identical instants bearing such close resemblance to one another that the flow may be regarded as essentially homogeneous.

In the first part of this chapter I will introduce the ideas needed to think about *extensive and intensive time*. The term 'extensive' may be applied to a flow of time already divided into instants of *a given extension or duration*, instants which may be counted using any device capable of performing regular sequences of oscillations. These cyclic sequences may be maintained mechanically, as in old clock-works, or through the natural oscillation of atoms, as in newer versions, but in either case *sequences of cycles* of different extension are used to measure stretches of time of different scales: seconds, minutes, hours, days. This idea, on the other hand, may be extrapolated from the measuring process to the very process which gives birth to time. I will discuss a theory by the nonlinear physicist Arthur Iberall according to which the measurable flow of time of our everyday experience is in fact a product of a *metrization* or a *quantization* of time into instants. Between the fastest vibrations of subatomic particles and the extremely long life-cycles of stars and other cosmic bodies, Iberall imagines a nested set of oscillations pulsating at increasingly longer time scales providing time with its metric structure.[7] This idea, of course, assumes that time is *not* like that of classical physics, that is, unaffected by the processes and transformations occurring within it.

After reviewing Iberall's theory and showing how it relates to Deleuze's, I will move on to discuss some of the intensive characteristics of time, those relating to the individuation of the stable oscillators which collectively create a metric temporality. I will describe the work of the nonlinear biologist Arthur Winfree who pioneered a method to study the birth and death of oscillations, or more exactly, a method to locate the *sensitive point* in an oscillation at which an external shock of the right intensity and duration can completely annihilate it. He has also investigated the opposite phenomenon, how a stimulus of the right intensity and timing can give birth to self-sustained oscillations. What Winfree's work shows is that the sequences of oscillations at different scales making up metric time cannot be viewed as composed of identical instants. Rather, each sequence will exhibit a distribution of *singular and ordinary* instants bearing witness to their intensive origin. Winfree's concepts of *critical timing*, *duration and intensity* will play a crucial role in defining the intensive or nonmetric aspects of time.[8]

Let's begin then with the question of extensive time. A nested set of cycles of different temporal scales would seem to offer the right form of temporality for the flat ontology of individuals I proposed before. In this ontology, individual organisms are component parts of species, much as individual cells are parts of the organisms themselves, so that cells, organisms and species form a nested set of individuals at different *spatial* scales. But clearly, each of these individuals also operates at a different temporal scale so that something like a nested set of cycles would be needed to complete the picture. On the other hand, to think of species, organisms or cells as possessing a single characteristic spatial scale is too simplified. As I said, between the cell and the organism there are a variety of spatial structures (tissues, organs, systems of organs) bridging the two scales. A species, in turn, is typically composed of several reproductive communities (demes) inhabiting different ecosystems, each community constituting an individual operating at a intermediate spatial scale between that of organism and species.

A similar point applies to temporal scales, an individual typically displaying a *spectrum of time scales*. Many individual organisms, for example, possess internal clocks which establish one of their temporal scales (their sleep–awake cycle), but may also have monthly and yearly cycles and even longer ones, like the length of time needed to achieve sexual maturity (reproductive cycles). They also possess many shorter cycles displayed in different types of rhythmic behaviour: breathing, mastication, locomotion. This means that actual time, rather than being a simple nesting of cycles, may include overlaps between the multiplicity of temporal scales associated with each level of individuality. In the present context, however, it will be more expedient to assume a simple embedding of time scales. For this purpose we can assign (by convention) a particularly prominent time scale to each individual level, such as the cycle which measures *the maintenance of their identity*: the length of time after which all (or most) of the individual cells in an organism have been replaced by new ones without affecting the organism's identity, or the length of time after which all the individual organisms that form a species have died and new ones have taken their place, thereby preserving the continuity of the species' own identity. This simplified nested set of cycles will constitute my working model of extensive or actual time. The question now is whether this metric temporality can be accounted for in the same way as metric space, that is, as the product of a symmetry-breaking event.

Nonlinear dynamics, in fact, allows a natural approach to the quantization or metrization of time in terms of spontaneous broken symmetry. In particular, there is a well-studied bifurcation, *the Hopf bifurcation*, which converts a steady state attractor into a periodic one.[9] To see in what sense this bifurcation implies a broken time symmetry we can use a spatial analogy. I said before that the phase transition from a gas to a crystalline state offered an example of a loss of invariance under spatial displacement. While the pattern of distribution in space for the gas remains basically the same under all displacements (if we imagine the gas stored in an infinite container) a regular arrangement of crystals loses some of this invariance and remains visually unchanged only for a specific number of displacements (those matching the length of individual crystals, or multiples of that length). Similarly, the time distribution of a process caught in a steady state attractor displays invariance for all time displacements, but after a Hopf bifurcation only displacements by a multiple of the period (or duration) of the cycle will leave the time distribution unchanged, all others will create a sequence of cycles that is *out of phase* with the original one. As Prigogine and Nicolis put it, a process 'in the regime of uniform steady state . . . ignores time. But once in the periodic regime, it suddenly "discovers" time in the phase of the periodic motion . . . We refer to this as the *breaking of temporal symmetry*.'[10]

Unlike linear oscillators (those most prevalent in classical and relativistic physics), a nonlinear oscillator born from a Hopf bifurcation displays a *characteristic period* (and amplitude). By contrast, the periods and amplitudes of linear oscillators (typically modelled as sinusoidal oscillations) are not intrinsic but depend on contingent details about their initial conditions.[11] Arthur Iberall uses this idea of an *intrinsic time scale* not dependent on extrinsic constraints as a basis for his theory of the quantization of time. As he puts it, such a theory should be based on

> . . . the mathematics of *sequences of pulses unfolding in time as distinguished from sustained sinusoidal oscillations*. The basic idea is that each pulse of action, in a nonlinear system embedded in a real universe, emerges as a new creation out of its past. It is the sustained linear instability in the local environment [which caused the Hopf bifurcation in the first place] that ensures the repetitive quality of the action. On the other hand, in the idealized lossless [i.e. conservative] linear isochronous system, with its characteristic sustained sinusoidal oscillation, causality for the action would be yoked irrevocably to the endless past and to an unending future.[12]

Iberall argues that, given that nonlinear oscillators have a characteristic time scale, ranging from the very short scales of atomic oscillators, to the intermediate scales of biological oscillators, to the very long lifecycles of stars and other cosmic bodies, we may view them as forming a nested set of levels. This embedded set would ensure 'the unfolding of time, pulse by pulse . . . Time is not a universal unity for all levels of organization. Yet levels are nested within one another and, within limits, are referable to each other.'[13] In other words, rather than assuming that time exists as an already quantized flow (divided into uniform, identical instants) we should *account for this metric structure* using the embedded set of differently scaled oscillations. In a sense, each oscillation would 'synthesize' a pulse of metric time, many nested sequences of these pulses yielding the familiar form of time which we humans can measure using a variety of chronometers. This concept of time is remarkably close to that of Deleuze's for whom each of these pulses of action would constitute a synthesis of 'present time' (the 'lived present' of atomic, biological and cosmic oscillators), a synthesis that would work by *contracting* an immediate past and future into a living present. He refers to this metric or extensive time by the name of 'Chronos', and writes:

> In accordance to Chronos, only the present exists in time. Past, present and future are not three dimensions of time; only the present fills time, whereas past and future are two dimensions relative to the present in time. In other words, whatever is future or past in relation to a certain present (a certain extension or duration) belongs to a more vast present which has a greater extension or duration. There is always a more vast present which absorbs the past and the future. Thus, the relativity of past and future with respect to the present entails a relativity of the presents themselves in relation to each other . . . Chronos is *an encasement, a coiling up of relative presents* . . .[14]

Let me explain in what sense each cycle would constitute only a present, and not a past or a future. Given an oscillator at a particular scale (a biological clock, for instance), what is immediate past and future for such an entity would still be part of the 'lived' present of an oscillator operating at longer time scales, at the level of geological or stellar dynamics, for example. Conversely, the minimum living present for a biological oscillator already includes many past and future events for oscillators operating at atomic and sub-atomic scales. Metric, extensive time would then be fundamentally cyclical and 'composed only of interlocking presents'.[15] I must emphasize at this point that, despite the reference to a 'lived present', this account of time has nothing to do with psychological time. It is true that Deleuze sometimes presents his theory of the synthesis of the present by contraction of immediate past and future, as a psychological theory, but this is simply a matter of convenience of presentation and not fundamental to his account.[16]

The idea that it is not subjective experience but the objective time scale of oscillators that matters may be further illustrated with a well-known example from relativity theory, an example which has sometimes led to confusion due to a mistaken psychological interpretation. The example concerns two twin brothers one of which stays on earth while the twin travels in a spaceship at a speed close to that of light. The relativistic conclusion that the twin on the spaceship would age much less than the one who stayed on earth has sometimes been challenged on the grounds that the difference between the two situations is a matter of subjective convention: while the twin in the spaceship may be said to be moving forwards relative to the one on the earth, it is also possible to say that, taking the spaceship as our frame of reference, it is the earth that is moving backwards relative to the ship, so that the situation is strictly symmetric. Given this symmetry, the shrinkage of time would be an illusion, similar to the apparent shrinkage in size which observers experience as they get further away from each other.[17] This conclusion is, of course, false. As the philosopher Hans Reichenbach argued long ago, the situation for the two twins is *not symmetric*. To see this, however, we must go beyond the psychological time of the observer to the time scale of the oscillators *of which the observer is composed*, not only the biological oscillators defining metabolic cycles at the cellular scale, but also the atomic oscillators of which the cells themselves are made. It is these oscillators that are *objectively affected* in the case of the rapidly moving twin, slowing down and hence retarding the aging process, but not in the case of his earthbound counterpart.[18]

A better way of explaining in what sense we may speak of the 'lived present' of a particular oscillator is through the relations between objective time scales, on one hand, and the resulting capacities to affect and be affected, on the other. I said in Chapter 2 that what one individual may afford another may depend on their relative spatial scales: the surface of a lake affords a walking medium to a small insect but not to a large mammal. A similar point applies to time scales. Each level of temporal scale defines what oscillators at that level 'perceive' as *relevant change*: certain cycles are simply *too slow* for them to appear as changing or moving relative to a faster level, and vice versa, certain oscillations are much *too fast* for them to even count as existing for oscillators operating at longer time scales. Subjective human time, our psychologically lived present with its experienced duration, would become in this interpretation a particular case of these objective relations of mutual relevance between the affordances of oscillators. Indeed, we may generalize this point to include physical phenomena which cannot be characterized as periodic. What matters for this argument is the existence of *characteristic time scales*, whether one thinks of these in terms of the intrinsic period of cyclic attractors or, more generally, in terms of the *relaxation time* associated with any kind of attractor.

An example of what is meant by 'relaxation time' is the time taken by a radio transmitter to settle into a stable periodic state after being turned on, what engineers refer to as 'transient behaviour'. These transients occur in many phenomena and in each case they display a characteristic time scale.[19] In state-space terminology this can be explained as follows. As I said before, all trajectories within a particular basin of attraction will be deterministically drawn to the attractor. Once there they may be temporarily dislodged from the attractor by an external shock but as long as the shock is not intense enough to expel them from the basin, they will return to the attractor. In this case, *the time taken for the trajectory to return to its attractor* is its relaxation time. How this relates to the question of affordances may be illustrated with an example adapted from Arthur Iberall. There are some solid materials, referred to generically as 'glasses', which unlike their crystalline counterparts, do not have a well-defined phase transition from the liquid state. In a sense, glasses are 'arrested liquids', that is, they retain the amorphous spatial arrangement of molecules that a liquid displays but *flow much more slowly*. Roughly, the distinction between the glass and liquid states can be made in terms of relaxation times: these are relatively long for glasses and relatively short for liquids.

Iberall argues that whether a particular body *appears solid or liquid to a given observer* will depend on the ratio between relaxation and observational time scales, in the sense that for sufficiently long observational times the glass will appear to the observer as a flowing liquid.[20] The inclusion of the observer in this description may give the wrong impression that something psychological is being discussed, but this impression dissolves once we realize that 'observation' is simply one particular instance of 'interaction'. In other words, what counts here is the *ratio of relaxation time to interaction time*, a ratio that can be defined without including a human observer in the picture. In particular, we can let the liquid and glass interact with each other and speak of how solid the glass 'appears' to the liquid, and vice versa. The glass, given its long relaxation time scale relative to the scale of interaction with the liquid, will behave as a solid, affording the liquid, for instance, an obstacle to its flow, or affording it a channel in which to flow. The flowing liquid, in turn, will afford erosion to the glass. In short, what capacities the glass has to affect and be affected by the liquid will depend on their relative time scales, the characteristic durations of their relaxation to equilibrium.

The objective relativity of affordances with respect to temporal scales makes them the ideal candidate to define the 'lived present' of a particular individual, that is, what this individual 'perceives' within its own time scale as the relevant capacities of the other individuals interacting with it. It is in this sense that Deleuze affirms, quite literally, that even inorganic things 'have a lived experience'.[21] To summarize the main conclusion of this section: material and energetic processes give time its metric and measurable form by their possession of a characteristic time scale, specified either through relaxation times, or as I will do in the rest of this section, through the intrinsic period of nonlinear oscillations. To phrase this conclusion in Deleuze's words, at any one of these embedded time scales the present is 'cyclical, measures the movement of bodies and depends on the matter that limits it and fills it out'.[22]

Having sketched how extensive time should be conceived in a Deleuzian ontology I would like to move on to discuss the ideas needed to think about the *intensive* aspects of temporality. In this book questions of intensity have been mostly related to the problem of the genesis of individuals. In the case of the nonlinear oscillators which quantize time Arthur Winfree's experimental and theoretical work gives us, as I said, the means to explore the intensive properties involved in the birth and death of oscillations. Winfree's best-known work deals with populations of biological oscillators (the internal clocks of fruit flies or mosquitoes, for instance) which he isolates from their surroundings to perform controlled experiments on their reaction to shocks of different timing, duration and intensity. Winfree's main result is, basically, that a *singular, critical stimulus* applied at a *singular, sensitive moment* has a destructive effect on the sleep–awake cycle of organisms, giving a population of mosquitoes, for example, permanent insomnia.[23] The stimulus itself needs to be of the right duration and intensity in order to act as an annihilating shock, but it nevertheless acts not as a direct cause of the death of an oscillation but merely as a trigger. What effect the shock will have will depend on the internal intensive structure of the oscillator itself.

For example, if the oscillation is governed by a periodic attractor which contains within it a stable steady-state attractor (what Winfree calls a 'black hole') then the critical stimulus will completely annihilate the oscillation.[24] On the other hand, the result of the stimulus may be not steady-state, atemporal behaviour but arrhythmic, ambiguous temporal behaviour, if the periodic attractor is associated with a set of states (called a 'phaseless set') bounded by a *phase singularity*.[25] In addition to these results related to the extinction of oscillations, Winfree has studied the complementary problem of what gives rise to these oscillations in the first place. Basically, he has found that by changing the experimental conditions he can transform an annihilating stimulus into a *conjuring stimulus*, that is, a critical shock that can create oscillations, the phase singularity in this case becoming an organizing centre for temporal structures.[26] Winfree's results display many of the traits that we have found characterize intensive processes, in particular, *mechanism-independent tendencies*. The tendency to be annihilated by a critical shock, for example, is not limited to the temporal behaviour of animals with nervous systems but is also exhibited by the behaviour of much simpler oscillators, ranging from yeast cells to inorganic chemical reactions.[27]

Other aspects of Winfree's work on oscillators illustrate a different feature of the intensive: the ability of nonlinear oscillators to *synchronize or entrain* one another's temporal behaviour. I said in Chapter 2 that the definition of 'intensive' may be expanded to include *capacities*, and in particular, the capacity of an individual to form assemblages with individuals very different from itself. Unlike the quantitative or qualitative properties of an individual, which as emergent properties refer to an individual's *inside* (that is, to the interactions among the lower scale individuals which compose it), an intensive property in the expanded sense refers to 'an *adequate outside* with which to assemble in heterogeneity', as Deleuze puts it.[28] The capacity of nonlinear oscillators to entrain one another's temporal behaviour is a particularly striking example of this other aspect of the intensive, allowing biological oscillators, for instance, to synchronize their sleep–awake cycles with cycles *outside themselves*, such as the day–night cycle of the planet. Entrainment is another phenomenon which Winfree has studied in detail, partly because of the need to prevent it from happening while studying the effects of annihilating stimula. Only if mosquito or fruit fly populations are isolated from the effects of the Earth's rotation will their internal clocks display their intrinsic duration or period. This period varies for different animals, from twenty-three hours for mosquitoes to twenty-five for humans, explaining the name 'circadian' given to these clocks, a term meaning 'nearly a day's length'.

When not in isolation, circadian clocks become entrained with the planet's own rotational period of twenty-four hours, a synchronizing capacity with obvious adaptive value since it allows a flexible coordination of internal rhythms and seasonally changing day lengths. Thanks to entrainment, biological oscillators can mesh, or form a heterogeneous assemblage, with the daily and seasonal rhythms of their external environment. Entrainment displays the typical characteristics of an intensive process, stimulus-independence and mechanism-independence. Synchronization of temporal behaviour is triggered rather than caused by relatively weak coupling signals which may be optical, chemical or mechanical. The exact nature of the signals serving as stimuli is not as important as their intensity: these signals must be maintained at a critical threshold of strength else the synchronization will abruptly stop.[29] A similar indifference is displayed towards the mechanisms implementing oscillating behaviour: entrainment occurs in populations of purely physical oscillators, such as the vibrating components of laser light, in inorganic chemical reactions, and in a large variety of biological oscillators, including the menstrual cycles of humans.[30]

The theory of metric time in terms of a nested set of cycles which I sketched above involves a kind of temporality which is *inherently sequential*, each individual life being a linear sequence of oscillations. The first part of Winfree's work shows that these linear sequences are not, in fact, homogeneous series of identical moments or instants. There are, in each series, a *distribution of singular and ordinary moments* and this distribution implies that there exist relations of critical timing between the sensitive points of oscillators and external shocks. The second part of his work displays a different aspect of intensive time, an aspect which takes us beyond sequential and into *parallel temporal structures*. The phenomenon of entrainment allows many independent sequences of oscillations to act in unison, to become in effect a single parallel process. The most dramatic and well-studied example of this phenomenon is perhaps the slime mould *Dictyostelium*. The lifecycle of this creature involves a phase where the organisms act as individual amoebae, the behaviour of each constituting an independent sequential process. At a critical low point of availability of nutrients, however, we witness the spontaneous aggregation of an entire population of these amoebae into a single field of parallel oscillators, eventually leading to their fusing together into a single organism with differentiated parts. As one scientist has remarked, witnessing this phenomenon 'one may really be watching a replay of the basic kinds of events responsible for the appearance of the first multicellular organisms.'[31]

In the next section of this chapter I would like to extend these ideas about critical duration and timing as well as parallelism to more complex processes of individuation than those exemplified by the slime mould. But let me first summarize what I have said about the birth of metric or extensive time. I gave before an example of how each of the embedded cycles making up this form of temporality may be said to be born through a symmetry-breaking event (a Hopf bifurcation). This was, however, a purely formal example leaving out the details of process which constitute the substance of the intensive. Adding to this formal model Winfree's experimental results mitigate but do not completely solve the problem. We can compare this simplified model of the birth of metric time to the metaphor I used in the last chapter to illustrate the birth of metric space. The neat picture of a symmetry-breaking cascade transforming a topological space into a metric one had to be comprehensively reworked to make it physically plausible: the nonmetric aspects of intensive processes turned out to be subtle and complex, as did the metric aspects of the extensive products; moreover, the least metric level of the embedded set had to be replaced with a virtual continuum whose description required yet another set of complex concepts.

A similar complexification is now in order to put some flesh on the rather skeletal formal model of a Hopf bifurcation. I will return to my two examples of individuation processes (the genesis of organisms and species) not only to add detail to Winfree's ideas about critical timing and parallelism, but more importantly, to show how intensive temporality may be crucial to the emergence of *novelty* in biological evolution. The process of embryogenesis, for instance, involves the parallel development of many simultaneous sequences of events, the relations between these sequences determined in part by the relative duration of these processes with respect to one another, and by the relative timing of the onset or cessation of one process relative to another. At this scale, as I will argue in a moment, the emergence of brand new designs may come about through *relative accelerations* in these parallel processes. A different source of novelty may be illustrated by moving up in scale to a discussion of ecosystems, which as individuation environments may be said to play relative to species the role which an egg or a womb play for individual organisms. In this other case too, relative accelerations in the tempo of evolution may lead to radical innovations. Unlike the temporality of the embryo, however, where the term 'intensive' has its original meaning, ecosystems will involve the expanded meaning, that is, the source of acceleration and innovation in this case is the assemblage of heterogeneous species in the process known as symbiosis.

Let me begin with the temporal aspects of the genesis of organisms. In the last chapter I emphasized the role of rates of change and couplings between separate rates as key to understanding embryological development. Although a rate of change does not need to involve time (we may be interested in the rate of change of pressure relative to oceanic depth or atmospheric height, for example), time does enter into the formulation of many important rates. These rates of change display the same interplay between characteristic time scale and affordances which I mentioned before in connection to relaxation times (the latter are, in fact, nothing but rates of approach to equilibrium). A process may change too slowly or too fast in relation to another process, the relationship between their temporal scales determining in part their respective capacities to affect one another. Even when two processes operate at similar scales, the result of their interaction may depend on their coupled rates of change. For example, the graphic patterns which many organisms display in their skins (e.g. zebra stripes or leopard spots) may be explained as the result of the variable concentration of chemical substances, a concentration which depends on the rates at which substances react with each other relative to the rates at which the products of such reaction diffuse through an embryo's surfaces. Different patterns may be achieved by controlling these relative rates, a task performed by genes and gene products (enzymes).

As the physicist Howard Pattee has convincingly argued, in the developing organism we find an interplay between *rate-dependent* phenomena (like chemical reaction and diffusion effects) and *rate-independent* phenomena. While the formation of self-organized patterns of chemical concentration does depend on the relative rates of diffusion and reaction, the information contained in genes does *not* depend on the rate at which it is decoded. On the other hand, this rate-independent information, once translated into enzymes, acts by *controlling rates*.[32] Enzymes are catalysts, and the latter are defined precisely as chemical elements capable of accelerating or decelerating a chemical reaction. The fact that embryological development is all about rates of change which are coupled or uncoupled through the action of genes and gene products, suggests that the processes underlying embryological development may be viewed as a kind of 'computer program'. But this metaphor should be used carefully because there are different kinds of computer programs presupposing *different forms of time*, some using sequential or serial time, others departing sharply from these linear forms of temporality. As Stuart Kauffman puts it:

> It is a major initial point to realize that, in whatever sense the genomic regulatory system constitutes something like a develop-

> mental program, it is almost certainly not like a serial-processing algorithm. In a genomic system, each gene responds to the various products of those genes whose products regulate its activity. All the different genes in the network may respond at the same time to the output of those genes which regulate them. In other words, the genes act in parallel. The network, in so far as it is like a computer program at all, is like a *parallel-processing network*. In such networks, it is necessary to consider the *simultaneous activity* of all the genes at each moment as well as the *temporal progression of their activity patterns*. Such progressions constitute the integrated behaviors of the parallel-processing genomic regulatory system.[33]

Thinking about the temporality involved in individuation processes as embodying the parallel operation of many different sequential processes throws new light on the question of the emergence of novelty. If embryological processes followed a strictly sequential order, that is, if a unique linear sequence of events defined the production of an organism, then any novel structures would be constrained to be added *at the end of the sequence* (in a process called 'terminal addition'). On the contrary, if embryonic development occurs in parallel, if bundles of relatively independent processes occur simultaneously, then *new designs may arise from disengaging bundles*, or more precisely, from altering the duration of one process relative to another, or the relative timing of the start or end of a process. This evolutionary design strategy is known as *heterochrony*, of which the most extensively studied case is the process called 'neoteny'.[34]

In neoteny the rate of sexual maturation is disengaged from the rate of development of the rest of the body, indeed, *accelerated* relative to somatic development, resulting in an adult form which is a kind of 'grown-up larva'.[35] Neoteny illustrates that novelty need not be the effect of terminal *addition* of new features, but on the contrary, that it can be the result of a *loss* of certain old features. Humans, for example, may be regarded as juvenalized chimpanzees, that is, primates from which a developmental stage (adulthood) has been eliminated. More generally, the loss of a feature made possible by the uncoupling of rates of change may provide an escape route from morphologies that have become too rigid and specialized allowing organisms to explore new developmental pathways.[36] To Deleuze this aspect of individuation processes (an aspect which must be added to population thinking to complete the Darwinian revolution) is highly significant because it eliminates the idea that evolutionary processes possess an inherent drive towards an increase in complexity, an idea which reintroduces teleology into Darwinism. As he writes, 'relative progress . . . can occur by formal and quantitative simplification rather than by complication, by a loss of components and syntheses rather than by acquisition . . . It is through populations that one is formed, assumes forms, and through loss that one progresses and picks up speed.'[37]

The flexibility with which parallel processes endow embryological development may be said to come to an end once the final organism acquires a more or less fixed anatomy. That is, at this point the intensive becomes hidden under the extensive and qualitative. Yet, anatomical features are never fully fixed even in adulthood. Many parts of the body retain their capacity to self-repair, and in some animals even the capacity for complete regeneration. Additionally, even if relative to the flexibility of an embryo the *anatomical* properties of a finished organism are indeed rigid, its *behavioural* properties may not be, particularly if such an organism is endowed with flexible skills beside its hard-wired reflexes and behavioural routines. At any rate, even the most anatomically and behaviourally rigid individual, even the most extensive of finished products, is immediately caught up in larger-scale individuation processes where it becomes part of other intensities, such as the intensive properties characterizing ecosystems.

One of the most important factors considered in studies of ecosystems is changes in the *population density* of each of the interacting species. Population density, like temperature or pressure, is an intensive property that cannot be divided in extension. But like other intensities it may be divided by phase transitions. In particular, there are critical thresholds at which the state of a population changes in kind, such as minimal values of density (sometimes called 'nucleation thresholds') below which a population goes extinct.[38] Similarly, much as a population of molecules will spontaneously tend to relax, after a certain characteristic time, to an equilibrium value for its temperature, so population density will exhibit a characteristic relaxation time after being subjected to an environmental shock, such as a particularly harsh winter. The ecologist Stuart Pimm argues that this rate of return to equilibrium characterizes a population's *resilience* to shocks: short rates of return to equilibrium signal a robust population, that is, one capable of recovering rapidly after a shock, while long relaxation times betray poor resilience and hence, vulnerability to extinction. Given that extinction means the death of a species as an individual, and that the extinction of one species may mean the rapid birth of others to occupy the vacant niche, these intensive properties may be said to partly characterize processes of individuation at this scale.

Ecosystems involve processes operating at several simultaneous time scales. One factor affecting population density is internal to a species, that is, determined by the birth and death rates of a population. This factor displays a relatively short time scale of return to equilibrium. When the densities of several populations are coupled in parallel, as when a population of plants, hervibores and carnivores is coupled into a food chain, relaxation times become longer: when the density of a predator population affects that of its prey, and this, in turn, the density of the plants it consumes, re-equilibration after a shock may be delayed until the cascading effects stop. This longer time scale of recovery is determined by the degree of connectivity which one species has to other species, that is, by the *length of the food chain* to which the species belongs. Finally, there are even longer-term processes determined by non-biological factors such as the rate of availability of mineral nutrients in an ecosystem during recovery from a catastrophe, such as the effects of the onset or cessation of an Ice Age.[39] Given the importance of resilience as protection against extinction, and given the key role which the degree of connectivity plays at intermediate time scales, an ecosystem may also be considered a parallel-processing network in which changing relationships of *fitness* (between predators and prey, or hosts and parasites) propagate at different rates throughout the network influencing both the emergence of new, and the disappearance of old, individual species.[40]

Relations between population densities, however, give us only a rough idea of the complex *temporal structure* of an ecosystem. Considered as a network in which the flesh (or biomass) of plants and animals circulate, an ecosystem will display a variety of temporal rhythms characterizing each of its alimentary couplings, these rhythms, in turn, associated with the spectrum of oscillatory behaviour at different scales exhibited by every organism. But considered as an individuation environment there is a particular rhythm which must be singled out: the *evolutionary rates* of each of the coupled species. Evolutionary rates used to be thought as basically uniform, characterized by a linear and gradual accumulation of genetically coded beneficial traits. This rate of accumulation would vary from species to species, due to their different generation times, but within each species it was supposed to be basically uniform. Today we know that this picture is incomplete given that for a variety of reasons there occur accelerations and decelerations in these evolutionary rates. (The very large time scales involved in evolution means, however, that even an accelerated rate will still characterize a very long process, one between 5000 and 50,000 years, for example.[41])

As in the case of embryological development where loss of a particular process or component may lead to the emergence of novel features, in an ecosystem losses may also lead to accelerations in evolutionary rates and rapid spread of novel designs. An extinction event, for example, may eliminate a set of species and vacate their niches, leading in turn to an explosion of new designs by other species (an adaptive radiation) to occupy the vacant positions in the food chain.[42] A different example of events leading to accelerated evolution and rapid emergence of new capacities is *symbiosis*. Although traditionally the term 'symbiotic relationship' refers to a particular kind of alimentary coupling (one in which both partners benefit from the association) the difficulty in defining and establishing mutually beneficial relations has led to a new view of its nature and function. Today symbiosis is defined as an assemblage of heterogeneous species which *persists for long periods*, relative to the generation times of the interacting organisms, and which typically leads to the *emergence of novel metabolic capabilities* in at least one of the partners.[43] The emphasis on long duration is due to the need for *coevolution* between the partners, both of which need to have exerted selection pressures on each other biasing the long-term accumulation of their genes and bodily traits. (Given that some members of an ecosystem may have arrived through recent invasions or colonizations, not all interacting couples in a food chain need to have coevolved.)

Symbiosis as a source of evolutionary innovation occurs at many levels of scale. At the cellular level, for example, two of the key capacities at the basis of food chains may have emerged through an assembly of heterogeneities. Photosynthesis, the ability to 'bite' into solar radiation to produce chemical energy stored in sugars, and respiration, the ability to tap into a reservoir of oxygen as fuel to burn these sugars, are both thought to have emerged through cellular level symbioses with micro-organisms.[44] At larger scales, examples include the autonomous communities of micro-organisms which line the guts of hervibores allowing them to digest cellulose, the bacteria that allow legumes to fix nitrogen, and the fungi which permit many plant roots to get access to phosphorous. In all these cases, novel capabilities to exploit otherwise unavailable resources have come about not through a slow and gradual accumulation of favourable mutations but through an accelerated process: meshing the capabilities of two or more heterogeneous populations of organisms followed by the subsequent coevolution of the partners.[45]

When discussing intensive processes Deleuze usually divides the subject into singularities and affects, but sometimes he uses an alternative and equivalent formulation in terms of speeds and affects: *speeds of becoming and capacities to become*.[46] The many parallel processes which define a developing embryo, for example, are defined by their relative speeds, and by the accelerations and decelerations these may undergo resulting in the production of novel forms. In Deleuzian terms, such an individuation environment would be characterized in part by relations of 'speed and slowness, rest and movement, tardiness and rapidity'.[47] As I said, changes in these relative speeds may be used as an evolutionary strategy (heterochrony) allowing an organism an escape route from an over-specialized design. Ecosystems also display relations of relative speed between parallel processes but in this case the emergence of novelty depends more on the capacity to join in with a heterogeneous partner in a common coevolutionary line of flight, or as Deleuze puts it, on 'a composition of speeds and affects involving entirely different individuals, a symbiosis'.[48] To phrase this in Prigogine's terms of being and becoming: whereas embryogenesis is a process through which a yet unformed individual *becomes what it is*, acquiring a well-defined inside (the intrinsic properties defining its being), symbiosis represents a process through which a fully formed being may cease to be what it is to *become something else*, in association with something heterogeneous on the outside.

This description of more complex forms of intensive temporality was intended as a complement to the simpler formulation in terms of the individuation of oscillations. Questions of critical timing and duration, as well as of parallelism, are still prominent but have acquired a subtler form. Similarly, the problem of the metrization or quantization of time, which also had a simple formulation in terms of a nested set of sequences of oscillations, needs now to lose some of that simplicity. In particular, for the sake of ease of presentation I have artificially separated issues related to time and space, but in reality we are always confronted with complex spatio-temporal phenomena. Even the simple oscillators studied by Winfree are nonlinear spatio-temporal oscillators where the spatial and temporal aspects interact. For this reason, the question of the emergence of metric or extensive properties should be treated as a single process in which a continuous *virtual spacetime* progressively differentiates itself into actual discontinuous spatio-temporal structures operating at different scales. In other words, the emergence of a metric spacetime involves the entire flat ontology of individuals, each nested level of scale contributing to the metrization of space and time simultaneously.

I would like to conclude this chapter with a more detailed discussion of this virtual spacetime. In Chapter 2 I described the elements which, according to Deleuze, constitute the content of a nonmetric continuum: changing populations of virtual multiplicities (conceived as complex ideal events) and a quasi-causal operator which assembles this heterogeneous population into a plane of consistency. This particular breakdown of the contents of the virtual is, of course, speculative, and as such, it may very well turn out to be wrong. There is, as I said, an empiricism of the virtual, even if it does not (and should not) resemble the empirical study of the actual. But while the specific *solution* which Deleuze proposes may turn out to be inadequate, he should get credit for having adequately *posed the problem*. In order to get rid of essentialist and typological thinking it is not enough to denounce the transcendent and affirm the immanent. Replacing Plato's transcendent essences with Aristotle's immanent natural states, for example, gets us out of essentialism but not of typological thought. One must also give *mechanisms of immanence* (however speculative) to explain the existence, relative autonomy and genetic power of the virtual.[49] Let me first summarize what I said before about the quasi-causal operator, the manner in which it meshes multiplicities by their differences, since this constitutes the first immanence mechanism. I will then describe the second task which Deleuze ascribes to this virtual entity: *to generate* the multiplicities by extracting them from actual intensive processes. Together, these two tasks ensure that the resulting virtual spacetime does not have the form of a transcendent space filled with timeless essences.

I described the first task of the quasi-causal operator as that of giving virtual multiplicities a minimum of actualization by prolonging their singularities into series of ordinary ideal events, and establishing relations of convergence and divergence between these series. I said that to specify how these *immaterial linkages* between series are established Deleuze borrows from the most abstract version of communication theory the concept of transmission of information in a channel (a sign/signal system, in his terms,). An information channel (signal) exists whenever two heterogeneous series of events are coupled by changing probability distributions. No reference needs to be made to either a causal mechanism or to anything actually flowing in the channel. Quanta of information (signs) may be said to pass from one series to another whenever a change in the probability distribution in one series is correlated to a change in the other one. Such a linkage of series of events through signs occurs spontaneously in some intensive systems, such as systems *poised at the edge* of a phase transition. Even when such poised systems are inorganic, that is, even in the absence of specialized biological hardware, they can coherently transmit information as long as they manage to remain in the vicinity of the critical event without actually crossing the threshold.

The embryological and ecological individuation processes I have just discussed, at least when modelled as parallel-processing networks, display this emergent ability in the neighbourhood of a *critical point of connectivity*. Stuart Kauffman argues, for example, that the many food chains that form an ecosystem must not exceed a certain critical length (typically of four species: a plant, a hervibore, a predator, and a predator of the predator) for the parallel network to display complex behaviour.[50] This sensitive value may be achieved via the coevolution of the members of a food chain. Similarly, the parallel network formed by genes and gene products which constitutes the informational backbone of a developing embryo also needs to keep its degree of connectivity near a critical value. Kauffman explicitly compares this critical value (not too low but not too high) to the singular zone of intensity existing at the phase transition between a gas and a solid (that is between states with too little and too much order, respectively) and argues that embryos and ecosystems may need to be poised at the edge in order to maximize their emergent computational capacities.[51]

Unlike actual poised systems, however, where information transmission takes the form of correlations between the *numerical* probabilities of occurance of two series of events, virtual series must exclusively involve changing distributions of the singular and the ordinary, given that virtual series and the space they form cannot presuppose any metric or quantitative notion without begging the question. In particular, virtual series must be conceived as dense *ordinal* series which, as I argued, are logically and genetically prior to already quantized numerical series and can be regarded as one-dimensional nonmetric continua. In addition, the requirement of not presupposing any notion to which the virtual is supposed to give rise implies that the statistical distributions involved in an information channel cannot be conceived as fixed (or 'sedentary') like the famous Gaussian or bell-shaped distributions characterizing the statistical properties in many actual populations. Unlike these familiar equilibrium distributions which refer to already individuated populations occupying a metric space, Deleuze *designs* the quasi-causal operator to produce *mobile and ever-changing* ('nomad') distributions in the virtual series, establishing both convergent and divergent relations between them.[52]

In short, the first task of the quasi-causal operator is what Deleuze calls a *condensation of singularities*, a process involving the continuous creation of communications between the series emanating from every singularity, linking them together through non-physical resonances, while simultaneously ramifying or differentiating the series, ensuring they are linked together only by their differences.[53] The mesh of one-dimensional continua that results would constitute the spatial aspect of the virtual. To this, a temporal dimension, which Deleuze calls 'Aion', should now be added. As he writes, the specification of the virtual

> implies, on the one hand, a space of nomad distribution in which singularities are distributed (Topos); on the other hand, *it implies a time of decomposition whereby this space is subdivided into sub-spaces*. Each one of these sub-spaces is successively defined by the adjunction of new points ensuring the progressive and complete determination of the domain under consideration (Aion). There is always a space which condenses and precipitates singularities, just as there is always a time which progressively completes the event through fragments of future and past events.[54]

Deleuze borrows the term 'adjunction' from the mathematician Evariste Galois, the creator of group theory. I will return in the next chapter to the work of this pioneer, but at this point it is enough to say that the operation Galois defined as 'adjunction of fields' is an abstract operation very closely related to the idea of the progressive differentiation of a space through a cascade of symmetry-breaking transitions. In other words, the successive determination of sub-spaces to which Deleuze refers is simply the progressive unfolding of multiplicities through a series of symmetry-breaking events. The form of temporality involved in this unfolding, however, should be conceived in a very different way from that in which actual bifurcation events occur. The latter involve a *temporal sequence* of events and stable states, the sequence of phase transitions which yields the series of stable flow patterns conduction–convection–turbulence, for example. Moreover, as each bifurcation occurs, *only one* of the several alternatives available to the system is actualized. For example, in the transition to the convection regime, either clock or anti-clockwise rotating convection cells may emerge, but not both. Indeed, at every bifurcation there are alternatives that are *physically unstable* (unlike the two options for convection cells both of which are stable) which means that even if they are actualized they will not last very long and will be destroyed by any destabilizing fluctuation.[55] In a virtual unfolding, on the other hand, the symmetry-breaking events not only *fully coexist* with one another (as opposed to follow each other), but in addition, each broken symmetry produces *all the alternatives simultaneously*, regardless of whether they are physically stable or not.

This virtual form of time, involving the idea of *absolute simultaneity* (or absolute coexistence) would seem to violate the laws of relativity. In relativistic physics two events cease to be simultaneous the moment they become separated in space, the dislocation in time becoming all the more evident the larger the separating distance.[56] There are two reasons, however, why this should not be an objection to Deleuze's conception of virtual time. The first and most obvious reason is that in virtual space there are *no metric distances*, only ordinal distances which join rather than separate events. Much as the notions of spatial 'length' or 'area' lose their meaning when we move away from Euclidean geometry to other ways of specifying the relations of proximity defining a space, so should the notions of 'stretch' or 'lapse' of time separating non-simultaneous events be meaningless in the context of a nonmetric temporality. But there is a second and more important reason why relativistic constraints on absolute simultaneity, such as the constraint on the maximum speed at which causal signals may travel, should not apply to the virtual. The temporality of the virtual should not be compared to that of the processes governed by the laws of relativity, but to *the temporality of the laws themselves*. Unlike experimental laws (like Boyle's law of ideal gases) which simply record laboratory regularities, fundamental laws (such as Newton's or Einstein's) are not mere mathematical re-descriptions of experience.[57] Although physicists do not usually speculate about the ontological status of fundamental laws, to philosophers these laws are supposed to be *eternal*, and to be valid *simultaneously* throughout the universe. In other words, in philosophical discussions fundamental laws enjoy the same form of timelessness as immutable essences. And it is *this* form of time that the virtual is supposed to replace.

Nevertheless the question remains, what form of temporality would allow the absolute coexistence of virtual events? Or what amounts to the same thing, how should we conceive of a nonmetric form of time? It clearly cannot be any present time, however long, since the very concept of a present assumes that of a stretch or lapse of time of a particular characteristic scale. But it cannot be a timeless dimension either if we are to avoid the trappings of essentialism. The solution which Deleuze proposes to escape these alternatives is ingenious. Unlike a transcendent heaven inhabited by *pure beings without becoming* (unchanging essences or laws with a permanent identity) the virtual needs to be populated exclusively by *pure becomings without being*. Unlike actual becomings which have at most an intensive form of temporality (bundles of sequential processes occurring in parallel) a pure becoming must be characterized by a *parallelism without any trace of sequentiality, or even directionality*. Deleuze finds inspiration for this conception of time in phase transitions, or more exactly, in the critical events defining *unactualized* transitions. When seen as a pure becoming, the critical point of temperature of 0°C, for example, marks neither a melting nor a freezing of water, both of which are actual becomings (becoming liquid or solid) occurring as the critical threshold is crossed in a definite direction. A pure becoming, on the other hand, would involve both directions at once, a melting–freezing event which never actually occurs, but is '*always forthcoming and already past*.'[58]

The events involved in the construction of virtual space, the progressive unfolding of virtual multiplicities as well as the stretching of their singularities into series of ordinary points, need to be thought as pure becomings in this sense. In this construction, as Deleuze says, '*Time itself unfolds . . . instead of things unfolding within it . . .* [Time] ceases to be cardinal and becomes ordinal, a pure *order* of time.'[59] Unlike actual time, which is made exclusively out of presents (what is past and future relative to one time scale is still the living present of a cycle of greater duration), a pure becoming would imply a temporality which always *sidesteps the present*, since to exist in the present is to be, no longer to become. This temporality must be conceived as an *ordinal continuum unfolding into past and future*, a time where nothing ever occurs but where everything is endlessly becoming in both unlimited directions at once, always 'already happened' (in the past direction) and always 'about to happen' (in the future direction). And unlike actual time which is *asymmetric* relative to the direction of relative pasts and futures, a pure becoming would imply a temporality which is *perfectly symmetric* in this respect, the direction of the arrow of time emerging as a broken symmetry only as the virtual is actualized.[60]

I said in Chapter 2 that multiplicities, being incorporeal effects of material causes, are impassible or causally sterile entities. The time of a pure becoming, always already passed and eternally yet to come, forms the temporal dimension of this impassibility or sterility of multiplicities.[61] But I also said that the quasi-causal operator, far from being impassible, is defined on the contrary by a pure capacity to affect, acting in parallel with physical causality in the production of the virtual. In particular, the quasi-cause must be capable of weaving multiplicities into a heterogeneous continuum and to do so constantly so as to endow the latter with a certain autonomy from their corporeal causes.[62] What temporal aspect would correspond to the exercise of this capacity? Here again, we cannot presuppose any metric concepts, that is, we cannot assume that this performance occurs in any present stretch of time, however short. This other time must indeed be conceived as *instantaneous*. As Deleuze writes:

> Corporeal causes act and suffer through a cosmic mixture and a universal present which produces the incorporeal event. But the quasi-cause operates by doubling this physical causality – it embodies the event in the most limited possible present which is the most precise and the most instantaneous, the pure instant grasped at the point it divides itself into future and past.[63]

In what sense would a temporality characterized by a instant which unfolds itself into past and future be nonmetric? Actual time, as I said, may be seen as the product of a metrization or quantization performed by a nested set of presents with characteristic time scales. Whether one views the latter in terms of relaxation times or in terms of the intrinsic period of nonlinear oscillations, the processes occurring in actual time always have a time scale of *limited duration* and yet are potentially *infinite*, in the sense that a particular sequence of cycles may go on pulsing for ever. Virtual time, on the other hand, would be nonmetric in the sense that it is *unlimited* in the past and future directions in which it unfolds, but always *finite* like the instant without thickness that performs the unfolding.[64] The time of the virtual would be constituted entirely by what, from the point of view of metric time, cannot be but singularities: a maximum and a minimum, events of *unlimited duration* (the unfolding of multiplicities) and events of *zero* *duration* (the operation of the quasi-cause). The quasi-causal operator would have to

> bring about the correspondence of the minimum time which can occur in the instant with the maximum time which can be thought in accordance with Aion. To limit the actualization of the event in a present without mixture, to make the instant all the more intense, taut, and instantaneous since it expresses an unlimited future and an unlimited past.[65]

No doubt, this description of the temporal aspect of virtuality lacks the precision of its spatial counterpart. The latter has the advantage of over a century of mathematical work on the nature of nonmetric spaces and their broken symmetry relations to metric ones, whereas similar formal treatments of time do not really exist. Moreover, even if we disregard time and focus only on space, Deleuze's description of the virtual continuum goes beyond the resources available from those formal theories and may therefore seem much too speculative and complicated. Why, one may ask, go through so much trouble to specify the immanence mechanisms through which a virtual continuum is constructed when it is simpler and more natural to assume that the entities revealed by nonlinear mathematics (attractors, bifurcations) are of the same type as our more familiar Platonic entities? A leading figure in the theory of dynamical systems, the mathematician Ralph Abraham, for example, phrases his evaluation of the merits of the field this way:

> The benefits of using dynamical concepts at the present stage of development of self-organization theory fall in two classes: permanent ones – the acquisition of concepts to be embedded in morphodynamics, guiding its development; and temporary ones – the practice of new patterns of thought. In the first category I would place the attractors, the stable bifurcations, and their global bifurcation diagrams, as essential features of morphodynamics. These may be regarded as guidelines, exclusion rules and topological restrictions on the full complexity of morphodynamic sequences . . . I see [the

> importance of dynamicism] for self-organizing system theory as temporary and preparatory for a more complete morphodynamics of the future. And yet, dynamicism even now promises a permanent legacy of restrictions, a taxonomy of legal, universal restraints on morphogenetic processes – a Platonic idealism.[66]

Deleuze would agree with much of what is expressed in this passage, particularly the characterization of the role of virtual entities as topological restrictions or constraints, that is, as quasi-causal relations which complement causal ones in the determination of a given self-organizing or intensive process. On the other hand, to view the set of topological restrictions discovered so far as forming some kind of fixed, eternal taxonomy, would seem to him to defeat the very point of postulating such constraints in the first place. No doubt, it is much simpler to assume the existence of Platonic entities than to define a complex operation through which these entities are meshed into a continuum thereby acquiring a certain autonomy from actual events. The preference for simplicity here, however, has less to do with the elimination of redundant features (the legitimate use of simplicity arguments, as in Occam's razor) and more to do with *familiarity*. Arguments based on the latter, as physicists concerned with the conceptual foundations of their subject are aware, make an illegitimate use of simplicity.[67] In the present context, it seems to me, to espouse a Platonic idealism on the basis that it is a more familiar thesis would be misguided. Given that no philosopher (or scientist) has ever before specified mechanisms of immanence, our lack of familiarity with the latter should be seen merely as a contingent fact about intellectual history not as a basis to reject a new theory.

I emphasize this point about simplicity because however complex the description of the virtual may seem so far, it is only half the story. In particular, we may grant that the above description is a reasonable specification of how a nonmetric spacetime continuum may be built *given* a population of virtual multiplicities and still demand to know *where these multiplicities come from*. Clearly, they cannot be simply assumed to exist on their own since this would make them into entities hardly distinguishable from immutable essences. There is, in fact, another task which the quasi-causal operator must perform, another immanence mechanism which accounts for the very existence of multiplicities. As Deleuze says, the quasi-cause '*extracts singularities from the present*, and from individuals and persons which occupy this present'.[68] This *extraction operation*, recovering a full multiplicity from a partial spatio-temporal actualization, defines the second immanence mechanism. Deleuze sometimes uses a geometric characterization of this operation, describing it as the extraction of *a section or slice*. Ordinarily, this mathematical operation simply reduces the dimensionality of the object to which it applies. A slice of a three-dimensional volume, for example, is a two-dimensional surface, while the volume itself may be viewed as a slice or section of a four-dimensional hypervolume. The analysis of attractors in state space, particularly strange or chaotic attractors, makes extensive use of this operation (a 'Poincaré section') to extract information from a complex topological shape and display it in a way which is easier to study.[69] Deleuze, however, has a more elaborate operation in mind, one that does not have a counterpart in mathematics.

To see what this original slicing operation amounts to let's return to the example of the sequence of flow patterns conduction–convection–turbulence. Let's imagine a concrete physical system in a state of convection, that is, actualizing one of the available flow patterns (a periodic attractor). In this case, the virtual component (the attractor) exists merely as an effect of actual causes, such as relations between temperature and density differences or competition between gravitational and viscous forces, causal relations which account for the emergence and maintenance of convection cells. Deleuze's hypothesis is that such an actual system may be 'sampled' or 'sliced through' to obtain its full quasi-causal component, the entire set of attractors defining each flow pattern and the bifurcations which mediate between patterns. In other words, a Deleuzian section would not consist in a mere reduction of the original dimensionality, but in an elimination of every detail of the actual event except its *topological invariants*: the distribution of its singularities, as well as the full dimensionality of its state space.

Let me spell out the details of this important idea. I said in Chapter 1 that Deleuze borrows from Riemann the concept of an N– dimensional manifold which does not need to be embedded in a space of

N+1 dimensions to be studied, but that constitutes a space on its own, each one of its dimensions defining a relevant degree of freedom of, or a relevant way of changing for, a given dynamical system. Each multiplicity extracted or sampled from actual intensive processes would possess a definite dimensionality (a specific value for the N variable) since the process it governs is capable of changing in only a finite number of relevant ways. This finite number of dimensions would constitute a key characteristic defining the virtual multiplicity as a concrete universal entity, and this finite number would vary for different multiplicities extracted from different processes. In other words, the population of multiplicities would be *dimensionally heterogeneous*. Given that the plane of consistency must assemble multiplicities together by their differences, this 'plane' cannot be conceived as a two-dimensional surface but as a space of variable dimensionality, capable of bringing a dimensionally diverse virtual population into coexistence. As Deleuze writes:

> It is only in appearance that a plane of this kind 'reduces' the number of dimensions; for it gathers in all the dimensions to the extent that *flat multiplicities* – which nonetheless have *an increasing or decreasing number of dimensions* – are inscribed upon it . . . Far from reducing the multiplicities' number of dimensions to two, the *plane of consistency* cuts across them all, intersects them in order to bring into coexistence any number of multiplicities, with any number of dimensions. The plane of consistency is the intersection of all concrete forms . . . The only question is: Does a given becoming reach that point? Can a given multiplicity flatten and conserve all its dimensions in this way, like a pressed flower which remains just as alive dry?[70]

Deleuze sometimes phrases his description as if the quasi-causal operator was the agent performing the extraction or section operation, some other times ascribing this agency to the plane of consistency itself. The difference between the two formulations is, I believe, unimportant. What is important, on the other hand, are the details of the operation and their justification. In particular, the fact that each multiplicity defines a space of its own, that is, the *absence of a space of*

*N+1 dimensions* where they would be embedded, is key to the task of conceiving a virtual space which does not unify multiplicities, that is, a space composed by the coexisting multiplicities themselves in their heterogeneity. Similarly, the quasi-causal operator is often referred to as a 'line' but not because it would be a one-dimensional entity. Rather, the quasi-cause would operate at *N–1 dimensions*, unlike a transcendent source of unity which must operate from a supplementary (e.g. N+1) dimension. In Deleuze's words:

> Unity always operates in an empty dimension supplementary to that of the system considered (overcoding) . . . [But a] multiplicity never allows itself to be overcoded, never has available a supplementary dimension over and above its number of lines [or dimensions] . . . All multiplicities are flat, in the sense that they fill or occupy all of their dimensions: we will therefore speak of a plane of consistency of multiplicities, even though the dimensions of this 'plane' increase with the number of connections that are made on it. Multiplicities are defined by the outside: by the abstract line, the line of flight . . . according to which they change in nature and connect with other multiplicities . . . The line of flight marks: the reality of a finite number of dimensions that the multiplicity effectively fills; the impossibility of a supplementary dimension, unless the multiplicity is transformed by the line of flight; the possibility and necessity of flattening all of the multiplicities on a single plane of consistency or exteriority, regardless of their number of dimensions.[71]

Let me summarize what I have said about the two immanence mechanisms. The operator's first task, to assemble multiplicities together by creating convergent and divergent relations among the ordinal series emanating from them, may be considered a *pre-actualization*. It would endow multiplicities with a minimum of actuality and, in this sense, it would represent the first broken symmetry in the cascade that culminates in fully formed actual beings. The second task of the quasi-causal operator, to extract virtual events from intensive processes may, in turn, be seen as a veritable *counter actualization* since it would follow a direction opposite to that which goes from the virtual to the intensive, and from there to the extensive and qualitative.[72] Counter-actualization would, in fact, complement pre-actualization: while the former extracts flat (or folded) multiplicities from actually occurring events, the latter would take these and 'unflatten' them, that is, it would allow them to progressively unfold and differentiate without fully actualizing them. Each of these two operations would possess a temporal dimension: the quasi-causal operator would sample or section all actual events, at all different time scales, instantaneously; then, each flat multiplicity would be immediately unfolded in two unlimited directions at once, past and future, distributing the singularities which define each of the unfolding levels on both sides of the instant at once, 'in the manner of a pod which releases its spores'.[73]

The operation of pre-actualization would give multiplicities not only a certain autonomy from the intensive processes acting as their real causes, it would also endow these impassive and sterile effects with whatever morphogenetic power they enjoy.[74] In other words, pre-actualization would not only explain how an unactualized singularity belonging to a physical system with multiple attractors would subsist as a potential alternative state, it would also explain how the singularity that *is* actualized gets its power to attract in the first place. To the extent that linking multiplicities together and endowing them with productivity foreshadows the intensive processes which follow down the symmetry-breaking cascade, the quasi-causal operator is referred to as a 'dark precursor'.[75] The operation of counter-actualization, on the other hand, would operate in the opposite direction, up the cascade from the intensive towards the virtual. I said in Chapter 2 that some areas of the world, those defined by processes which are nonlinear and which operate far from equilibrium, do not conceal the virtual underneath extensities and qualities but rather reveal it, or allow it to express itself.[76] These areas would represent a spontaneous movement towards the virtual which is still physical and corporeal but which may be given a boost making it reach the level of a pure virtuality. To the extent that counter-actualization *accelerates an escape* from actuality which is already present in some intensive processes, the quasi-causal operator is referred to as a 'line of flight'.[77]

In conclusion, I would like to repeat that whatever the merits of Deleuze's particular proposals for the implementation of the quasi-causal operator, we should at least credit him with having elucidated the overall constraints that any implementation would have to meet. If we are to get rid of essentialist and typological thought we need some process through which virtual multiplicities are derived from the actual world and some process through which the results of this derivation may be given enough coherence and autonomy. Deleuze himself gave several different models for each one of these tasks, a fact that shows that he did not think he had achieved a final solution to the problem, only its correct formulation. On the other hand, he clearly thought that *the problem itself was worth posing*, regardless of its particular solutions. That this is indeed the case may be glimpsed from the fact that Deleuze's description of his constructivist method in philosophy closely matches the two tasks which the operator is supposed to accomplish: creating virtual events (multiplicities) by extracting them from actual processes and laying them out in a plane of consistency.[78] This methodology, moreover, is what in his view would distinguish philosophy from science. As he writes:

> It could be said that science and philosophy take opposed paths, because philosophical concepts have events for consistency whereas scientific functions have states of affairs or mixtures for reference: through concepts, *philosophy continually extracts a consistent event from the states of affairs* . . . whereas through functions, science continually actualizes the event in a state of affairs, thing, or body that can be referred to.[79]

It matters little whether we describe this method as involving two separate operations (to extract ideal events *and* to give them consistency) or as a single one (to extract a consistent event). The important point is that Deleuze conceives of pre-actualization and counter-actualization, however implemented, as defining an *objective movement* which a philosopher must learn to grasp. As he puts it, we philosophers must invent devices to allow us to become 'the quasi-cause of what is produced within us, the Operator'.[80] Spelling out the details of Deleuze's methodology will involve connecting the results of his ontological analysis with questions of epistemology. In epistemological terms to extract an ideal event from an actually occurring one is, basically, to define what is *problematic* about it, to grasp what about the event *objectively stands in need of explanation*. This involves discerning in the actual event what is relevant and irrelevant for its explanation, what is important and what is not. That is, it involves correctly grasping the *objective distribution of the singular and the ordinary* defining a well-posed problem. To give consistency to these well-posed problems, in turn, means to endow them with a certain autonomy from their particular solutions, to show that problems do not disappear behind their solutions, just like virtual multiplicities do not disappear behind actualized individuals. The epistemological side of a Deleuzian ontology is constituted by such a philosophy of problems and this will form the subject matter of the following chapter.

CHAPTER 4

# *Virtuality and the Laws of Physics*

In a flat ontology of individuals, like the one I have tried to develop here, there is no room for reified totalities. In particular, there is no room for entities like 'society' or 'culture' in general. Institutional organizations, urban centres or nation states are, in this ontology, not abstract totalities but concrete *social individuals*, with the same ontological status as individual human beings but operating at larger spatio-temporal scales. Like organisms or species these larger social individuals are products of concrete historical processes, having a date of birth and, at least potentially, a date of death or extinction. And like organisms and species, the relations between individuals at each spatio-temporal scale is one of parts to whole, with each individual emerging from the causal interactions among the members of populations of smaller scale individuals. Although the details of each individuation process need to be described in detail, we can roughly say that from the interactions among individual decision-makers, institutions emerge; from interactions among institutions, cities emerge; and from urban interactions, nation states emerge.[1] The population serving as substratum for the emergence of a larger whole may be very heterogeneous or, on the contrary, highly homogeneous. But even in those cases where the degree of homogeneity at different scales is high enough to suggest the existence of a single 'culture' or 'society', the temptation to postulate such totalities must be resisted, and the degree of homogeneity which motivated such postulation must be given a concrete historical explanation.

Thus far I have used the term 'science' as if its use was unproblematic, but given the requirements of a flat ontology it is clear that this term should not be used since it refers to an abstract totality, and moreover, to a totality defined by an essence. Instead, we must strive to identify the specific processes which have given rise to *individual scientific fields*, which like any other individual, must be conceived as composed of populations of entities at a smaller scale. In the case of the field of classical mechanics, for example, these components are, roughly: populations of mathematical models and techniques for the individuation of predictions and explanations; populations of phenomena produced in laboratories and populations of machines and instruments which individuate and measure those phenomena; populations of experimental skills, theoretical concepts and institutional practices. Like an organic species, the degree to which an individual scientific field has a well-defined identity will depend on contingent historical facts such as its degree of internal homogeneity and its degree of isolation from other fields. Similarly, the degree to which several fields resemble each other should be given a historical explanation, such as one field serving as exemplar for the construction of another, or the export of instruments and techniques from one field to another, or the sharing of institutional components among different fields. This way the question of whether there is such a thing as 'science' in general becomes an empirical question, one which, I believe, should receive a negative answer. Many contemporary analysts do indeed seem to think that, as a matter of empirical fact, science displays a deep and characteristic *disunity*.[2]

In the first part of this chapter I would like to develop the ideas needed to think about individual scientific fields, using classical mechanics as a concrete example, but also to review some of the traditional philosophical obstacles which have historically prevented a correct assessment of the disunity, heterogeneity and divergent development of 'science'. At this point it should come as no surprise that in my view the main obstacle has been the entrenchment of essentialist and typological thought in philosophical studies of scientific practice. Many philosophers in the past have taken the essence of classical mechanics to be its *exceptionless laws*. This is particularly true when fundamental laws, such as Newton's laws, are viewed as general truths from which everything else *follows mechanically*, that is, by simple logical deduction. When species are viewed not as individual entities but as general categories, the productive or genetic processes which yield these individuals tend to be ignored. Similarly, the view of laws as general truths has tended, historically, to eliminate from philosophical discussion the *productive or genetic* connections involved in the physical processes governed by those laws.

More specifically, the essentialist view of laws has concealed the productive power of *causal connections*, that is, the fact that events acting as causes actually produce their effects. Contrary to a popular misconception, philosophical approaches to scientific practice have thrived, from the seventeenth century on, in a world devoid of causes and ruled exclusively by laws stating *constant regularities*.[3] Part of what made possible the replacement of causes by laws was a view of causality as an inherently *linear relation*, such that, given a particular cause, the same effect was bound to be produced. Clearly, if causality always exhibited this simple form, if effects always followed mechanically and necessarily from their causes, postulating a separate productive power of causes distinct from the exceptionless laws governing their operation would be redundant. But more complex forms of causality do exist, nonlinear and statistical causality, for instance, and these are involved in all the intensive production processes which I have described in previous chapters. Hence a crucial task for a Deleuzian epistemologist involves rescuing these genetic links between events from the limbo where general laws have cast them.

Besides concealing productive relations behind static categories, the traditional philosophical approach to laws may be criticized for subordinating mathematical models to *linguistic statements*. Much of what I have argued in this book depends on treating mathematical models in their specificity, that is, as displaying a certain *behaviour* which is crucial for their successful application to scientific tasks. The most obvious example is the tendency of solutions to an equation to approach an attractor, a tendency which is not displayed by linguistic translations of the content of the equation but which depend on the specific mathematical form of both the equation and the operators that act on it. Thus, a second task for a Deleuzian epistemologist is to rescue models and their dynamic behaviour from static linguistic renderings of laws. These two related errors, *elimination of causes and subordination to language* (and deductive logic) are the basic characteristic of essentialist approaches to classical physics, and their criticism will form the subject matter of the first section of this chapter. Let me begin with the dismissal of productive causes in favour of constant regularities. As the philosopher of science Ian Hacking puts it:

> Hume notoriously taught that cause is only constant conjunction. To say that A caused B is not to say that A, from some power or character within itself, *brought about* B. It is only to say that things of type A are *regularly followed* by things of type B . . . Hume is in fact not responsible for the widespread philosophical acceptance of a constant-conjunction attitude towards causation. Isaac Newton did it, unintentionally. The greatest triumph of the human spirit in Hume's day was held to be the Newtonian theory of gravitation . . . Immediately before Newton, all progressive scientists thought that the world must be understood in terms of mechanical pushes and pulls. But gravity did not seem 'mechanical', for it was action at a distance . . . For empirically minded people the post-Newtonian attitude was, then, this: we should not seek for causes in nature, but only regularities . . . The natural scientist tries to find universal statements – theories and laws – which *cover all phenomena as special cases*. To say that we have found the explanation of an event is only to say that the event can be *deduced from a general regularity*.[4]

Hacking argues that this elimination of productive causes in favour of statements of regularities (and deductive relations between those statements) is characteristic not of physics in general, but only of philosophies of physics which concentrate exclusively on the theoretical component of a field at the expense of its experimental component. The day to day practice of experimental physicists, consisting as it does in specific *causal interventions* in reality, is much too rich and complex to be reduced to logical relations between statements. The experimentalist is directly involved in productive relations, whether these involve the creation of an apparatus to individuate phenomena or the use of instruments to produce individual measurements of properties of those phenomena. It is only theory-obsessed philosophies, whether held by physicists or professional philosophers, that can afford to forget about causal connections and concentrate exclusively on logical relations. The ultimate expression of this essentialist stance is a model of scientific explanation developed in the twentieth century which takes the

Humean reduction of causes to linguistic statements of regularities to an extreme.

In this epistemological theory, known as the *deductive-nomological* approach, scientific explanations are treated as logical *arguments* consisting of several propositions, one of which must be an exceptionless law. The term 'proposition' refers to the meaning of declarative sentences, that is, to what two sentences in different languages, expressing the same state of affairs, have in common. In this model, to explain a particular laboratory phenomenon is to deduce it from a set of propositions: from a linguistically stated law (such as 'two bodies are gravitationally attracted to each other in direct proportion to the product of their masses, and in inverse proportion to the square of their distance') and a set of propositions describing initial (and other) conditions, we derive further propositions which may be treated as predictions to be tested for their truth or falsity in a laboratory. If the behaviour of the phenomenon conforms to these predictions we can claim to have explained it, not, of course, by having given causal mechanisms for its production, but in the way one explains things in a typological approach: *subsuming it as a particular case under a general category*. Although hardly any working physicist would accept that his or her complex explanatory practices are captured by this simplistic theory, the deductive-nomological approach has dominated much of twentieth-century philosophy of science and continues to have many defenders in this field.[5]

When one accepts this model of explanation the structure of the theoretical component of a scientific field takes the form of an *axiomatic*: from a few true statements of general regularities (the axioms) we deduce a large number of consequences (theorems) which are then compared to the results of observations in the laboratory to check for their truth or falsity. Given that deduction is a purely mechanical way of *transmitting* truth or falsity, it follows that whatever truth one may find in a theorem must have *already been contained* in the axioms. It is in this sense that axioms are like essences. To counter this essentialist conception, a new generation of philosophers has developed an alternative characterization of what a theory is, reintroducing productive causal relations as an integral part of explanations, as well as rejecting the linguistic characterization of explanatory practices. In the view of these philosophers, explanations, rather than being simply logical arguments, involve a complex use of mathematical models of different types: models of general relations, models of particular experimental situations, as well as statistical models of the raw data gathered in laboratories. One of the defenders of this new view, Ronald Giere, puts it this way:

> Even just a brief examination of classical mechanics as presented in modern textbooks provides a basis for some substantial conclusions about the overall structure of this scientific theory as it is actually understood by the bulk of the scientific community. What one finds in standard textbooks may be described as a cluster (or cluster of clusters) of models, or, perhaps better, as a *population of models consisting of related families of models*. The various families are constructed by combining Newton's laws of motion, particularly the second law, with various force functions – linear functions, inverse square functions, and so on. The models thus defined are then multiplied by adding other force functions to the definition. These define still further families of models. And so on.[6]

Giere emphasizes the point that, despite the fact that some members of this population of models (Newton's laws of motion) serve to generate the various branching families, the relation between a fundamental model and those derived from it is *not* like that between axioms and theorems. Far from being a mechanical process of deduction, the complex modelling practices which have historically generated these families involve many judicious approximations and idealizations, guided by prior achievements serving as exemplars.[7] I will return to this question in a moment but for now I would like to add that the basic idea of thinking of a physical theory as a population of models fits well with the ontological stance I am defending. Such a population is easily conceived as the product of a historical accumulation, subject to all the contingencies of such historical processes, and hence with no pretence that it represents a complete or final set of models. At any rate, the completeness or closure of the set becomes an empirical matter, not something to be assumed at the outset as in axiomatic treatments. Certain populations (like those of the sub-field of classical dynamics) may seem to have achieved closure at a certain point in history only to be reopened later giving rise to a new round of accumulation, as when computer-driven developments in nonlinear dynamics reopened what was widely considered a closed field. As Ilya Prigogine puts it: 'Unfortunately, many college and university text-books present classical dynamics as a closed subject . . . [but] in fact, it is a subject in rapid evolution. In the past twenty years, [physicists] have introduced important new insights, and further developments can be expected in the near future.'[8]

The philosopher of science Nancy Cartwright has proposed a set of distinctions that may be used to describe the non-axiomatic structure of this population of models. Somewhat paradoxically, she argues that the fundamental laws of physics, those laws which in axiomatic treatments are assumed to be the highest truths, are indeed *false*. The laws of physics lie, as she puts it. What she means is that a fundamental law *achieves its generality at the expense of its accuracy*. A fundamental law, such as Newton's law of gravity, is strictly speaking true only in the most artificial of circumstances, when all other forces (like electromagnetic forces) are absent, for instance, or when there is no friction or other nonlinearities. In other words, the law is true but only if a very large 'all other things being equal' clause is attached to it.[9] We can compensate for the shortcomings of fundamental laws by adding to the basic equation other equations representing the action of other forces or the complex causal interactions between forces. But then we lose the generality that made the original law so appealing to essentialists. The model becomes more true, describing with increased accuracy the structure of a given experimental phenomenon, but for the same reason it becomes less general. In short, for Cartwright the objective content of physics does not lie in a few fundamental laws, but in a large number of *causal models* tailored to specific situations. (Giere does not speak of 'causal models' but of 'hypotheses' linking the abstract models and the world, but the overall thrust of his argument is very close to that of Cartwright.[10])

The essentialist may object that, given that the specialized causal models are derived from the fundamental laws, they must inherit whatever degree of truth they have from those laws. But Cartwright (like Giere) replies that this oversimplifies the description of the modelling practices of real physicists. The causal models are not logically deduced from the general laws, but constructed from them using a complex set of approximation techniques which cannot be reduced to deductive logic. As Cartwright says, the content of the causal models 'we derive is not contained in the fundamental laws that explain them.'[11] In short, the population of models which constitutes the theoretical component of classical mechanics may be roughly divided into two sub-populations: a large number of causal models closely adapted to particular experimental situations, and a few fundamental models corresponding to basic laws from which branching families of other abstract models are derived. This breakdown of the contents of the population leaves out a different class of models, *statistical models of the data*, which is also very important. Positivist philosophers used to think that the predictions deduced from axioms and auxiliary premises (those describing initial conditions) were confronted directly with observations in a laboratory, that is, with raw data. But for at least two hundred years physicists have used statistical models to organize the raw data, and, in particular, to attempt to capture the *distribution of measurement errors* in the data.[12] Beside ignoring this important kind of model, the positivist emphasis on 'the observer' is misleading because it reduces to a subjective phenomenon what is in fact a complex practice of data gathering, involving not passive observations but active causal interventions.

Leaving aside the experimental side for a moment, what are we to think of the few fundamental laws? Is it correct to say that they lie, or is it more accurate to say that they are not the kind of mathematical objects that can be true or false? Cartwright suggests that the function of these laws is to *unify and organize* the rest of the population.[13] This is, I believe, a step in the right direction but we cannot simply take this unifying capability for granted; we must at least try to account for it. Historically, the unification of the different branches of classical mechanics was achieved by a series of physicists and mathematicians, starting with the work of Leonard Euler in the mid-eighteenth century and culminating a hundred years later with that of William Hamilton. It may be said that, together with other important figures (Maupertuis, Lagrange), these scientists transformed classical mechanics from a science of forces to one of *singularities*. In the words of the historian Morris Kline:

> Hamilton's principle yields the paths of falling bodies, the paths of projectiles, the elliptical paths of bodies moving under the law of gravitation, the laws of reflection and refraction of light, and the more elementary phenomena of electricity and magnetism. However, the chief achievement of the principle lies in showing that the phenomena of all these branches of physics satisfy *a minimum principle*. Since it relates these phenomena by a common mathematical law, it permits conclusions reached in one branch to be reinterpreted for another. Hamilton's principle is the final form of the least-action principle introduced by Maupertuis, and because it embraces so many actions of nature it is the most powerful single principle in all of mathematical physics.[14]

The history of minimum principles, the idea that, for example, light moves along the path that *minimizes* travelling distance, is indeed a long one having roots in Greek antiquity and medieval philosophy.[15] In the seventeenth century, Pierre de Fermat created the first application of this idea in the context of early modern physics, the Principle of Least Time governing the behaviour of light in geometrical optics. For much of its history the principle carried strong theological overtones as it was associated with the belief that it reflected the economy of thought of a Creator. Maupertuis even went as far as to state that his Least Action principle was the first scientific proof of the existence of God. Eventually the theological connection was lost, as scientists realized that what mattered was not the ideological interpretation but the mathematical technology that was created around these ideas: *the calculus of variations*. This was the first technology ever to deal directly with singularities and it rivals in importance, as far as its effects on nineteenth- and twentieth-century physics, the other mathematical fields I have discussed in this book (differential geometry, group theory).[16]

One way of looking at the calculus of variations is as a novel way of *posing mechanical problems*. Instead of looking at a problem in physics as a problem of the causal effects of forces, one looks at it as a problem of finding, among the many possible processes that may change a physical system from one state to another, the actual process. More exactly, the techniques developed by Euler and Lagrange allow the construction of a set of possibilities (for example, a set of possible paths which a light ray might follow) and supply the resources needed to sort these possibilities into two groups, one of *ordinary* and one of *singular* cases. As it happens, the results of experiments show that the singular cases (a minimum or a maximum) are the ones that are in fact actualized.[17] Although the singularities uncovered by the calculus of variations are not, strictly speaking, attractors, its creators did seem to think that they played a similar role. Attractors are described as defining the long-term tendencies of a system, that is, the state the system will adopt if we wait long enough to allow it to settle down. This emphasis on the final state suggests that one way to look at the difference between attractors and causes is through the old distinction made by Aristotle between *final and efficient* causes. Euler himself, when introducing his variational technology, used this Aristotelian distinction:

> Since the fabric of the universe is most perfect, and is the work of a most wise Creator, nothing whatsoever takes place in the universe in which some relation of maximum and minimum does not appear. Wherefore there is absolutely no doubt that every effect in the universe can be explained as satisfactorily from final causes, by the aid of the method of maxima and minima, as it can from the effective causes themselves . . . Therefore, two methods of studying effects in Nature lie open to us, one by means of effective causes, which is commonly called the direct method, the other by means of final causes . . . One ought to make a special effort to see that *both ways of approach to the solution of the problem* be laid open; for thus not only is one solution greatly strengthened by the other, but, more than that, from the agreement between the two solutions we secure the very highest satisfaction.[18]

In a Deleuzian ontology final causes would have to be replaced by quasi-causes in order to avoid ascribing teleological or goal-seeking behaviour to physical systems. But the important point for my argument is that it was precisely the ability to pose a problem not in terms of specific efficient causes (forces) but in a way which by-passed causal details, that allowed the variational version of classical mechanics to play a unifying and organizational role in the population of models. The singularities which the calculus of variations uncovered represented, in my terminology, a mechanism-independent reality. On the other hand, as Euler himself acknowledged, this method was complementary not exclusive to the causal one. One may know that a given classical mechanical process will tend to minimize some quantity, but the full explanation of the process will also involve a correct description of the causal mechanisms that achieve such minimization. This other task, however, must be performed by other models, less general and more specifically tailored to the details of an experimental situation.

To summarize the argument of this section, far from being mere mathematical expressions of linguistic truths, laws must be viewed as models from which the mathematical form cannot be eliminated. The unification brought about by the calculus of variations, for example, cannot be understood otherwise since its techniques do not apply to linguistically stated laws. These irreducibly mathematical models form a growing and heterogeneous population, some members of which carry causal information about productive relations between events, others embody *quasi-causal* relations between singularities. In other words, the population of models making up the theoretical component of classical mechanics contains a large number of specific causal models which are the vehicles for truth (the part of the population that *interfaces with the actual world*), and fewer models which do not refer to the actual world (hence are neither true nor false) but which nevertheless do *interface with the virtual world* by virtue of being well-posed problems. For Deleuze a problem is defined precisely by a distribution of the singular and the ordinary, the important and the unimportant, the relevant and the irrelevant. A well-posed problem gets these distributions right, and a solution always has the truth it deserves according to how well specified the corresponding problem is.[19] In these terms Newton's achievement would consist not in having discovered general truths about the universe, but in having *correctly* *posed an objective problem* defined by the simplest distribution of singularities (unique minima or maxima). This interpretation preserves the objectivity of Newton's laws but it deflates his achievement somewhat, in the sense that, if the insights of nonlinear dynamics about multiple attractors are correct, the single minimum problem is not the most general one.

This conclusion assumes, however, that the traditional axiomatic approach to physics can be replaced by a *problematic approach*, that is, that problems can replace fundamental law statements. But this replacement needs more justification given that it goes against the grain of the traditional ontology of physics. Hamilton's Least Action principle, for example, is still interpreted by most physicists as an axiom expressing a general truth from which many particular truths in physics follow mechanically. As Morris Kline puts it:

> To the scientists of 1850, Hamilton's principle was the realization of a dream . . . From the time of Galileo scientists had been striving to deduce as many phenomena of nature as possible from a few fundamental physical principles . . . Descartes had already expressed the hope that all the laws of science would be derivable from a single basic law of the universe.[20]

And, I should add, this hope for a single law statement from which everything else follows has displayed a considerable resilience and longevity, still animating the dream for a final theory among some contemporary physicists. Therefore the task for the next section of this chapter will be to describe in more detail the *extra-propositional and sub-representative* nature of these distributions of the important and the unimportant which are supposed to replace law statements as well as essences. In Deleuze's words:

> It will be said that the essence is by nature the most 'important' thing. This, however, is precisely what is at issue: whether notions of importance and non-importance are not precisely notions which concern events or accidents, and are much more 'important' within accidents than the crude opposition between essence and accident itself. The problem of thought is tied not to essences but to the

> evaluation of what is important and what is not, to the distribution of the singular and regular, distinctive and ordinary points, which takes place entirely within the unessential or within the description of a multiplicity, in relation to the ideal events that constitute the conditions of a problem.[21]

I will focus first on a particular kind of problem, *explanatory problems*, to show the role which the causal and the quasi-causal play in the explanation of physical phenomena. As Ian Hacking has argued, the same positivist biases which promote the belief that causality is not an objective relation also promote the downplaying of explanation as an epistemological activity, that is, promote the positivist thesis that 'explanations may help organize phenomena, but do not provide any deeper answer to *Why* questions . . .'[22] To the non-positivist philosophers who are reviving the study of causality, on the contrary, questions as to *why a phenomenon occurs* are crucial since they require as answers more than a mere description of regularities. Answering a Why question typically demands supplying a causal explanation, perhaps in the form of a causal model of a mechanism. In addition, I will argue that these questions sometimes require supplying a quasi-causal factor to explain whatever regularity there is in the behaviour of the mechanisms, that is, to capture the mechanism-independent aspect of the phenomenon.[23] Despite the fact that questions and answers are, indeed, linguistic entities, Why questions involve as part of the conditions that make them *answerable*, or well-posed, a non-linguistic or extra-propositional aspect which is properly problematic: a distribution of the relevant and the irrelevant. Let me begin this new section with a quote from the philosopher Alan Garfinkel who has developed an original approach to these matters:

> When Willie Sutton was in prison, a priest who was trying to reform him asked him why he robbed banks. 'Well,' Sutton replied, 'that's where the money is.' There has been a failure to connect here, a failure of fit. Sutton and the priest are passing each other by . . . Clearly there are different values and purposes shaping the question and answer. They take different things to be *problematic* or stand in need of explanation. For the priest, what stands in need of

> explanation is the decision to rob at all. He does not really care what. But for Sutton, that is the whole question. What is problematic is the choice of what to rob.[24]

Garfinkel suggests that requests for explanations may be modeled as questions having the form 'Why did event X (as opposed to Y or Z) occur?' with the clause in parenthesis constituting what he calls a *contrast space*. The misunderstanding between the thief and the priest in his example is due to the fact that each is using the same question but with different contrast spaces. While for the thief the question is 'Why rob banks?' (as opposed to gas stations or retail stores) for the priest the question is 'Why rob banks?' (as opposed to making an honest living). The thief's answer is indeed a true answer, but as far as the priest is concerned, it is an *irrelevant* answer, a fact that suggests that the relevancy and validity of an explanation is relative to a particular contrast space. These spaces capture both what is *presupposed* in a question (*Given* that one must rob, why banks?), and hence considered to be not in need of explanation, as well as the relevant *explanatory alternatives*. Garfinkel argues that characterizing contrast spaces involves going beyond the resources of language, even in cases (like the thief and priest example) where the situation is mostly linguistic. As he puts it:

> These contrast spaces are still not well-understood objects. Their structure is not readily identifiable with any of the traditional objects of logic, for example. They have some similarities with 'possible worlds', for instance, but they are not simply spaces of possible worlds. They are more like equivalence classes of possible worlds (under the relation 'differs inessentially from') with almost all possible worlds excluded altogether from the space. (Contrast spaces are typically quite small.) . . . Basically, these spaces are similar to what physicists call *state spaces*. A state space is a geometric representation of the possibilities of a system; a parametrization of its states, a display of its repertoire.[25]

I have already discussed why linguistically specified possible worlds fail to break with essentialism, and how bringing in mathematical entities (such as state spaces and their attractors) can eliminate the need to characterize relevant alternatives (equivalence classes) through relations like 'differs inessentially from'. In a typical nonlinear state space, subdivided by multiple attractors and their basins of attraction, the structure of the space of possibilities depends not on some extrinsically defined relation (specifying what is an inessential change) but on the *distribution of singularities* itself. The trajectories in state space, defining possible sequences of states, are spontaneously broken into equivalence classes by the basins of attraction: if the starting point or initial condition of two different trajectories falls within a given basin both trajectories are bound to end up in the same state, and are equivalent in that respect. Garfinkel, in fact, acknowledges the role which attractors may play in structuring the contrast spaces of physical and biological explanations. As he says, 'What is necessary for a true explanation is an account of how the underlying space is partitioned into basins of irrelevant differences, separated by ridge lines of critical points.'[26]

How does a distribution of singularities objectively define the correctness or truth of a problem? The answer is that, as Deleuze says, 'there are problems which are false through indetermination, others through overdetermination'.[27] In other words, a problem may be false or badly posed if the alternatives which structure a contrast space are *too sharply defined*, since in that case the validity of the explanation becomes too dependent on the occurrence of precisely those events (overdetermination). On the contrary, the problem may fail to be true if it is *so vaguely defined* that it is impossible to tell whether an actually occuring event belongs to one or another of the relevant alternatives (indetermination). Let me give an example of a problem which is not well posed due to its conditions being overdetermined. Garfinkel illustrates this case with a well-known ecological phenomenon, the rhythmic or periodic changes in the overall numbers of coupled populations of prey and predators (rabbits and foxes, in his example). As the population of rabbits increases the foxes' numbers also increase due to the extra available food. But at some point, there are too many foxes so that the population of rabbits is reduced. This, in turn, brings down the number of foxes, which allows the rabbit population to recover and start the cycle again. This cyclic behaviour of the coupled populations is what is *ecologically problematic* about the situation, that is, what demands an explanation.[28]

We may pose the problem in two alternative ways, one at the level of interactions between individual rabbits and foxes, which gives an overdetermined contrast space with too many alternatives, and another at the level of the overall density of the populations yielding a well-posed problem. To put this in linguistic terms, if we posed the problem 'Why was this rabbit eaten?', one answer may be framed at the population level (because of the large number of foxes) and another at the organism level (because it passed through the capture space of a specific fox at a specific time). In other words, one problem is 'Why was this rabbit eaten (as opposed to not eaten)?' while the other is 'Why was this rabbit eaten (by this particular fox as opposed by this or that other fox)?'. The second contrast space includes much that is *irrelevant* to the question since, given a high enough density of foxes, if this rabbit had not been eaten by this fox it would have been eaten by another. In other words, there is a certain degree of *redundant causality* operating at the micro-level, so that framing the question at that level is bound to yield the wrong distribution of the important and the unimportant.[29] The second way of framing the question is, as Garfinkel says, *explanatorily unstable*:

> The general criterion in the cases we are dealing with is that an object of explanation should be chosen which is stable under small perturbations of its conditions. In the whole microspace of the foxes and rabbits system there is a point corresponding to the death of that rabbit at the hands of that fox, at that place and time, and so forth. Now imagine a kind of mesh laid over the space, which determines what is to count as relevantly the same as that event. [This is, in effect, the contrast space of the explanation.] If the mesh is very fine, the resulting causal relations will be relatively unstable. Perturbing the initial conditions slightly [say, making the rabbit pass not so near that fox] will result in a situation which is different, inequivalent. [The rabbit not being eaten by that fox.] If however, we choose a mesh large enough (and cleverly enough) we can capture a stable relation, like the one between high fox populations

and high likelihoods of rabbit deaths. [Where changing the path of the rabbit still results in its being eaten but by another fox.[30]]

Using the notion of explanatory stability, Garfinkel develops an application of contrast spaces to differentiate the validity of explanations operating at different scales of reality. In the context of a flat ontology of individuals this differentiation is crucial since we would like to have objective criteria to tell when an explanation is valid at the level of individual organisms, for example, and when we need an explanation at the spatio-temporal scale of an individual species. In the example just mentioned, a population-level intensive property (density) can furnish a more stable explanation of the cyclic behaviour of the prey–predator system than an organism-level one. Similarly for explanations of social phenomena, some will be adequate at the scale of individual subjects, others will serve to answer Why questions at the scale of individual institutions, and yet others will capture the relevant causal effects of individual cities or nation states.

In short, causal problems should be framed at the correct level given that each emergent level has its own causal capacities, these capacities being what differentiates these individuals from each other. But what about quasi-causal factors, how do they affect the success or failure of explanations? To return to our example, if the properties of the cyclic dynamics of the prey–predator system, the duration of the cycle, for example, are not stable, that is, if external shocks can easily change this duration, then there is no need for quasi-causal factors. But, on the other hand, if such shocks only temporarily change the duration and the cycle spontaneously returns to its original period, then there will be an aspect of the dynamics not explained by the causal model, a mechanism-independent aspect which still demands explanation. Population biologists have in fact observed such stable or robust cycles both in the field and in the laboratory, a fact that has influenced the introduction of attractors as part of their explanatory models.[31]

I should emphasize that, despite my choice of example, there is nothing specifically biological about this argument. The exact same ideas apply to systems of causally interacting populations of inorganic entities. I have mentioned several times the regimes of flow of convection and turbulence. When explaining such phenomena one has to frame the problem at the correct level so as not to introduce irrelevant differences. Given a convection cell and its coherent cyclic behaviour, for example, there are a large number of micro-causal descriptions (of individual molecules colliding with one another) which are irrelevant to its explanation. In other words, there is a large causal redundancy at the micro-level, with many collision histories being compatible with the same macro-level effect: a coherent cyclic flow pattern. Here the proper level of explanation will involve macro-causal factors: temperature and density gradients, competition between gravitational and viscous forces, and so on. Moreover, the existence of critical thresholds recurring at regular values for the gradients (structural instabilities) and the robustness of the recurring flow patterns to shocks (asymptotic stability) will call for additional quasi-causal factors: bifurcations and periodic attractors. (Or, in the case of turbulence, chaotic attractors.)

Let me pause for a moment to bring the different lines of the argument together, and then link the conclusions to those reached in previous chapters. I argued first that the axiomatic approach to classical mechanics, exemplified here by the deductive-nomological model of explanation, views laws as the main carriers of objective truth, a truth which is then mechanically transmitted to theorems via deduction. Explaining a given phenomenon is modelled as a logical argument, subsuming the truth of a theorem describing the phenomenon under the truth of a law. An alternative approach, a problematic approach, rejects the idea that fundamental laws express general truths and views them instead as posing correct problems. Problems are defined by their presuppositions (what is not being explained) as well as by their contrast spaces (defining what the relevant options for explanation are). In the particular case of explanations in classical physics, where the laws are expressed by differential equations, the presuppositions are the physical quantities chosen as relevant degrees of freedom (which make up the different dimensions of a state space) while the contrast space is defined by a distribution of singularities in state space, that is, by a particular partition of possibilities into distinct basins of attraction. As the example of hydrodynamic regimes of flow shows, however, a contrast space may have a more complex structure: a cascade of symmetry-breaking bifurcations may link several such spaces in such a way that *a problem may gradually specify itself* as the different contrast spaces it contains reveal themselves, one bifurcation at a time.

These conclusions are directly connected with the ontological ideas I explored before, but to see this connection we must expand the conception of problems beyond those involving scientific explanations. In Deleuze's approach the relation between well-posed explanatory problems and their true or false solutions is the epistemological counterpart of the ontological relation between the virtual and the actual. Explanatory problems would be the counterpart of virtual multiplicities since, as he says, 'the virtual possesses the reality of a task to be performed or a problem to be solved'.[32] Individual solutions, on the other hand, would be the counterpart of actual individual beings: 'An organism is nothing if not the solution to a problem, as are each of its differenciated organs, such as the eye which solves a light problem.'[33] Let me illustrate this idea with a simple example I used before: soap bubbles and salt crystals, viewed as the emergent result of interactions between their constituent molecules. Here the problem for the population of molecules is to *find (or compute its way to) a minimal point* of energy, a problem solved differently by the molecules in soap films (which collectively solve a minimization problem stated in surface-tension terms) and by the molecules in crystalline structures (which collectively solve a bonding energy problem). It is as if an ontological problem, whose conditions are defined by a unique singularity, 'explicated' itself as it gave rise to a variety of geometric solutions (spherical bubbles, cubic crystals).[34]

This intimate relation between epistemology and ontology, between problems posed by humans and self-posed virtual problems, is characteristic of Deleuze. A true problem, such as the one which Newton posed in relatively obscure geometric terms and which Euler, Lagrange and Hamilton progressively clarified, would be *isomorphic* with a real virtual problem. Similarly, the practices of experimental physicists, which include among other things the skilful use of machines and instruments to individuate phenomena in the laboratory, would be isomorphic with the intensive processes of individuation which solve or explicate a virtual problem in reality. This conception of the task of theoretical and experimental physicists runs counter to the traditional realist picture which views it as that of producing a corpus of linguistic propositions expressing true facts which mirror reality. In this old and tired view, the relation between the plane of reality and that of physics would be one of similarity. Yet, as Deleuze says, there is 'no analytic resemblance, correspondence or conformity between the two planes. But their independence does not preclude isomorphism . . .'[35] Indeed, as I said in the conclusion of the previous chapter, there is a further isomorphism which must be included here: the philosopher must become isomorphic with the quasi-causal operator, extracting problems from law-expressing propositions and meshing the problems together to endow them with that minimum of autonomy which ensures their irreducibility to their solutions.

In the second part of this chapter I would like to discuss the details of these isomorphisms, one involving the experimental, the other the theoretical component of classical physics. This will imply dealing with both sides of the relation, that is, not only the laboratory and modelling practices of physicists, but also the *behaviour* of the material phenomena and machinery which inhabit laboratories as well as the *behaviour* of the mathematical models with which the theorist makes contact with the virtual. I will begin with a discussion of how the capacity of material and energetic systems to self-organize and self-assemble, a capacity which reveals a properly problematic aspect of matter and energy, is concealed when physicists or philosophers focus on *linear causality* at the expense of more complex forms. Yet, I will also argue that even if a material system under study has been fully linearized and domesticated, the causal relations *between experimentalist, machines, material phenomena and causal models* are still nonlinear and problematic. Indeed, the physics laboratory may be viewed as a site where heterogeneous assemblages form, assemblages which are isomorphic with real intensive individuation processes.

I will then move on to questions of quasi-causality and compare Deleuze's epistemological approach to state space, an approach that emphasizes the singularities that define the conditions of a theoretical problem, to those of analytical philosophers who stress the solutions to the problem, that is, who see not the singularities but the trajectories in state space as the conveyors of theoretical knowledge.

While trajectories bear a relationship of geometric similarity to quantities measured in the laboratory, the singularities defining a problem in physics are isomorphic with those defining the conditions of a virtual multiplicity. Here too, I will argue that it is the behaviour of *linear equations* that conceals the problematic aspect of mathematical models. In short, whether we are dealing with causes or quasi-causes, with experimental or theoretical physics, the crucial task is to avoid *the subordination of problems to solutions* brought about by the search for simple linear behaviour. Let me begin with a quote from the philosopher of science Mario Bunge on the conception of matter brought about by excessive concentration on linear causes:

> Before atoms, fields and radioactivity became pieces of common knowledge, even scientists could be found that shared the belief that 'brute matter' is a *homogeneous, unorganized and quiescent stuff entirely lacking spontaneity* – the matter, in short, dreamt by immaterialist philosophers. From the fact that every experiment is an encroachment on matter, they jumped to the Aristotelian conclusion that matter is nothing but the *barren receptacle of forms* – a belief still held in esteem by those quantum theorists who hold that it is the experimenter who produces all atomic-scale phenomena.[36]

And, I could add, still held in esteem by those critics of science who think that all phenomena are socially constructed. This conception of matter as basically inert is directly linked to the defining characteristics of classical causality, the most important of which is the simple *additivity* of the effects of different causes. This apparently innocent assumption is indeed full of consequences, some of which are fatal for the philosophical project which I have sketched in these pages. In particular, a flat ontology of individuals assumes that, at every spatio-temporal scale, there are properties of a whole which cannot be explained as a mere *sum* of the properties of its component parts, but which *emerge* from their causal interactions. Without stable emergent properties, and the novel causal capacities these, in turn, give rise to, the concept of a larger scale individual collapses.

The idea of additive causes became dominant in physics for the apparent simplicity with which it endows a system under study.[37] In traditional laboratory practices, *isolating and separating* causal influences in order to study them, is an indispensable operation. Although perfect isolation is indeed a myth (as is its opposite, the complete interdependence of every event in the universe), relatively linear causal chains may be created in the laboratory by singling out at every link a particular causal factor or one of its consequences, and ignoring the rest. As Bunge notes, this procedure may be ontologically objectionable but is in many cases methodologically indispensable.[38] It is hard to visualize, for example, how else classical physics could have taken off without introducing this simplification in the causal behaviour of matter. Similarly, the very possibility of performing a causal *analysis* seems to call for a separation of different factors followed by a simple additive synthesis. Early physicists can hardly be criticized for assuming additivity when it was a pragmatic precondition for analysis. What is objectionable, however, is the reification of this pragmatic constraint into a principle of nature.

There are several components of the classical conception of causality which account for the property of additivity: *uniqueness, necessity, unidirectionality and proportionality*. Together, these four components account for the radical impoverishment of material agency typical of clockwork world views. Let me briefly explain what each of these components involves starting with uniqueness: *the same cause leads to the same effect*. Uniqueness is to be contrasted to two alternatives, one in which several different causes lead to the same effect, and the other in which the same cause may lead to a variety of different effects. Bunge gives as an example of the former the production of heat (which as an effect may be produced by a variety of causes: friction, combustion, nuclear chain reactions, microwaves) and as illustration of the latter the action of certain hormones (such as *auxin*) which 'is growth stimulating when located in the tips of a plant but inhibits growth when placed in the roots'.[39] In either case, without a unique connection between cause and effect, simple additivity is compromised: if several different causes can independently produce an effect, adding a second cause after one is already operating may have no effect at all; and conversely, if the same cause may produce different effects depending on circumstance, two occurrences of this cause will not necessarily add up to a sum of their independent effects.

The second characteristic of classical causality is necessity: the same cause will produce the same effect *always, without exception*. The alternative here is to eliminate the necessary linkage and replace it with one of *enhanced probability*: the occurrence of a cause does not necessarily imply the occurrence of its effect only an increase in the probability of its occurrence.[40] This type of probabilistic causality is often illustrated with examples like 'cigarette smoking causes cancer' as applied to populations of human beings. The toxins contained in cigarettes do increase the probability of disease but in different ways for different people, with the consequence that not every smoker actually dies of cancer. While necessity implies additivity, enhanced probability does not: cigarette smoking, considered as a causal process, carries with it probability distributions for various types of interactions with the human body, and these propensities to interact do not add up in a simple way at the level of the entire population.[41]

The third and fourth characteristics are uni-directionality and proportionality. In a linear causal chain, *effects do not react back on their causes*, that is, in these chains causal influence is not reciprocal. Yet, even in classical mechanics the uni-directionality of causes is only an approximation, since every action involves a reaction, however small. If the feedback of effects on their causes cannot be eliminated (by making it so weak that it becomes irrelevant) the absence of uni-directionality may imply the failure of proportionality: *small causes always produce small effects*. In other words, without feedback the intensity of the effect will tend to be proportional to that of the cause, while in the presence of reciprocal interaction causal influence may be reduced or increased. There are cases where the effect acts so as to inhibit the cause (negative feedback), in which case large causes may have relatively small effects. In other cases (positive feedback) the effect amplifies the cause, so that small causes may have large effects.[42] It seems clear that a failure of uni-directionality and proportionality also compromises additivity: given a set of causes which interact with their effects, some being inhibited others stimulated by them, their joint effect will be a simple sum only in the improbable case where the inhibitions exactly cancel out the stimulations.

While these four properties account for additivity, they themselves presuppose another characteristic of classical causality, one inherited from the Aristotelian concept of efficient cause: *externality*. In this view, causes are taken to be external agents operating on relatively passive targets, hence being solely responsible for whatever effects are produced. The previous four traits of linear causality presuppose externality to the extent that they break down precisely when the body being acted upon *ceases to be a mere patient*. A failure of uniqueness occurs whenever one cause can produce several effects depending on the tendencies of the body it acts upon, and similarly for the case in which the same effect can be triggered by a variety of causes. The elimination of necessity in favour of enhanced probability and the different probabilities of achieving an effect which a causal process may transmit also depend on the probabilities to be affected carried by the target of the cause. And, of course, the failure of uni-directionality and proportionality are directly linked to the fact that the bodies acted upon by causes are not passive but can react back and exercise their own causal powers.[43]

The flat ontology of individuals I have defended in these pages depends crucially, as I said, on the elimination of linear causes, or, at least, on cutting them down to size by showing them to be special limiting cases. In this ontology individuals always exist as part of populations in which the most meaningful and relevant causal relations are of the statistical or probabilistic kind. None of these individuals is ever a passive receptacle for external causal influences since their internal causal structure always plays a part in determining the final effect. The lack of uniqueness and uni-directionality is further strengthened by the existence of quasi-causal relations. If the internal dynamic of an individual is such that several alternative stable states are available to it, it is hardly surprising that the same effect (a switch between two attractors, for example) may be brought about by a variety of causes, and conversely, one and the same external cause may trigger different effects depending on how close an individual is to a bifurcation, or to the border of a basin of attraction.

In short, while linear causality makes the response of a material system to an external cause basically *unproblematic* (given the cause, there is nothing else in the effect that demands explanation), nonlinear and statistical causality *re-problematize* material systems, showing them capable of self-organization and self-assembly, with many things left unexplained in the effect after the mere citation of an external cause. In addition, linear and nonlinear causality imply two different models for the relationship between matter and form. Additivity and externality presuppose, as I said, a matter obedient to laws and constituting an inert receptacle for forms imposed from the outside. Matter under nonlinear and non-equilibrium conditions is, on the other hand, intensive and problematic, capable of spontaneously giving rise to form drawing on its inherent tendencies (defined by singularities) as well as its complex capacities to affect and be affected. As Deleuze says, the first model:

> assumes a fixed form and a matter deemed homogeneous. It is the idea of the law that assures the model's coherence, since laws are what submits matter to this or that form, and conversely, realize in matter a given property deduced from the form . . . [But that] model leaves many things, active and affective, by the wayside. On the one hand, to the formed or formable matter we must add an entire energetic materiality in movement, carrying *singularities* . . . that are already like implicit forms that are topological, rather than geometrical, and that combine with processes of deformation: for example, the variable undulations and torsions of the fibers guiding the operations of splitting wood. On the other hand, to the essential properties of matter deriving from the formal essence we must add *variable intensive affects*, now resulting from the operation, now on the contrary, making it possible: for example, wood that is more or less porous, more or less elastic and resistant. At any rate, it is a question of surrendering to the wood, then following where it leads by connecting operations to a materiality instead of imposing a form upon a matter . . .[44]

Although Deleuze is referring here to artisans (carpenters in this example, but also blacksmiths) similar conclusions apply to experimental physicists. As Ian Hacking has forcefully argued, experimental physics, far from being a mere appendage of theoretical physics (supplying tests to confirm or disconfirm predictions from formal models), has in fact a life of its own. For example, the experimentalist must individuate in a stable and repeatable way *laboratory phenomena*.

Rather than being a mere by-product of theoretical knowledge of laws, the individuation of phenomena involves, as Hacking says, 'a keen ability to get nature *to behave in new ways*'.[45] In the traditional interpretation, these material and energetic phenomena were supposed to be unintelligible outside a theoretical framework, but Hacking shows that, on the contrary, laboratory phenomena (such as polarization of light, the photoelectric effect, Brownian motion) typically survive the birth and death of new theories, or what amounts to the same thing, the switching from one to another incommensurable theoretical paradigm. Many times the individuation of a phenomenon not only precedes the development of a theory that will explain it, but it remains in this problematic state, crying out for an explanation, for many decades.[46]

Beside individuating phenomena that may or may not occur naturally, experimental physicists must develop techniques and procedures to isolate, identify and manipulate entities which have been individuated by objective processes occurring outside the laboratory. In this case too, it is a question of connecting operations to a materiality instead of deducing the form of the entities in question from a theoretical law. As Hacking argues, physicists individuate entities like electrons by intervening causally in the world, interacting with real electrons so as to determine their mass (as was done by Thompson in 1897), or their charge (as performed by Millikan around 1908), as well as other of their properties.[47] The individuation of electrons (as well as other formerly theoretical entities) is even more complete when experimentalists move beyond their properties to study their capacities. We learn from electrons, we *acquire expertise* about them, by making them part of heterogeneous assemblages where they affect and are affected by other entities, and it is this causal know-how more than anything related to general laws, which gives us confidence that these individuals actually exist. As Hacking writes:

> There are an enormous number of ways in which to make instruments that rely on the causal properties of electrons in order to produce desired effects of unsurpassed precision . . . We do not make instruments and then infer the reality of the electrons, as when we test a hypothesis, and then believe it because it passed the

> test. That gets the time-order wrong. By now we design apparatus relying on a modest number of home truths about electrons, in order to produce some other phenomenon that we wish to investigate . . . We spend a lot of time building prototypes that don't work. We get rid of innumerable bugs . . . The instrument must be able to isolate, physically, the properties of entities that we wish to use, and damp down all the other effects that might get in our way. *We are completely convinced of the reality of electrons when we regularly set out to build – and often enough succeed in building – new kinds of device that use various well-understood causal properties of electrons to interfere in other more hypothetical parts of nature*.[48]

It is in the context of these complex laboratory practices that the causal models I mentioned before (the part of the population of models that interfaces with the actual world) are deployed. As the sociologist of science Andrew Pickering has argued, experimentalists, machines, causal models and electrons (or other material entities) form, in the context of a particular experimental project, a heterogeneous assemblage. Each of these distinct components retains its heterogeneity but they are meshed to one another in a complex process in which causal modes are fine tuned to better adapt to the results of an experiment, machines and procedures redesigned to change the way they affect and are affected by phenomena, and skills sharpened to cope with unforeseen difficulties. In this assemblage each of the component parts plays a role *interactively stabilizing* the whole. As Pickering writes, 'Scientific knowledge should be understood as sustained by, and as part of, interactive stabilizations situated in a multiple and heterogeneous space of machines, instruments, conceptual structures, disciplined practices, social actors and their relations, and so forth.'[49]

Following Deleuze we may think about these complex assemblages as the epistemological counterpart of the intensive in ontology. Much as virtual multiplicities (viewed as self-posed ontological problems) depend on intensive assemblages like ecosystems to progressively give rise to ontological solutions, so experimental problems must first be embodied in an intensive assemblage prior to their being solved. In learning by doing, or by interacting with and adjusting to materials, machines and models, experimentalists *progressively discern* what is relevant and what is not in a given experiment. In other words, the distribution of the important and the unimportant defining an experimental problem (what degrees of freedom matter, what disturbances do not make a difference) are not grasped at a glance the way one is supposed to grasp as essence (or a clear and distinct idea), but slowly brought to light as the assemblage stabilizes itself through the mutual accommodation of its heterogeneous components. In this assemblage the singularities and affects of the experimentalist's body are meshed with those of machines, models and material processes in order for learning to occur and for embodied expertise to accumulate.[50] On the other hand, besides this expertise (which may be applied in the design and performance of other experiments and which, therefore, remains intensive) there are also extensive or formal products of laboratory practices: individual pieces of data, individual facts, individual solutions, which take their place in the corpus of accumulated knowledge. As Deleuze writes, 'Learning is the appropriate name for the subjective acts carried out when one is confronted with the objectivity of a problem . . . whereas knowledge designates only the generality of concepts or the calm possession of a rule enabling solutions.'[51]

To summarize, there are two different ways of subordinating problems to solutions in the causal realm. One involves the elimination of the nonlinear causal capacities of the material systems under study either by homogenizing them or by focusing on low-intensity equilibrium situations. In either case, one studies a matter so obedient to laws that the productive aspect of causal connections may be disregarded and be reduced to a constant regularity. What makes a material system problematic, what continuously demands new explanations, is precisely the open-endedness of the assemblages it may form, or the multiple stable states in which it may exist and the abrupt transitions it may undergo. But if we assume that there is always a unique stable state, or that a cause always produces one and the same effect, we may forget about the problem and focus on the solution: the constant regularity itself as described by a law. On the other hand, one subordinates problems to solutions when the complex causal interventions in reality which the experimentalist must perform, as well as the mutual adjustments between machines, skills and 'a large number of interlocking low level generalizations',[52] are relegated to a secondary place and the formal cognitive products of this assemblage are taken as the only worthy objects of philosophical reflection. Once detached from their intensive individuation context, where the experimental learning of relevances and irrelevances takes place, these individual items of knowledge become significant only by reference to a theoretical framework of laws and abstract concepts.

Let me turn now to the subordination of problems to solutions in the realm of the quasi-causal. As I said before, the part of the population of models which interfaces with the virtual is not the one composed of detailed models of causal mechanisms but the one including the much simpler ones expressing fundamental laws. Unlike the case of complex causal models, the relation of problems to solutions in the case of basic laws (and models directly derived from them) may be approached using the results of Deleuze's ontological analysis of state space. State-space ideas do not apply to causal models for two reasons. One is their sheer complexity: the mathematical techniques needed to analyse state space are typically valid only for models with a few degrees of freedom, defining a state space with a low dimensionality, and are not at present sufficiently developed to apply to more complex cases. This limitation may be lifted one day as these techniques improve but there is a more important reason why they will still be of limited value to the experimentalist: *state spaces do not capture any information about causal processes*.

Let me explain. In some interpretations of state space the series of possible states which populate it (that is, the trajectories or solution curves) are erroneously endowed with causal significance, with each successive state viewed as the cause of the following one (or in some interpretations, the initial state is taken as the cause while the final state is the effect). This is, indeed, a mathematical expression of the positivist reduction of the productive or genetic aspect of causes to *a process of uniform succession* (another version of Hume's regular conjunction). But as critics of positivism have pointed out, only *actual events* can perform the genetic role of causes. As Mario Bunge argues, 'states cannot have a productive virtue of their own. The state of a material system is a system of qualities, not an event or a string of events. Every state is the *outcome* of a set of determiners . . . Consequently there can be no action of one state upon another state of a given system; in particular, *there can be no causal links among states*, nor among any other system of qualities.'[53]

On the other hand, while the analysis of the state space of a model may not provide us with causal information, it can be made to yield insight about quasi-causal relations. This epistemological result, however, depends on a particular ontological interpretation of the contents of state space. Deleuze, as I said, does *not* view the differential relations defining a model as expressing a law governing the generation of the series of states that make up a trajectory, but as defining a vector field which captures the overall tendencies of the system as a distribution of singularities. 'Beneath the general operation of laws' as he says 'there always remains the play of singularities.'[54] These singularities define the conditions of the problem, independently of its solutions, while each solution curve is the product of a specific individuation process guided at every point by the tendencies in the vector field:

> Already Leibniz had shown that the calculus . . . expressed problems which could not hitherto be solved or, indeed, even posed . . . One thinks in particular of the role of the regular and the singular points which enter into the complete determination of the species of a curve. No doubt the specification of the singular points (for example, dips, nodes, focal points, centres) is undertaken by means of the form of integral curves, which refers back to the solutions of the differential equations. There is nevertheless a complete determination with respect to the existence and distribution of these points which depends upon a completely different instance, namely, the field of vectors defined by the equation itself . . . Moreover, if the specification of the points already shows the necessary immanence of the problem in the solution, its involvement in the solution which covers it, along with the existence and distribution of points, testifies to the transcendence of the problem and its directive role in relation to the organization of the solutions themselves.[55]

To bring out the originality of Deleuze's analysis it will help to contrast it with the analyses performed by analytical philosophers who focus exclusively on the epistemological role played by trajectories. In one approach, for example, the role of the trajectories is to be used as *predictions* about the specific sequence of values which the relevant properties of the system being modelled will follow. The first step in the procedure, according to this approach, involves making measurements of the properties of a real system in a laboratory and plotting the resulting numerical values *as a curve*. If the laboratory system is prepared in such a way that it starts its evolution in the same initial conditions as the model, then this curve and the corresponding state-space trajectory should be *geometrically similar*. A perfect match between the two, with the state-space trajectory exactly tracking the plotted values, could then be interpreted as meaning that the model is true to the modelled system. Given that, due to empirical limitations, we cannot prepare a laboratory system to start at precisely the same initial conditions as an abstract model, the relation between plotted values and predicted trajectories will not be a perfect match, so that their relation will be one of *approximate truth*. Nevertheless, it is the geometrical similarity, or approximate similarity, between the two curves that matters for epistemological purposes.[56]

An alternative view would disregard this extrinsic resemblance between metric objects, and emphasize instead the *common possession of topological invariants*. As one physicist puts it,

> For present purposes, a system may be viewed both as a field of physical phenomena in which a class of elements exhibits its functions or behaviors in space and time, and as an abstract description which presumably may be isomorphic with the physical field . . . Two systems will be viewed as functionally isomorphic over a dynamic range *if they have the same singularities* of motion, in the stability sense, over that range.[57]

This would be the correct stance to adopt in a Deleuzian analysis. The epistemological value of state space would be to reveal a *topological isomorphism* between singularities in the model and singularities in the physical system being modelled. This isomorphism, in turn, would be explained by showing that the model and the physical system are *co-actualizations* of the same virtual multiplicity (or of part of the same multiplicity, given that the isomorphism is valid only within a range).

Deleuze's approach does not exclude the possibility that there can be similarities between trajectories and plotted values, but this resemblance must itself be explained as a result of the common topological properties of the systems producing the curves. The reply that possession of common properties is what makes a model and a real system similar is, as the philosopher Nelson Goodman argued long ago, redundant. As he put it, 'to say that two things are similar in having a specified property in common is to say nothing more than that they have that property in common'.[58]

There is another way of stating the difference between these two philosophical approaches to the epistemology of state space. In the analytical approach, the main epistemological relation is that between laws (expressed by differential equations) and the trajectories obtained as solutions to those equations. This relation is one of *general to particular*. In other words, if we ignore the role which the vector field plays in the individuation of trajectories, it seems natural to view laws as stating a *general rule* governing the evolution of series of states, and to see each trajectory as the result of applying that rule for a *particular initial condition*. In the Deleuzian approach, on the contrary, the particular state at which a trajectory starts becomes irrelevant, given that many different starting points within the same basin of attraction end up in the same place, the attractor. In other words, it is the distribution of singularities itself that determines what changes in initial conditions are relevant (relative to the end state) and which are irrelevant. On the other hand, the generality of the law (of which a given trajectory and plot of real values are particular instances) is replaced by the *universality* of virtual multiplicities of which both model and real system are divergent actualizations. As Deleuze writes, 'Singularity is beyond particular propositions no less than universality is beyond general propositions.'[59]

The subservience of problems to solutions in the analysis of state space is but one example of an error with a rather long history, a 'long perversion' which Deleuze traces back at least to Aristotle.[60] Originally, the subordination derived from the habit of thought of thinking about problems as if they were propositions, that is, from missing the non-linguistic and extra-propositional nature of their conditions (contrast space). But in more recent times, in the historical period when classical mechanics developed, the surrender to solutions took a more specific, more mathematical form. To Deleuze, mathematical problems are subordinated to their solutions whenever the well-posedness of a problem is approached in terms of its *solvability* (the possibility of finding a solution). In the final section of this chapter I would like to discuss two episodes in the history of mathematics where this traditional subordination was inverted, with solvability becoming *a consequence* of the well-posedness of a problem. As I will discuss in a moment, this inversion has for Deleuze revolutionary consequences whose impact has not been generally appreciated. One episode involves the history of algebraic equations, and the reversal of the subordination had, as one of its consequences, the birth of group theory. The other episode is more familiar, relating to the history of differential equations, having as a result the birth of the theory of dynamical systems, which is the source of the modern approach to state space.

Let me begin by describing in very rough form the technical issues involved in questions of solvability in the case of algebraic equations. There are two kinds of solutions to equations, *particular and general*. A particular solution is given by numerical values which, when used to replace an equation's unknowns, make the equation come out true. (For example, an algebraic equation like $x^2 + 3x - 4 = 0$ has as its numerical solution $x = 1$.) A general or exact solution, on the other hand, does not yield any specific value or set of values but rather *the global pattern of all particular solutions*. This general pattern is typically given by another equation or formula. The above example, which may be written as $x^2 + ax - b = 0$, has the general solution $x = \sqrt{a^2/2 + b} - a/2$. When mathematicians speak of the solvability of an equation they usually mean its exact solvability, and the subordination of problems to solutions stems from the demand that a well-posed problem have an exact solution, not just numerical ones. By the sixteenth-century mathematicians knew that exact solvability was an achievable goal, at least with equations where the unknown variable was raised up to the fourth power (that is, those including $x^2$, $x^3$ and $x^4$). But then a crisis ensued. Equations raised to the fifth power refused to yield to the previously successful method. Was this lack of exact solvability indicative that there was something wrong with the problem as it was posed by the fifth degree equation?

The answer came two centuries later when it was noticed that there was a pattern to the solutions of the first four cases, a pattern which might hold the key to understanding the recalcitrance of the fifth, known as the *quintic*. First Joseph-Luis Lagrange and Neils Abel, and then Evariste Galois, found a way to approach the study of this pattern using resources that today we recognize belong to group theory. In a nutshell we can say that Galois 'showed that equations that can be solved by a formula must have groups of a particular type, and that the quintic had the wrong sort of group'.[61] I cannot go here into the technical details of Galois's work but what he achieved was to invert the subordination of problems to solutions: rather than general solvability defining the correctness of a problem, *the form of the problem became the explanation of general solvability*. In other words, while before the exact solvability of the first four cases was taken for granted (as a property which problems must have) it now became something that could be explained by a *universal feature* of the problem which these four cases posed. This is what Deleuze means when he says that 'it is not the solution which lends its generality to the problem, but the problem which lends its universality to the solution',[62] a universality captured in this case by a group of transformations. But how exactly does a group of transformations capture the universal conditions that define a problem as a problem, that is, independently of its solutions?

To answer this question let me first take a different example, the use of transformation groups to study the invariants of physical laws. Two of the most typical transformations in this case are displacements in space or time. Given a law-governed physical process that can be reproduced in a laboratory, if we simply move it in space (for instance, by reproducing it in another, far away laboratory) we can expect the regular aspects of its behaviour to remain invariant. Similarly, if we simply change the time at which we begin an experiment, we can expect this time displacement to be irrelevant as far as the regularity of the process is concerned. It is only the difference in time between the first and final states of the process that matters, not the absolute time at which the first state occurs. Thus, via transformations applied to the equations expressing laws, we can discover those types of change to which *the law is indifferent*, that is, the types of changes which do not matter as far as the law-like process is concerned. The sense in which the group of an equation captures the conditions of a problem is then that it reveals distributions of the relevant and the irrelevant, the irrelevance of using absolute time or absolute position as inputs to a law for instance. It may be asserted without exaggeration that understanding this connection had profound implications in the history of physics playing a crucial role, for example, in the development of the general theory of relativity.[63]

Similarly, Galois's analysis of algebraic equations relied on the use of certain transformations (*substitutions or permutations* of the solutions) which, as a group, showed what changes were relevant to the validity of the equation (or more exactly, to the validity of the relations between solutions). More specifically, when a given permutation of one solution by another leaves the equation valid, the two solutions become, in a sense, *indistinguishable* as far as this validity is concerned. The equation is indifferent to the switch. As Morris Kline writes, 'The group of an equation is a key to its solvability because the group expresses the degree of indistinguishability of the [solutions]. It tells us what we do not know about the [solutions].'[64] Or as Deleuze would put it, the group reveals not what we know about the solutions, but *the objectivity of what we do not know about them*, that is, the objectivity of the problem itself.[65] Moreover Galois's method involves the equivalent of a symmetry-breaking cascade in that the solutions to the equation become increasingly more accurately defined as the original group gives rise to sub-groups which *progressively limit the substitutions* leaving the relations invariant. In other words, through a cascade which unfolds the original group, the problem itself becomes progressively better specified and, *as a by-product of this self-specification*, individual solutions emerge. As Deleuze writes:

> We cannot suppose that, from a technical point of view, differential calculus is the only mathematical expression of problems as such . . . More recently other procedures have fulfilled this role better. Recall the circle in which the theory of problems was caught: a problem is solvable only to the extent that is is 'true' but we always

> tend to define the truth of a problem by its solvability . . . The mathematician Abel [later followed by Galois] was perhaps the first to break this circle: he elaborated a whole method according to which solvability must follow from the form of a problem. Instead of seeking to find out by trial and error whether a given equation is solvable in general we must determine the conditions of the problem which progressively specify the fields of solvability in such a way that the statement contains the seed of the solution. This is a radical reversal of the problem–solution relation, a more considerable revolution than the Copernican.[66]

The reversal of the problem–solution relation also had revolutionary consequences in the case of differential equations. Although very different from their algebraic counterpart, equations in the calculus also have particular and general solutions, both produced by the integration operator. As it happens, most differential equations cannot be solved by integration in a general or exact way. Today we get around this limitation by using computers to generate a population of many numerical solutions, a population which may be used to discover the general pattern. In the eighteenth century, when the physics which Newton and others had created was first given differential form, this way out of the difficulty was not, of course, available. One consequence was the neglect of models whose constituent equations could not be solved exactly, given that without a way of knowing the overall pattern of particular solutions, physicists could not learn very much from a model. Thus, in a very real sense, the solvability of a problem was what made it worthy of study. As the mathematician Ian Stewart writes:

> The mathematicians of the eighteenth century ran headlong into a problem which has plagued theoretical mechanics to this day: to set up the equations is one thing, to solve them quite another . . . The eighteenth century's main achievements were in setting up equations to model physical phenomena. It had much less success in solving them . . . A process of self-selection set in, whereby equations that could not be solved were automatically of less interest than those that could.[67]

One can hardly blame these mathematicians and physicists for falling prey to this process of self-selection, since they were operating within the limits imposed by the mathematical technology of their time. On the other hand, the long-term effects of subordinating the choice of problems to their solvability did influence their (and their successors') world view, biasing it towards a clockwork picture of reality. The reason for this was that the equations that could be exactly solved happened to be the *linear equations*. The mathematical difference between linear and nonlinear equations is explained in terms of the *superposition principle*, which states that given two different solutions of a linear equation, their sum is also a valid solution. In other words, once we have discovered a few solutions to an equation many more can be obtained for free via the superposition principle. In an era characterized by the general scarcity of exact solutions, such a principle must have seemed like a gift from the optimizing rationality of God. Conversely, failure to obey this principle promoted the neglect of nonlinear equations.[68] In the terms I have been using in this chapter we may say that superposition, that is, a property of the behaviour of solutions, *biased the process of accumulation* that created the population of models making up the theoretical component of classical mechanics. The requirement of exact solvability promoted the accumulation of linear models at the expense of nonlinear ones, and even the few nonlinear models allowed to become part of the population were used only in a linearized form. (Linearization is achieved by using nonlinear models only for *very low intensities* of the recalcitrant variables.) As Stewart puts it:

> Classical mathematics concentrated on linear equations for a sound pragmatic reason: it could not solve anything else . . . So docile are linear equations, that classical mathematicians were willing to compromise their physics to get them. So the classical theory deals with *shallow* waves, *low*-amplitude vibrations, *small* temperature gradients [that is, linearizes nonlinearities]. So ingrained became the linear habit that by the 1940s and 1950s many scientists and engineers knew little else . . . Linearity is a trap. The behaviour of linear equations . . . is far from typical. But if you decide that only linear equations are worth thinking about, self-censorship sets in.

> Your textbooks fill with triumphs of linear analysis, its failures buried so deep that the graves go unmarked and the existence of the graves goes unremarked. As the eighteenth century believed in a clockwork world, so did the mid-twentieth in a linear one.[69]

The counterpart to Abel's and Galois's reversal of the problem–solution relation is represented by the work of Henri Poincaré on the qualitative (or topological) study of differential equations. His was a novel approach created, like the group-theoretic approach to algebraic equations, to break through the barrier of a recalcitrant problem: the *three body problem*, the problem of modelling the mutual interactions of three solar system bodies (such as the sun, the earth and the moon). Although other mathematicians had already approached the study of solutions by analysing their behaviour in the neighbourhood of singular points, Poincaré approached the wider question of the way in which the *existence and distribution* of singularities organized the space of all solutions. In other words, like Galois, Poincaré by-passed exact solvability as a way to get global information and instead used a novel method to investigate *the space defining the problem itself*, that is, he used the distributions of singular points as a way to gain qualitative information about the tendencies in the behaviour of all solutions.[70]

Poincaré's phase-portrait approach to state space has, of course, been the basis of much of what I have said in this book about the ontology of the virtual and the problematic. But Galois's approach has also been crucial since it provided the idea of a progressive specification of virtual multiplicities through symmetry-breaking cascades. In short, a theory of virtuality as has been pursued in these pages depends fundamentally on the results of the reversal of the problem–solution relation, and conversely, subordinating problems to solutions may be seen as a practice that effectively hides the virtual, or that promotes the illusion that the actual world is all that must be explained. Thus construed, this subordination joins the axiomatic treatment of classical physics as a barrier to a more satisfactory problematic approach.[71] In addition, there are the obstacles posed by the linearity of causes in experimental physics, and the linearity of models in theoretical physics, both of which are intimately related since the former's additivity is equivalent to the latter's superposition. Additivity and superposition characterize an unproblematic world, or at best, a world which is only temporarily problematic or in need of explanation, but which will eventually yield to a super-law or a theory of everything which will leave nothing unexplained. On the other hand, nonlinear models and their multiple attractors, as well as nonlinear causes and their complex capacities to affect and be affected, define a world capable of surprising us through the emergence of unexpected novelty, a world where there will always be something else to explain and which will therefore remain forever problematic. As Mario Bunge writes:

> If the joint action of several causes is always an external juxtaposition, a superposition, and in no case a synthesis having traits of its own, and if the hypothetical patients on which the causal agents act are passive things incapable of spontaneity or self-activity – incapable, in short, of adding something of their own to the causal bond – then it follows that, in a sense, *effects preexist in their causes*. According to this extreme but consistent doctrine on the nature of causation, *only old things come out of change*; processes can give rise to objects new in number or new in some quantitative respects, not however new in kind; or again, no new qualities can emerge. A world running on a strictly causal pattern [i.e. a linear pattern] is such as yogis, Thomists and eighteenth-century Newtonians imagined it, namely, a universe without a history . . .[72]

Unlike this linear world, the ontology I have developed in this book is fully historical. Each of the individuals which populates this other world is a product of a definite historical process of individuation and, to the extent that an individual's identity is defined by its emergent properties and that these properties depend on the continuing causal interactions among an individual's parts, each individual is itself a historical causal process. The realm of the quasi-causal is also fully historical but, as I explained in the previous chapter, it possesses its own original form of temporality and thus bears no resemblance to causal history. In other words, in a Deleuzian ontology there exist two histories, one actual and one virtual, having complex interactions with one another. On one hand there is a historical series of actual events genetically involved in the production of other events, and on the other, an equally historical series of ideal events defining an objective realm of virtual problems of which each actualized individual is but a specific solution. To conclude with Deleuze's own words,

> It is correct to represent a double series of events which develop in two planes, echoing without resembling each other: real events on the level of the engendered solutions, and ideal events embedded in the conditions of the problem, like the acts – or, rather, the dreams – of the gods who double our history.[73]

# *Appendix: Deleuze's Words*

Gilles Deleuze changes his terminology in every one of his books. Very few of his concepts retain their names or linguistic identity. The point of this terminological exuberance is not merely to give the impression of difference through the use of synonyms, but rather to develop a set of *different* theories on the *same* subject, theories which are slightly displaced relative to one another but retain enough overlaps that they can be meshed together as a heterogeneous assemblage. Thus, the different names which a given concept gets are not exact synonyms but near synonyms, or sometimes, non-synonymous terms defining closely related concepts. In this book I deliberately homogenized the terminology for the sake of clarity but giving a list of near synonyms will now prove useful to the reader as he or she moves back from my simplified presentation of Deleuze's ontology to his original ones. In fact, beyond providing a mere list I will try to map the connections between the different terminologies and discuss the different ways in which the ontology is conceptualized and articulated in each of the books. As I map these terminological connections I will use the following abbreviations of Deleuze's books, followed when necessary by a page number (chapter numbers refer to the present book):

| | |
|---|---|
| *Anti-Oedipus* | AO |
| *A Thousand Plateaus* | ATP |
| *Difference and Repetition* | D&R |
| *Logic of Sense* | LOS |
| *What is Philosophy?* | WIP |

The main sources used in my reconstruction were D&R, where the theory of multiplicities and the virtual continuum they form is most clearly articulated, and LOS which presents the most detailed description of the quasi-causal operator. I will begin this appendix with a list of the components of Deleuze's ontology (D&R, 277–8). I will then expand the description of each of the seven components of this 'ontological list', not only to relate them to the terminology used in my presentation, but also to add details which I left out for the sake of simplicity but which are now necessary in order to relate the items in the ontological list to those in other books. Finally, I will take three books, ATP, AO and WIP, and map each component of the list to their counterparts there.

## THE ONTOLOGICAL LIST

(1) the depth or *spatium* in which intensities are organized;
(2) the disparate series these form, and the fields of individuation that they outline (individuation factors);
(3) the 'dark precursor' which causes them to communicate;
(4) the linkages, internal resonances and forced movements which result;
(5) the constitution of passive selves and larval subjects in the system, and the formation of pure spatio-temporal dynamisms;
(6) the qualities and extensions . . . which form the double differenciation of the system and cover over the preceding factors;
(7) the centres of envelopment which nevertheless testify to the persistence of these factors in the developed world of qualities and extensities.

### 1. Intensive Spatium

This term refers to the virtual continuum formed by multiplicities. In this book I used the term 'plane of consistency' to refer to it, a term used throughout ATP. Other near synonyms include 'plane of immanence' (WIP), 'body without organs' (AO, ATP), 'machinic phylum' (ATP), and 'ideal or metaphysical surface' (LOS). A possible source of confusion here is the term 'intensive' which in my presentation was used in relation to individuation processes, not the virtual continuum. Deleuze uses the term in three senses:

a) Its original, thermodynamic sense in which it refers to intensive properties, like pressure, temperature or density. Differences in these quantities have a morphogenetic effect (they drive fluxes of matter or energy, for example) and when not allowed to get cancelled (as in non-equilibrium physics) display the full potential of matter-energy for self-organization.
b) A second derived sense in which it refers to the assembly of different components as such, that is, the creation of heterogeneous assemblages in which the components' differences are not cancelled through homogenization.
c) A third derived sense in which it refers to the properties of ordinal series. These series are constituted by the differences between their terms, that is, by asymmetrical relations such as 'in between'. When we consider more than one term between two others, this serial relation is called a 'distance', although this term must be qualified (Deleuze speaks of 'non-decomposable distances') to distinguish it from its non-technical meaning where it refers to a metric concept (such as 'length'). Finally, there are the uncancellable differences, or constitutive inequalities, which ordinal series present when compared to one another (only judgments of greater or lesser are possible, not of exact equality). It is mainly in this third sense that the term is used in the expression 'intensive spatium' as the following quote shows:

> Difference, distance and inequality are the positive characteristics of depth as intensive spatium (D&R, 238).

## 2. Multiplicities and Divergent Series

Although the term 'multiplicity' is not used in the list above, it is clear that it belongs in this entry since the 'disparate series' mentioned are nothing but the effect of expanding in a serial form the singularities defining each unfolding level of a multiplicity. The term has some near synonyms: 'partial objects' (AO); 'philosophical concepts' (WIP); 'ideal events' (LOS). Sometimes Deleuze refers to multiplicities indirectly via their components, such as 'nomadic singularities' and 'noematic attributes' (LOS), or 'vague essences' and 'becomings' (ATP).

The term 'disparate' means 'difference of difference' (D&R, 241). To speak of 'disparate series' is another way of expressing the idea that the ordinal series which form the nonmetric continuum must be related to one another via *affirmative divergence*, so that not only are the series made up of differences, their divergent relations further *differentiate these differences*:

> Difference must become the element, the ultimate unity; it must therefore refer to other differences which never identify it but rather differentiate it. Each term of the series, being already a difference, must be put into variable relations with other terms, thereby constituting other series devoid of center and convergence. Divergence and decentering must be affirmed in the series itself. (D&R, 56)

## 3. Dark Precursor

This term refers to what in my reconstruction I called the 'quasi-causal operator'. Its near synonyms include: 'quasi-cause', 'aleatory or paradoxical point' and 'nonsense' (LOS); 'line of flight' and 'abstract machine' (ATP); 'desiring machines' (AO); 'conceptual personae' (WIP); 'object = x ' (D&R, LOS).

## 4. Resonances and Forced Movements

This entry includes the effects which the quasi-causal operator has on the multiplicities and their series. In my reconstruction I used an information-theoretic model for these effects (in terms of emissions of signs or information quanta) but Deleuze also uses an alternative physical model in terms of resonances (D&R, LOS, WIP). The terms 'resonance' and 'forced movement' should not be taken as mere physical metaphors. Rather, we should think about resonance as *positive feedback*, a generic process which implies one or other form of *mutually stimulating couplings* (e.g. autocatalysis) inducing resonances among heterogeneous elements, as well as the *amplification of original differences* (forced movements).

The crucial idea is that the quasi-causal operator must couple the ordinal series emanating from multiplicities so as to weave these into a nonmetric continuum. Resonances are the means to effect couplings, while the resulting forced movement produces the continuum (LOS, 239–40). As I have just said, the couplings between series must ensure their affirmative divergence, keeping the continuum open and in constant variation. But also, as a separate operation (what I called 'pre-actualization' in Chapter 3), it must induce some *convergences* in the series, since it is in these centres of convergence that the process of actualization begins:

> To be actualized . . . means to extend over a series of ordinary points; to be selected according to a rule of convergence; to be incarnated in a body; to become the state of a body; and to be renewed locally for the sake of limited new actualizations and extensions. (LOS, 110)

## 5. Passive Selves and Spatio-Temporal Dynamisms

This entry contains the two components of what in my reconstruction I referred to as 'intensive individuation processes'. The first meaning of the term 'spatio-temporal dynamism' is straightforward, referring to the phenomena of self-organization which occur in many non-equilibrium systems. Self-organizing dynamics are typically governed by the singularities (attractors and bifurcations) which characterize differential relations (that is, coupled rates of change or relations of relative rapidity and slowness.) In this sense, the term relates to the first sense of the word 'intensive', as in a non-equilibrium material where intensive differences have not been cancelled. But the term also refers to 'affects', or the second sense of 'intensive', that is, to the capacities and dynamisms which produce heterogeneous assemblages. That the two senses are intimately connected is clear from the following:

> It is no longer a question of imposing a form upon a matter but of elaborating an increasingly rich and consistent material, the better to tap increasingly intense forces. What makes a material increas-

> ingly rich is the same as what holds heterogeneities together without their ceasing to be heterogeneous. (ATP, 329)

Unlike spatio-temporal dynamisms, the terms 'passive self' and 'larval subject' received very little elaboration in my reconstruction, mostly because I wanted to keep the description of Deleuze's ontology as free from anthropocentrism as possible. The first term is related to the 'passive synthesis' which forms the core of Deleuze's theory of time, the synthesis of 'living presents' which metricize or give measure to time. In his theory, this synthesis is directly related to the genesis of subjectivity (it is a contemplative subject who contracts instants into a present) but, as I explained in Chapter 3, these 'contemplations' occur everywhere, in the form of proto-perceptions and proto-feelings which even microscopic individual entities may be said to have. Hence, we not only contract instants to synthesize our psychological sense of present, we are *made out* of micro-contractions and *their* presents:

> We are made of contracted water, earth, light, and air – not only prior to the recognition or representation of these, but prior to their being sensed. Every organism, in its receptive and perceptual elements, but also in its viscera, is a sum of contractions, of retentions and expectations. (D&R, 73)

The term 'larval subject' is closely related to these ideas, referring to the 'voluptuous consumption' of the intensities which drive spatio-temporal dynamisms. The best example here is the developing embryo as it experiences the intensive foldings, migrations, and other transformations which will eventually turn it into a fully formed organism. Indeed, unlike my reconstruction where the term 'individual' refers to the final product (organisms, species, etc.) in Deleuze's work it refers to the larval subjects themselves. It often has the meaning of a Leibnizian 'monad', and it is said to be born during pre-actualization, that is, from the centres of convergence which occur in the virtual series:

> A world already envelops an infinite system of singularities selected through convergence. Within this world, however, individuals are

> constituted which select and envelop a finite number of the singularities of the system . . . An individual is therefore always in a world as a circle of convergence, and a world may be formed and thought only in the vicinity of the individuals which occupy or fill it. (LOS 109–10)

To avoid confusion, I will use the term 'intensive individual' to refer to these monads, and 'individual' without qualification to refer to the extended and qualified actual entities which form my flat ontology of individuals.

## 6. Extensities and Qualities

These are the two characteristics which define the realm of the actual, the fully constituted world of extended and qualified individuals. In ATP these two characteristics are referred to as 'substances' and 'forms' respectively. To see the connection one needs to think, on the one hand, of a substance without any other characteristic than its manner of occupying space (its extension), and, on the other hand, of the forms or structures which endow this substance with specific qualities (such as its mechanical or optical properties). Given that no actual substance is ever purely extensional, these two characteristics are 'not really distinct. They are the abstract components of every articulation.' (ATP, 502)

## 7. Centres of Envelopment

This concept was not discussed in my reconstruction. I introduce it here not only because it appears as the last item in the listing of ontological components under discussion, but also because its definition relates to aspects of the theory of the actual which bear on questions of terminology. The different spheres of the actual (roughly, the physico-chemical, organic and cultural spheres) need to be conceived without presupposing a teleological development or 'any kind of ridiculous cosmic evolutionism' (ATP, 49). There are, on the other hand, very real distinctions between these spheres. In particular, unlike the physico-chemical sphere where the 'code' that underlies forms or qualities is distributed throughout the three-dimensionality of a structure, in the organic sphere this code becomes detached as a separate one-dimensional structure: the linear sequence of nucleic acids constituting the genetic code. The genetic code, in Deleuze's view, represents an *interiorization of the intensive individuating factors* which in physico-chemical strata remain external to individuals. This interiorization, which characterizes the increase in complexity of living systems, is what is referred to by the term 'centres of envelopment':

> The function of these centres may be defined in several ways . . . we claim that complex systems increasingly tend to interiorize their constitutive differences: the centres of envelopment carry out this interiorization of the individuating factors. (D&R, 256)

**Summary**

Let me now summarize what I have just said about the contents of the ontological list. Items 1, 2, and 3 constitute the elements of the virtual: the continuum, the multiplicities and the quasi-causal operator. Items 4 and 5 may be made to correspond, with a bit of tweaking, to the intensive. The reason why some tweaking is necessary is that it involves separating the divergent and the convergent relations between the series, the former belonging to the virtual and the latter (as a kind of pre-actualization) to the intensive. Centres of convergence would correspond to what some scientists call 'morphogenetic fields', or what Deleuze calls 'fields of individuation'. Although Deleuze includes as part of Item 2 'fields of individuation', and the resonances of Item 4 also produce divergences, it will prove useful to keep the two Items apart and define the intensive both by the fields of individuation and the spatio-temporal dynamisms that perform the actualization of these fields. Finally, Items 6 and 7 form the contents of the actual. Precisely because the virtual, the intensive and the actual are aspects of one and the same process, or the different moments of a cascade of progressive differentiation, some Items (4 and 7) represent areas of overlap (something of the virtual, convergence, within the intensive; something of the intensive, envelopment centres, in the actual). Let me now show how the virtual, the intensive, and the actual are treated in other books.

## A THOUSAND PLATEAUS

In ATP the different spheres which make up the actual world (physico-chemical, organic, cultural and so on) are called 'strata'. The term 'stratification' is near synonymous with 'actualization'. The different extensities and qualities which characterize the actual world are referred to as 'substances' and 'forms', and also as 'territorialities' and 'codes'. Thus, Deleuze writes that strata 'proceed simultaneously by code and by territoriality' (ATP, 40). The intensive processes which give rise to strata, and which become hidden under strata, are therefore called 'territorialization' and 'coding'. Given that some parts of the world may be pushed away from their equilibrium state, thereby revealing the hidden intensive factors, the terms 'deterritorialization' and 'decoding' are used to refer to these departures from the rigidity of strata, or rather, to the intensive movements which animate strata from within. In D&R, Deleuze had already introduced the notion of 'de-differenciation' (D&R, 249) but it is only later that this notion acquires its full importance and that it is divided along the two components of actualization.

Indeed, as I argued in Chapter 3, the quasi-causal operator may be said to accelerate these departures from actuality in an operation called 'counter-actualization'. In ATP, Deleuze speaks of 'relative deterritorializations' to refer to movements away from the actual towards the intensive, and of 'absolute deterritorialization' to refer to counter-actualization, the acceleration of these movements allowing them to reach all the way into the virtual. The three components of the virtual (the continuum, the multiplicities that compose it and the quasi-causal operator which effects the composition) have exact counterparts in ATP as the following extract illustrates:

> There was a first group of notions: the Body without Organs or destratified Plane of Consistency; the Matter of the Plane, that which occurs in the body or plane (singular, nonsegmented multiplicities composed of intensive continuums, emissions of particle-signs, conjunctions of flows); and the Abstract Machine, or Abstract Machines, in so far as they construct that body or draw the plane or 'diagram' what occurs (lines of flight, or absolute deterritorialization). (ATP, 72)

Multiplicities are said to 'occur' in the plane of consistency because, as I argued, they are ideal *events or becomings*. The term 'nonsegmented' should be read as near synonymous with 'nonmetric', and 'intensive continuum' as 'ordinal continuum'. The 'emissions of particle-signs' are the resonances that couple the multiplicities, and the 'conjunctions of flows' correspond to mutual amplifications or forced movements. The quasi-causal operator, here called the 'abstract machine', is characterized in terms of 'lines of flight' which refer to the process of counter-actualization, and is said to 'draw the plane', that is, to extract ideal events from what actually occurs and to mesh these multiplicities into a heterogeneous continuum. As Deleuze writes 'the plane of consistency does not preexist the movements of deterritorialization that unravel it, the lines of flight that draw it or cause it to rise to the surface, the becomings that compose it' (ATP, 270). Finally, the 'centres of envelopment' are not given a special name but they are referred to indirectly when it is asserted that 'the abstract Machine exits simultaneously developed on the destratified plane it draws, and *enveloped* in each stratum whose unity of composition it defines . . .' (ATP, 70; my emphasis).

This is, roughly, the mapping from one set of terms to another. But in ATP we witness an elaboration of the original ontological components and this introduces new terms and ideas. In particular, in ATP the actual world is not defined simply in terms of extensities and qualities, but of very specific articulations of the extensive and the qualitative. As I discussed in my reconstruction, the actual consists exclusively of individual entities, each individual at a given level of scale emerging from the interactions of populations of smaller scale individuals. Deleuze refers to these two scales of every stratum as the 'molecular' and the 'molar'. Stratification consists in producing populations of 'molecules' and organizing them into 'molar', or large scale, aggregates. (Clearly, 'molecules' may be cells or even organisms, when the molar scale is that of the organism or the species, respectively.) Thus, every stratum needs a double articulation, a double play of substances and forms, of extensities and qualities, one at the level of molecular populations and another at the level of molar aggregates:

> The first articulation chooses or deducts, from unstable particle-flows, metastable molecular or quasi-molecular units (*substances*) upon which it imposes a statistical order of connections and successions (*forms*). The second articulation establishes functional, compact, stable structures (*forms*), and constructs the molar compounds in which structures are simultaneously actualized (*substances*). (ATP 40–1)

This process is called a 'double articulation'. Although the term 'double differenciation' already occurs in the ontological list, it refers only to the pair substance and form, not to this more elaborate interplay of territorialities and codes. A similar elaboration is evident in Deleuze's treatment of the intensive. As I argued in Chapter 2, even the most rigidly metric (or 'most stratified') individual still has unactualized capacities to affect and be affected, and may not be limited to a single stable equilibrium but have a variety of unactualized stable states available to it. These two aspects of the intensive, 'affects' and 'singularities', become further developed into 'parastrata' and 'epistrata' in ATP. On one hand, affects endow individuals with the capacity to establish novel connections with alien milieus, as with the evolution of the capacity to tap into a reservoir of oxygen, or other non-alimentary energy sources. Organisms may also have the capacity to actively shape their environment, as spider webs or beaver dams illustrate. These capacities are what Deleuze calls 'parastrata', the capacity to connect with an 'annexed or associated milieu' (ATP, 51). On the other hand, a fully formed individual may be capable of a variety of stable states which may be actualized by crossing critical points and give rise to 'variations that are tolerated below a certain threshold of identity' (ATP, 50). These 'intermediate states or milieus' are what Deleuze calls 'epistrata'. As he writes, even 'a single chemical substance (sulfur or carbon, for example) has a number of more or less deterritorialized states' (ATP, 53). The relations of the different terms for intensive factors can then be summarized like this:

> Forms relate to codes and processes of coding and decoding in the parastrata; substances, being formed matters, relate to territorialities

> and movements of territorialization and deterritorialization on the epistrata. (ATP, 53)

Finally, there is a term which refers to the actualization (or effectuation) of the quasi-causal operator itself. I did not discuss this in detail, but I did give an example in Chapter 2 of the neighbourhood of a phase transition (or 'edge of chaos'). Deleuze's own example is not critical *points* in a *line* of values, but critical *surfaces* in objects with *volume* (LOS, 103). (In both cases the quasi-cause operates at an N−1 dimension, as discussed in Chapter 3). In ATP, the organic membrane as a critical surface is kept as an instance of the quasi-cause as it exists effectuated in the actual, organizing the division of epistrata and parastrata (ATP, 49–50). But now a special term is coined for this actualized quasi-causal operator: 'machinic assemblage'. As he writes: 'The most important problem of all: given a machinic assemblage, what is its relation of effectuation with the abstract machine? How does it effectuate it, with what adequation?' (ATP, 71).

Much as the quasi-cause or abstract machine endows the virtual continuum with consistency, the machinic assemblage endows actual entities with consistency. 'What we term machinic is precisely this synthesis of heterogeneities as such' (ATP, 330). The machinic assemblage performs the different operations involved in stratification, such as articulating a stratum with whatever serves as its substratum (e.g. the pre-biotic soup for organic strata), as well as doubly articulating the different extensities and qualities, substances and forms, which define a given stratum (ATP, 71). But also, as an actualized quasi-cause, the machinic assemblage is the agent behind counter-actualization:

> The assemblage has two poles or vectors: one vector is oriented towards the strata, upon which it distributes territorialities, relative deterritorializations and reterritorializations; the other is oriented towards the plane of consistency or destratification, upon which it conjugates processes of deterritorialization, carrying them towards the absolute of the earth. (ATP, 145)

## ANTI-OEDIPUS

In this book the mapping of the items of the ontological list is less straightforward. In particular, the virtual and the intensive are grouped together in a process which is referred to as 'molecular' (in the sense just mentioned), while the actual is referred to as 'the molar'. Unlike ATP, where all kinds of strata are considered, in AO only the actualization of human societies is dealt with, so the molar seems to become synonymous with 'large social aggregates', such as stable persons, governmental or economic institutions, agricultural or industrial machines. But it should be kept in mind that this narrowing of the meaning of 'the molar' is a matter of focus and not a change in the underlying theory.

With some care, in fact, the different elements of the ontological list can be paired with their counterparts in AO. The virtual and the intensive processes of actualization are referred to as 'desiring production' and defined as consisting of three separate 'passive syntheses' (AO, 26). These are referred to as 'the connective', 'the disjunctive' and 'the conjunctive' syntheses. (This three-part classification first appears in LOS, 174.) The disjunctive synthesis involves the creation of divergent relations among series, and it is said to occur on the body without organs (AO, 13). It therefore refers to the virtual continuum, 'a pure fluid in a free state, flowing without interruption, streaming over the surface of a full body' (AO, 8). The conjunctive synthesis, in turn, involves the creation of convergent relations among series, an operation which as I said above, forms 'individuation fields' which already prefigure the intensive (pre-actualization). This synthesis captures one of the aspects of the intensive, the emergence of a larval or passive subject, 'a strange subject with no fixed identity, wandering about over the body without organs . . . being born of the [intensive] states that it consumes . . .' (AO, 16). Finally, the connective synthesis captures another aspect of the intensive, the machinic assemblage. It connects or couples together heterogeneous 'partial objects or organs' through the emission of 'energy flows' (AO, 323). Here the term 'partial' is not used in its extensive sense but in the sense of matter filling space to a given degree of intensity. 'The eye, the mouth, the anus as degrees of matter' (AO, 309).

This interpretation of the three syntheses gives us one of the elements of the virtual (the plane of consistency or body without organs), and two of the intensive (larval subjects, assemblages), but leaves several things out. In particular, the other two elements of the virtual, multiplicities and the quasi-causal operator, don't seem to be included. Multiplicities appear in AO as 'partial objects' when these 'attach themselves to the body without organs as so many points of disjunction between which an entire network of new syntheses is now woven marking the surface off into coordinates, like a grid' (AO, 12). This corresponds to the idea that multiplicities exist in the sphere of the intensive embodied in self-organizing processes, but may be extracted from these as 'flat multiplicities' or 'pure events' and deployed as such on the plane of consistency. The quasi-causal operator is, in turn, referred to as a 'desiring machine':

> Insofar as it brings together – without unifying or uniting them – the body without organs and the partial objects, the desiring machine is inseparable both from the distribution of partial objects on the body without organs, and of the leveling [i.e. flattening] effect exerted on the partial organs by the body without organs, which results in appropriation. (AO, 327)

The desiring machine is said to have 'chains' as its apparatus of transmission (AO, 327). The term 'chain' is used instead of 'series'. It has the meaning of a 'Markov chain' (AO, 39), a series of events in which the probability of occurrence of any event depends only on the previous one in the series. In other words, a 'chain' is a partially aleatory series. This corresponds to one of the effects of the quasi-cause, briefly discussed in Chapters 2 and 3, of injecting chance in the distributions of virtual singularities to create 'nomadic' distributions, as opposed to the 'sedentary' probability distributions which characterize populations in the actual world. This is also expressed by saying that the quasi-cause must affirm all of chance with every throw of the dice (LOS, 59–60). The term 'chain' is also used as in the expression 'signifying chain' but without any reference to a fixed code, linguistic or otherwise. Rather these heterogeneous chains are made of 'flying bricks . . . containing within [them] not only an inscription with signs from different alphabets, but also various figures, plus one or several straws, and perhaps a corpse' (AO, 40).

There is one more detail to be discussed which provides an important bridge to the next book to be deciphered (WIP). Much as multiplicities are woven into a virtual continuum through their divergences, but also form individuation fields when their series converge, 'the points of disjunction on the body without organs form circles that converge on the desiring machines; then the subject . . . passes through all the degrees of the circle, and passes from one circle to another' (AO, 20). The term 'passing' is used here as synonymous with 'becoming', and the 'degrees of the circle' are 'intensive quantities in their pure state' (AO, 18). The idea here is that this larval subject without identity can move about the plane, from one individuation field to another, becoming now this and now that intensive individual depending on the intensities it consumes. This is the key idea behind the process which in AO, ATP and WIP is referred to as 'becoming-animal' (as well as 'becoming-woman', 'becoming-molecule', etc.). The concept appears first in D&R, 254:

> We should not say that individuals of a given species are distinguished by their participation in other species: as if, for example, there was ass or lion, wolf or sheep, in every human being. There is indeed all that and metempsychosis retains all its symbolic truth. However, the ass and the wolf can be considered species only in relation to the fields of individuation . . . [It is true that someone's soul] never changed bodies, but its body could be re-enveloped or re-implicated in order to enter, if need be, other fields of individuation . . .

In other words, becoming-animal is an operation which cannot be performed within the actual, by a transformation from a fully constituted individual of one species to another of a different species. But if we move towards the virtual, towards those circles of convergence or fields of individuation where there are still communications between not-yet-actualized species, one can become 're-enveloped' in another field. This theme is elaborated in AO, 86 and in ATP, 238 and becomes a key component of Deleuze's theory of artistic practice as discussed in WIP.

## WHAT IS PHILOSOPHY?

Much as AO narrows the focus of the ontology and deals only with the actualization of social structures, WIP deals exclusively with the relations between the virtual, the intensive and the actual, on one hand, and the different forms which *thought* assumes in certain societies (philosophical, artistic and scientific forms of thought). The virtual appears here as 'the plane of immanence' explored by philosophical thought; the intensive as 'the plane of composition' as it appears in artistic thought; and the actual as 'the plane of reference' as it is investigated by scientific thought. Let me discuss each one of these 'planes' starting with the actual world.

One way of thinking about the plane of reference is as a flat ontology of individuals. The subject matter of science would be, in this interpretation, the world of fully constituted individuals and the metric and measurable spacetime they form. In other words, actual individuals would form the reference of scientific statements, and all referents would form a 'plane' precisely in the sense that, ontologically at least, they do not have a hierarchical structure but remain a 'flat' set, varying only in spatio-temporal scale. In Chapters 1 and 2, where I discussed the *philosophical* concept of 'multiplicity', I emphasized that the scientific ideas involved (differential relations, singularities) had to be detached from their original context where they are related to mathematical *functions*. The justification I gave for this transformation was that functions, as they are ordinarily used, presuppose individuation. Indeed, in some of their uses (as in their use to create state or phase spaces) they define procedures for the individuation of states within these spaces. These states of affairs constitute a referent, and the use of functions therefore follows the line which goes from the virtual to its actualization, retaining only the final product.

This is part of what Deleuze means when he asserts that the object of science is 'functions which are presented as propositions in discursive systems' (WIP, 117). I will return below to the question of whether one can characterize science in this way. As I said in Chapter 4, I do not think there is such a thing as 'science' in general, so I reject many of the details of the characterization given in WIP. Nevertheless, the part of it that I do keep is the assertion that most scientific fields tend to study the world in the direction of actualization, sometimes concentrating on the final product and disregarding the process (e.g. equilibrium thermodynamics), sometimes studying the process but always in the direction of the final product.

Art, on the other hand, may be said to study, or engage with, the intensive itself. The term 'intensive' is used in a variety of senses only some of which are relevant to this characterization. One of the components of the intensive given in the ontological list was the larval subject who consumes intensities as such, and is born and reborn of these voluptuous consumptions. In this case, the intensive state comes first or it is prior to the individual that lives it (AO, 20). In other words, objective intensities do not constitute psychological sensations but the very 'being of the sensible' (D&R, 140), a being which is itself imperceptible psychologically given that intensities become hidden underneath qualities and extensities (D&R, 230). In WIP this being of the sensible is divided into two components, 'percepts' and 'affects':

> By means of the material [e.g. paint, canvas, brush], the aim of art is to wrest the percept from perceptions of objects and the states of a perceiving subject, to wrest the affect from affections [e.g. feelings] as the transition from one state to another: to extract a bloc of sensations, a pure being of sensations. (WIP, 167)

Simplifying somewhat, we may say that 'percepts' are related to the passive selves involved in the synthesis of living presents at all scales of reality, in the organic and inorganic world. Even though these presents are constituted by 'contemplations' or 'contractions of past and future instants', they do not refer to a psychological reality. As Deleuze writes:

> The plant contemplates by contracting the elements from which it originates – light, carbon, and the salts – and it fills itself with colors and odors that in each case qualify its variety, its composition: it is sensation in itself. It is as if flowers smell themselves by smelling what composes them . . . before being perceived or even smelled by an agent with a nervous system and a brain. (WIP, 212)

On the other hand, affects refer to state transitions which must be understood as 'becomings', in the sense of a becoming-animal or becoming-plant discussed above. The artist must reach that intensive state where one can leave one individuation field to enter another, where one can reach 'a zone of indetermination, of indiscernibility, as if things, beasts, and persons . . . endlessly reach that point that immediately precedes their natural differentiation' (WIP, 173). Finally, having reached the very being of the sensible, the artist must place these percepts and affects in their own plane, a plane of composition, a bloc or compound of sensations whose 'only law of creation is that the compound must stand on its own' (WIP, 164).

Thus, in a very literal sense, art is concerned with making *perceptible* the usually hidden realm of the intensive. Similarly, philosophy must make the virtual *intelligible*. Philosophy must go beyond the centres of convergence where the larval subjects of percepts and affects undergo intensive becomings, to reach the virtual in its full divergence and difference, its continuous or 'inseparable variations' (WIP, 126). Philosophy cannot perform this task via a set of propositions which *refer* to the virtual, but rather, it must construct a thought which is *isomorphic* with the virtual. Therefore, any philosophy must be constructed out of the three components of the virtual: multiplicities, quasi-causal operator, and the continuum. In WIP these three components are referred to as 'concepts', 'conceptual personae', and 'plane of immanence', respectively.

The term 'concept' does not refer to a semantic entity, that is, to concepts in the ordinary sense, a sense in which there would also be scientific concepts (e.g. entropy). Rather, it is defined as an entity which would be isomorphic with virtual multiplicities.

> [A concept is] a multiplicity, an absolute surface or volume [e.g. a manifold] . . . made up of a certain number of inseparable intensive variations according to an order of neighborhood, and traversed by a point in a state of survey. (WIP, 32)

To say that a concept 'orders its components by zones of neighborhood' (WIP, 20) is to say that the relations it involves are nonmetric or ordinal. This refers to the third sense of 'intensive' as defined above, and to the definition of topological spaces in Chapter 1, and is also expressed by saying that a concept's components are 'intensive ordinates' (WIP, 20). Concepts, therefore, are not to be thought of semantically, but literally as state or phase spaces, that is, as spaces of possibilities structured by singularities and defined by their dimensions or intensive ordinates. As Deleuze writes, 'Every concept therefore has a phase space, although not in the same way as in science' (WIP, 25). For example, the Cartesian concept of 'the Cogito' would be a space with three dimensions (doubting, thinking and being) each divided by singularities into *phases* (e.g. perceptual, scientific, obsessional doubting, as different phases of doubt, as opposed to different species of the genus doubt).

The idea of a 'point in a state of survey' refers to an operation of the quasi-cause which I did not describe in my reconstruction. Much as multiplicities must be meshed together into a continuum while preserving their differences ('*exo*-consistency'), so the heterogeneous components of a multiplicity must themselves be meshed by a 'point of absolute survey' (WIP, 21) which continuously traverses them at infinite speed ensuring their '*endo*-consistency'. Exo-consistency is explained in WIP in terms of resonances between divergent series:

> Concepts which have only [endo-]consistency or intensive ordinates outside of any coordinates, freely enter into relationships of non-discursive resonance . . . Concepts are centers of vibrations, each in itself and every one in relation to all the others. This is why they all resonate rather than cohere or correspond to each other . . . They do form a wall, but it is a dry-stone wall, and everything holds together only along diverging lines. (WIP, 23)

The quasi-causal operator behind these effects of endo- and exo-consistency is referred to as a 'conceptual persona'. Thus, Deleuze writes: 'The conceptual persona is needed to create concepts on the plane, just as the plane needs to be laid out. But these two operations do not merge in the persona, which itself appears as a distinct operator' (WIP, 76). Conceptual personae are endowed with all the characteristics of the quasi-causal operator. Much as the latter must inject as much chance into the distributions of the singular and the ordinary in virtual series, 'the persona establishes a correspondence between each throw of the dice and the intensive features of a concept . . .' (WIP, 75). And much as the operator is said to extract ideal events from what actually occurs (that is, to perform counter-actualizations or 'counter-effectuations'), in philosophy 'it is precisely the conceptual persona who counter-effectuates the event' (WIP, 76).

But why the term 'persona'? A clue to the meaning of this expression may be glimpsed from some remarks in LOS. As I have just said, in the circles of *convergence* defined by pre-actualized multiplicities an intensive individual develops (larval subject), an individual which expresses the world which *convergent* series form. Similarly, in the divergent series a 'virtual person' develops, a person who expresses what is common to many different worlds (LOS, 115). A more detailed explanation, however, emerges from a discussion in D&R. Much as a larval subject is born from percepts and affects which do not refer to psychological phenomena, but are the very being of the sensible, so personae are intimately connected with what constitutes the very being of the intelligible (D&R, 141). Difference in intensity is the being of the sensible ('sentiendum') and simultaneously that which cannot be sensed (by fully actualized individuals) since it is normally covered by extensities and qualities (D&R, 144). Similarly, the being of the intelligible ('cogitandum') is what can only be thought and at the same time that which marks the impossibility of thought (again, impossibility from the point of view of a fully actualized thinker). Hence the need to invent a conceptual persona to capture these cogitanda or 'thought-events', a persona who 'lives intensely within the thinker and forces him to think' (WIP, 70).

Finally, there is the third component: the virtual continuum itself or the 'plane of immanence' of a philosophy. This refers to the presuppositions of a philosophy, the main one of which is an assumed 'image of thought' (WIP, 37), in other words, a pre-conceptual intuition of what it is to think: 'Every philosophy depends upon an intuition that its concepts constantly develop through slight differences of intensity . . .' (WIP, 40). One way of understanding what this means is to think of the relation between concepts and the plane of immanence as that between solutions and problems. As I discussed in Chapter 4, problems are not reducible to their solutions but rather are defined by their conditions: a given distribution of the singular and the ordinary, the important and the unimportant. As such, problems are inherently 'obscure yet distinct' and only acquire clarity in the process which progressively specifies each of their solutions. The intuition referred to above would refer to the grasping of a problem as such, as distinct and obscure (as opposed to grasping an essence, or a clear and distinct idea), an intuition which can only reveal itself progressively as concepts are created as cases of solution:

> If the concept is a solution, the conditions of the philosophical problem are found on the plane of immanence presupposed by the concepts . . . and the unknowns of the problem are found in the conceptual personae that it calls up . . . Each of these three instances is found in the others, but they are not of the same kind, and they coexist and subsist without one disappearing into the other . . . [T]he three activities making up [the philosophical method] continuously pass from one to the other, support one another, sometimes precede and sometimes follow each other, one creating concepts as a case of solution, another laying out a plane and a movement on the plane as the conditions of a problem, and the other inventing a persona as the unknown of the problem. (WIP, 81)

In my reconstruction of Deleuze's ontology I used as a guiding constraint the avoidance of the categories of typological thought: resemblance, identity, analogy and contradiction. But I could have as well said that what guides this construction is the avoidance of the image of thought implied by these categories: 'a natural capacity for thought endowed with a capacity for truth or an affinity with the true . . .' (D&R, 131). This image which, Deleuze argues, haunts the history of philosophy, has the result of turning the plane of immanence into a plane of transcendence. Or what amounts to the same thing, to trap philosophy within the plane of reference, linking it to linguistic propositions which are either true of or false of their referents. This manoeuver, of course, closes the road to the virtual or the problematic. If, on the contrary, the image of thought leads to a plane of immanence, then philosophy 'does not consist in knowing and it is not inspired by truth. Rather it is categories like Interesting, Remarkable, or Important that determine success or failure' (WIP, 82). The image of thought that has this problematic effect is one in which thought is born from the *violent shock* of an encounter with pure intensive differences (being of the sensible), a shock which a philosopher may then be capable of communicating to his or her other faculties, leading all the way to pure virtual differences (being of the intelligible) (D&R, 140).

This is not the place to argue for or against this view of philosophy. Whether or not *all* philosophical systems may indeed be analysable in terms of the three components of the virtual remains an open question. On the other hand, I must take issue with the image of science which WIP develops, particularly because my disagreement with it bears not just on narrowly scientific questions but on deep ontological matters. Specifically, my main divergence from Deleuze's ontology occurs at the level of the flat ontology of individuals. I mentioned above that I broke with Deleuze's terminology by using the term 'individual' for extended and qualified actual beings, while he reserves it for intensive beings (larval subjects). But the break is more than just terminological. Although a flat ontology meshes well with many of Deleuze's ideas (his theory of actual time as a nested set of cyclic presents of different durations, for example), it is unclear to what extent he subscribed to such a view. In particular, in a flat ontology as I have developed here there is no room for totalities, such as 'society' or 'science' in general. But Deleuze does not seem to mind such entities. For example, while I would never speak of a virtual multiplicity corresponding to all of society (i.e. a 'social Idea' or 'social multiplicity') he does so without hesitation (D&R, 186).

In the case of 'science' as defined in WIP, that is, in terms of functions working as discursive propositions, the problem is that the image invoked is one too close to that created by Anglo-American philosophers of science of the first half of the twentieth century. All the examples of 'functives' (the components of functions) given in WIP come from classical mechanics. No mention is made, for instance, of the operators of quantum physics, which use functions themselves as inputs and outputs. And, of course, the question of what chemical or biological functions are is left mostly unspecified. This amounts to defining science as if its 'essence' was classical mechanics. Furthermore, much as old-school analytical philosophers disregarded the actual mathematical models used by physicists and focused exclusively on set theory, so Deleuze views set theory as the tool which constitutes the plane of reference of science (WIP, 121). My analysis in Chapter 4 of classical mechanics (as an *individual* field) broke with all this. It preserved the idea that classical physics (as many other scientific fields) is mostly concerned with the plane of reference (actual beings, metric spaces) but it uses a very different conception of how reference (or the fixing of reference) is achieved, placing more emphasis on causal interventions than on representations. Similarly for my treatment of mathematical models, which are not reduced to linguistic entities (functions as propositions) but tackled in their specificity.

On the other hand, my analysis of classical physics meshes well with Deleuze's views on science as developed elsewhere. The requirement of avoiding the categories of typological thought to prevent the plane from becoming a plane of transcendence may also be expressed by saying that we must avoid the 'classical image of thought, and the striating of mental space it effects' (ATP, 379). The term 'striated space' refers to a metric space, while nonmetric spaces, 'vectorial, projective, or topological' (ATP, 361) are referred to as 'smooth'. The transformation of thought itself into a metric space is not, however, an internal affair of philosophy, but on the contrary, it's directly linked to the relations between individual philosophers (e.g. Hegel) and individual State or Royal institutions. It is these intitutions which first striate or metricize real space (e.g. agricultural lands, urban areas), and later perform the same operation on mental spaces. The opposite transformation, to create a nonmetric space for thought is performed by philosophers (e.g. Spinoza) who operate outside of the State.

A similar distinction is made between scientific fields, or even among the different practices (theoretical as opposed to experimental) within one field. We have, on one hand, 'Royal science' (the science of the great Royal Societies or Academies at the service of the State), and, on the other, the 'minor sciences' operating in less prestigious surroundings. Roughly, the distinction is between scientific practices which are axiomatic or theorematic, as opposed to problematic; that operate within metric and exactly measurable spaces, as opposed to dealing with anexact yet rigorous nonmetric ones; that focus on the simple behaviour of matter, as in ideal solids or gases, as opposed to confronting the complex behaviour of liquids (e.g. turbulence); and that stress constant and homogeneous laws, as opposed to becomings and heterogeneities (ATP, 361). My account of classical physics, which is clearly at odds with the Royal and legalistic image which that field has of itself, may be seen as an account *from the point of view of minor science*. But for the same reason, it makes the distinction which WIP establishes between science and philosophy pass right through the middle of science itself. This, it seems to me, is the 'more Deleuzian' approach to the subject.

# *Notes*

## 1 THE MATHEMATICS OF THE VIRTUAL: MANIFOLDS, VECTOR FIELDS AND TRANSFORMATION GROUPS

1. The term 'multiplicity' makes its first appearance, as far as I can tell, in 1966 in Deleuze's book on Bergson, Gilles Deleuze, *Bergsonism* (Zone Books, New York, 1988), p. 39. Its final appearance occurs in Deleuze's last book in collaboration with Félix Guattari, Gilles Deleuze and Félix Guattari, *What Is Philosophy?* (Columbia University Press, New York, 1994), p. 15.
2. Morris Kline, *Mathematical Thought from Ancient to Modern Times*, Vol. 3 (Oxford University Press, New York, 1972), p. 882. (My emphasis)

   Making surfaces into spaces, by eliminating the supplementary dimension, allowed the differentiation and study of different metric geometries. As Morris Kline writes:

   > Thus if the surface of the sphere is studied as a space in itself, it has its own geometry, and even if the familiar latitude and longitude are used as the coordinates of points, the geometry of that surface is not Euclidian . . . However the geometry of the spherical surface is Euclidian if it is regarded as a surface in three-dimensional space. (p. 888)

   For the details on Gauss coordinatization procedure, which is what guarantees this absence of a supplementary dimension or embedding space, see Lawrence Sklar, *Space, Time, and Space–Time* (University of California Press, Berkeley, 1977), pp. 27–42.
3. Kline, *Mathematical Thought*, p. 890.
4. Gilles Deleuze, *Difference and Repetition* (Columbia University Press, New York, 1994), p. 182. On page 183, for example, he says: 'In all cases the multiplicity is intrinsically defined, without external reference or recourse to a uniform space in which it would be submerged.' See also Gilles Deleuze and Félix Guattari, *A Thousand Plateaus* (University of Minnesota Press, Minneapolis, 1987), pp. 8–9,

   > Unity always operates in an empty dimension supplementary to that of the system considered (overcoding) . . . [But a] multiplicity never allows

itself to be overcoded, never has available a supplementary dimension over and above its number of lines, that is, over and above the multiplicity of numbers attached to those lines.

5. Deleuze and Guattari, *A Thousand Plateaus*, p. 266. The remark quoted is made about the 'plane of consistency' not about multiplicities. But the former is nothing but the space formed by the multiplicities themselves, as I will explain in detail in the next chapter.
6. When Deleuze defines his multiplicities he always seems to be referring to manifolds whose dimensions are used to represent degrees of freedom (or independent variables) of some dynamic, and not to manifolds as mere geometric objects. Thus, in his first introduction of the term he says,

   > Riemann defined as 'multiplicities' those things that could be determined by their dimensions or their independent variables. He distinguished between discrete multiplicities and continuous multiplicities. The former contain the principle of their own metrics . . . The latter found a metrical principle in something else, even if only in phenomena unfolding in them or in the forces acting in them. (*Bergsonism*, p. 39)

   And elsewhere he says, using the word 'Idea' to refer to concrete universals or multiplicities as replacements for essences,

   > An Idea is an n-dimensional, continuous, defined multiplicity. Colour – or rather, the Idea of colour – is a three dimensional multiplicity. By dimensions, we mean the variables or coordinates upon which a phenomenon depends; by continuity, we mean the set of relations between changes in these variables . . . by definition, we mean the elements reciprocally determined by these relations, elements which cannot change unless the multiplicity changes its order and its metric. (*Difference and Repetition*, p. 182)

7. I take this rather simplified description from Ian Stewart. *Does God Play Dice? The Mathematics of Chaos* (Basil Blackwell, Oxford, 1989), Chapter 6.

8. > Looking for relationships between the different solution curves [i.e. trajectories] of the same differential equation, Poincaré began with a local analysis and examined the behavior of these curves in the neighborhood of a singular point . . . He showed that there were four possible different types of singular points and classified them by the behavior of the nearby solution curves: *nœuds* (nodes), through which an infinite number of solution curves pass; *cols* (saddle points), through which only two solution curves pass . . . *foyers* (foci), which the solution curves approach in the

> manner of a logarithmic spiral; and *centres* (centers), around which the solution curves are closed, enveloping one another. Having used direct algebraic computation to show that these four types necessarily exist, he studied their distribution. He found that in the general case only three types prevailed – nodes, saddle points and foci – with centers arising in only exceptional circumstances. (June Barrow-Green, *Poincaré and the Three Body Problem* [American Mathematical Society, 1997], p. 32)

Roughly, we can say that Poincaré discovered not only the *existence* of certain recurrent 'topological forms' which are bound to appear in a large class of different physical models, but also that some of these forms are 'more generic' than others, that is, that if we study the *distribution* of singularities in many different models some of them (centers) are less likely to occur than others. See also discussion of the term 'generic', a technical term whose meaning is still evolving, in Ralph Abraham and Christopher Shaw, *Dynamics: The Geometry of Behavior*, Vol. Three (Aerial Press, Santa Cruz, 1985), pp. 19–34.

9. Deleuze and Guattari, *A Thousand Plateaus*, p. 408.
10. 'To reverse Platonism', as Deleuze says, we need 'first and foremost to remove essences and to substitute events in their place, as jets of singularities' (Gilles Deleuze, *Logic of Sense* [Columbia University Press, New York, 1990], p. 53).
11. Speaking of the image of the light of reason (or of rationality as a faculty capable of grasping the essential truth of things) Deleuze says,

> The very conception of a natural light is inseparable from a certain value supposedly attached to the Idea – namely, 'clarity and distinctness' . . . The restitution of the Idea in the doctrine of the faculties requires the explosion of the clear and distinct, and the discovery of a Dionysian value according to which *the Idea is necessarily obscure in so far as it is distinct*, all the more obscure the more it is distinct.' (Emphasis in the original; Gilles Deleuze, *Difference and Repetition*, p. 146)

The term 'Idea' here refers to multiplicities, and the fact that Deleuze uses that Platonic term shows he means to replace essences with multiplicities,

> Ideas are by no means essences. In so far as problems are the object of Ideas, problems belong on the side of events, affections, or accidents, rather than of theorematic essences . . . Consequently the domain of Ideas is that of the inessential. (p. 187)

12. Self-assembly during [the early stages of] embryonic development is not mediated by direct gene intervention. When all the transcriptions have

> been prevented [through the use of an inhibitor] the regular cleavage patterns are retained. However, the polarity of molecular organization of both the egg's cytoplasm and its nucleus . . . are essential for normal development. Hence the main features of [early] embryogenesis – cell differentiation, induction, determination of pattern formation – all stem from the oogenetically originated, spatial distribution of preformed informational macromolecules. The initial condition of embryogenesis is oogenesis. The epigenetics of embryonic development is built on the topological self-organization and orientation of macromolecules of the total egg. (Vladimir Glisin, '*Molecular Biology in Embryology. The Sea Urchin Embryo*', in *Self-Organizing Systems. The Emergence of Order*, ed. Eugene Yates [Plenum, New York 1987], p. 163)

The term 'oogenesis' refers to the process which creates the egg in the first place.

13. Joe Rosen, *Symmetry in Science* (Springer-Verlag, New York, 1995), Chapter 2.

    Besides closure, a collection of entities together with a rule of combination needs to display associativity, and possession of identity and inverse elements. The set of positive integers (including zero, and using addition as a combination rule) displays associativity because the result of adding two numbers first, and then adding a third one is the same as that of adding the first to what results from adding the last two. It also contains an 'identity element', that is, an element which added to any other leaves the latter unchanged (in this case the identity element is the number zero). But it fails to be a group because it lacks inverse elements, those which when composed with certain others yield the identity element. For instance, the number '−3' when composed with the number '+3' does yield zero (which is the identity element) but '−3' is not part of the set of positive integers. Thus, for the integers to form a group we must also include negative numbers in the set.

14. This dynamic aspect of symmetry-based classifications is obscured in standard presentations of the subject by the fact that the emphasis is not placed on the transformation as an event, but on its input and output. That is, the transformation is a process but all that matters mathematically is the initial and final states of the object transformed. See Ian Stewart and Martin Golubitsky, *Fearful Symmetry* (Blackwell, Oxford, 1992), pp. 32–3.

15. *Ibid*., p. 97.

    Besides assuming ideal solids and gases, this illustration of broken symmetry assumes that the gas container and the crystal lattice are infinite in all directions. The use of an 'observer' to define invariance is just a convenience. The subjective point of view can, in fact, be avoided. See Joe Rosen, *Symmetry in Science*, pp. 173–4.

16. Stewart and Golubitsky, *Fearful Symmetry*, Chapter 7.
17. Ralph Abraham and Christopher Shaw, 'Dynamics: A Visual Introduction', in *Self-Organizing Systems*, ed. Yates, p. 576.
18. Stewart and Golubitsky, *Fearful Symmetry*, Chapter 5. See also, Gregoire Nicolis and Ilya Prigogine, *Exploring Complexity* (W. H. Freeman, New York 1989), pp. 12–15.
19. Brian C. Goodwin, 'The Evolution of Generic Forms', in *Organizational Constraints on the Dynamics of Evolution*, ed. J. Maynard Smith and G. Vida (Manchester University Press, Manchester 1990), pp. 113–14.
20. Deleuze, *Difference and Repetition*, p. 187.

    Although Deleuze does not explicitly use the term 'symmetry-breaking cascade', he does refer to an 'embedding of groups' (p. 180) precisely in the context of explaining how a multiplicity may be progressively determined. Unfortunately, his brief discussion of groups uses a very obscure aspect of Galois's method, the originator of group theory, called the 'adjunction of fields'. The two formulations are, nevertheless, equivalent, fields of numbers and groups being two related nineteenth-century abstract objects. An algebraic problem, specified progressively as its field is completed by successive adjunctions, is the equivalent of an abstract smooth space being specified by a progressive series of broken symmetries, yielding increasingly more differentiated, more striated spaces. Deleuze's discussion of Galois is correct technically, but it is not as clear and intuitive as the equivalent formulation in terms of 'embedding of groups'. Hence in this reconstruction I will stick with the clearer alternative. But whether one uses fields or groups, it is clear that some form of *progressive differentiation* is a key component of the concept of a Deleuzian multiplicity.

21. What distinguishes a space as opposed to a mere set of points is some concept that binds the points together. Thus in Euclidean space the distance between points tells how close points are to each other . . . As Frechet [a pioneer in the development of topology] pointed out, the binding property need not be the Euclidean distance function. In particular he generalized the notion of distance by introducing the class of metric spaces. In a metric space, which can be a two-dimensional Euclidean space, one speaks of the neighborhood of a point and means all those points whose distance from the point is less than some quantity . . . However, it is also possible to suppose that the neighborhoods, certain subsets of a given set of points, are specified in some way, *even without the introduction of a metric*. Such spaces are said to have a

> neighborhood topology. (Morris Kline, *Mathematical Thought*, p. 1160; my emphasis)

I will use the terms 'metric space' and 'nonmetric space' throughout this book in the sense in which they are defined in this quote but I will take some liberties. I will speak of topological spaces, for example, as the 'least metric' and of Euclidean as the 'most metric', even though it would be more technically correct to differentiate *features of spaces that do or do not depend on any strictly metric property*.

22. Deleuze usually speaks (following Bergson) of two different *types* of multiplicities, metric and nonmetric, which he calls 'striated' and 'smooth'. For the purposes of ensuring the correct interpretation of Deleuze's position here it would have been very useful if he had ever discussed Felix Klein's work, thereby clarifying the relations between the metric and the nonmetric as one of group inclusion. Unfortunately, as far as I can tell, Deleuze never discusses Klein. On the other hand, Deleuze is perfectly aware of the existence of several nonmetric geometries and uses *a single term* ('smooth space') to refer to all of them:

> It is the difference between a smooth (*vectorial*, *projective*, *or topological*) space and a striated (*metric*) space: in the first case 'space is occupied without counting' and in the second case 'space is counted in order to be occupied'. (Deleuze and Guattari, *A Thousand Plateaus*, p. 361; my emphasis)

The definitions given in the extract are his own, but are linked to the more orthodox definitions. A metric space is counted in order to be occupied in the sense in which sedentary cultures divide the land into measured (or counted) plots in order to inhabit it:

> Good sense is . . . agricultural, inseparable from the agrarian problem, the establishment of enclosures, and the dealings of middle classes the parts of which are supposed to balance and to regulate one another. The steam engine and livestock, but also properties and classes, are the living sources of good sense, not only as facts that spring up at a particular period, but as eternal archetypes. (Deleuze, *Logic of Sense*, p. 76)

To the sedentary way of metricizing space, of dealing with it as essentially extensive, Deleuze opposes an intensive way of occupying space the way a liquid does, that is, occupying it without dividing it or counting it. This alternative he calls a 'nomadic distribution'. The distinction between sedentary and nomadic distributions is first made in *Difference and Repetition*, pp. 36–7, in relation to questions of typological thinking, but is taken further in an actual comparison of nomad and sedentary cultures

> . . . even though the nomadic trajectory may follow trails or customary routes, it does not fulfill the function of the sedentary road, which is to *parcel out a closed space to people*, assigning each person a share and regulating the communication between shares. The nomadic trajectory does the opposite: it *distributes people (or animals) in an open space* . . . sedentary space is striated [i.e. metricized], by walls, enclosures and roads between enclosures, while nomadic space is smooth [i.e. non-metric], marked only by 'traits' that are effaced and displaced with the trajectory. (Deleuze and Guattari, *A Thousand Plateaus*, p. 380; emphasis in the original)

23. Morris Kline, *Mathematical Thought*, p. 917.
24. David A. Brannan, Matthew F. Esplen, Jeremy J. Gray, *Geometry* (Cambridge University Press, Cambridge, 1999), p. 364.
25. This way of describing the subject oversimplifies things somewhat. First of all, the actual relations between the different geometries are more complex than the simplified hierarchy 'topological–differential–projective–affine–Euclidean geometries' may suggest. For the details of Klein's original classification see *ibid*., p. 919.

    My friend the mathematician Andreas Dress (personal communication) summarizes Klein's programme (called the Erlanger Program) like this,

> The Erlanger Program by Felix Klein is based on the fact that depending on which (bijective) transformations you need to deal with (isometries keeping distances invariant, similarities scaling all distances by the same factor and, hence, keeping ratios of distances invariant, affine maps keeping ratios of distances of points on parallel lines invariant, projectivities keeping cross-ratios of distances invariant, differential transformations respecting infinitesimal straightness, homeomorphisms respecting nothing but infinitesimal closeness), it always makes sense to ask (1) which features of configurations within the space of interest do remain invariant, and (2) whether a basic family of such features can be found so that every other such feature can be expressed as a function of those basic ones.

26. Morris Kline, *Mathematical Thought*, p. 921. There are important exceptions to this statement. Some mathematicians, like Riemann himself, but also William Clifford, did see an ontological connection between the metric and

nonmetric properties of spaces. As one historian of twentieth-century physics writes,

> [Riemann] asserted that space in itself was nothing more than a three-dimensional manifold devoid of all form: it acquired a definite form only through the material content filling it and determining its metric relations . . . Riemann's anticipation of such a dependence of the metric on physical data later provided a justification for avoiding the notion of absolute space whose metric is independent of physical forces. For example, more than sixty years later, Einstein took Riemann's empirical conception of geometry using it as an important justification for his general theory of relativity.
>
> (Tian Yu Cao, *Conceptual Development of Twentieth-Century Field Theories* [Cambridge University Press, Cambridge, 1997], p. 373)

27. Gordon Van Wylen, *Thermodynamics* (John Wiley & Sons, New York, 1963), p. 16.

28. What is the significance of these indivisible distances that are ceaselessly transformed and cannot be divided or transformed without their elements changing in nature each time? Is it not the intensive character of this type of multiplicity's elements and the relations between them? Exactly like a speed or a temperature, which is not composed of other speeds or temperatures, but rather is enveloped in or envelops others, each of which marks a change in nature. The metrical principle of these multiplicities is not to be found in a homogeneous milieu but resides elsewhere, in forces at work within them, in physical phenomena inhabiting them . . . (Deleuze and Guattari, *A Thousand Plateaus*, pp. 31–3)

The term 'distance' is used as if it was a nonmetric property, though in its usual meaning it certainly denotes something metric. Deleuze takes this special intensive meaning of 'distance' from Bertrand Russell as I will discuss in detail later in the next chapter. On distances as intensive magnitudes, or as 'indivisible asymmetrical relations' see Deleuze, *Difference and Repetition*, p. 237. Deleuze does not explicitly give phase transitions as examples of 'changes in kind'. But one of the very few illustrations he does give is indeed a symmetry-breaking transition, 'For example, one can divide movement into the gallop, trot, and walk, but in such a way that what is divided changes in nature at each moment of the division . . .' (Deleuze and Guattari, *A Thousand Plateaus*, p. 483).

On phase transitions in animal movement as broken symmetries see, Stewart and Golubitsky, *Fearful Symmetry*, Chapter 8.

29. Cao, *Conceptual Development of Twentieth-Century Field Theories*, p. 283.

30. The essential idea of grand unified theories . . . [is] the general form of hierarchical symmetry breaking: an underlying large gauge symmetry of all interactions is broken down in a succession of steps, giving a hierarchy of broken symmetries. (*ibid*., p. 328)

31. It is beyond the scope of this chapter to analyse Einstein's use of differential manifolds in technical detail. But I should at least mention the way in which his usage differs from that of Deleuze. In Einstein's theory a gravitational field constitutes the metric structure of a four-dimensional manifold (spacetime), and to this extent, the metric properties of space (rather, spacetime) are indeed connected to the physical processes which occur within it. However, as the philosopher of science Lawrence Sklar reminds us, despite the fact that Einstein's field equation does relate the metric of a manifold to the distribution of mass and energy, the relation between the two is not genetic: the metric is *not caused* by the mass–energy distribution, it is only associated with it in a lawlike way. See Sklar, *Space, Time, and Space–Time*, pp. 50–1.

32. The move away from metamathematics (set theory) and back to the actual mathematics used by scientists was initiated by the philosopher Patrick Suppes. Yet the credit for the introduction of state space into modern analytical philosophy, as well as the credit for emphasizing physical modality in the analysis of that space, goes to another philosopher, Bas Van Fraasen. See Bas Van Fraasen, *Laws and Symmetry* (Clarendon Press, Oxford, 1989), Chapter 9.

33. Ralph Abraham and Christopher Shaw, *Dynamics: The Geometry of Behavior*, Vol. 1 (Aerial Press, Santa Cruz, 1985), pp. 20–1. My description is merely a paraphrase of the following description:

> The modeling process begins with the choice of a particular state space in which to represent the system. Prolonged observations lead to many trajectories within the state space. At any point on any of these curves, a velocity vector may be derived [using the differentiation operator]. It is useful in describing an inherent tendency of the system to move with a habitual velocity, at particular points in the state space. The prescription of a velocity vector at each point in the state space is called a *velocity vector field*. The state space, filled with trajectories, is called the *phase portrait* of the dynamical system. The velocity vector field has been

derived from the phase portrait by *differentiation* . . . The phrase *dynamical system* will specifically denote this vector field. (Emphasis in the original)

34. Albert Lautman, quoted in Gilles Deleuze, *Logic of Sense* (Columbia University Press, New York, 1990) p. 345. (My emphasis)
    Lautman's *Le Problème du Temps* (from which this extract is taken) and 'Essai sur le Notion de Structure et d'Existence en Mathematiques', are Deleuze's main sources on the ontological analysis of state space. Deleuze paraphrases Lautman's description in other books, but given the centrality of these ideas in his work I prefer to quote Lautman's own words.
35. Abraham and Shaw, *Dynamics: The Geometry of Behavior*, pp. 35–6.
36. Nicolis and Prigogine, *Exploring Complexity*, pp. 65–71.
37. Abraham and Shaw, *Dynamics: The Geometry of Behavior*, pp. 37–41.
38. Abraham and Shaw, *Dynamics: A Visual Introduction*, p. 562.
39. Deleuze, *Difference and Repetition*, pp. 208–9. (Emphasis in the original.) Deleuze borrows the ontological distinction of the actual and the virtual from Bergson. See Deleuze, *Bergsonism*, pp. 96–7.
40. Willard Van Orman Quine, quoted in Nicholas Rescher, 'The Ontology of the Possible', in *The Possible and the Actual*, ed. Michael J. Loux (Cornell University Press, Ithaca, 1979), p. 177.
41. For a brief account of the recent history of modal logic, see Michael J. Loux, 'Introduction: Modality and Metaphysics', in *Loux, The Possible and the Actual*, pp. 15–28.
42. Ronald N. Giere, 'Constructive Realism', in *Images of Science. Essays on Realism and Empiricism with a Reply by Bas C. Van Fraasen*, eds. Paul M. Churchland and Clifford A. Hooker (University of Chicago Press, 1985), p. 84.
43. Bas Van Fraasen, *Laws and Symmetry*. p. 223. Van Fraasen discusses the two standard types of laws, laws of succession (which govern the evolution of trajectories, and are exemplified by Newton's laws) and laws of coexistence (which restrict position in state space, and are illustrated by Boyle's law for ideal gases).
44. Exactly matching initial conditions in the laboratory and the model is not possible, so we normally deal with *bundles of trajectories* in state space. The statistical distribution of a small population of initial states in the model is made to match that of the errors which the experimenter may have made in preparing the real system in a particular initial condition. In what follows this point will not make much difference so I stick to the simpler case of a single trajectory.
45. Giere argues that the regularities exhibited by the possible histories reveal something about the *causal regularities* in the real physical system:

> For the modal realist, the *causal* structure of the model, and thus, to some degree of approximation, of the real system, is identical with the *modal* structure. For any real system, the functional relationship among the actual values of [the degrees of freedom] are causal not because they hold among the *actual* values in *all* such real systems but because they hold for all *possible* values of *this* particular system. (*Constructive Realism*, p. 84; emphasis in the original)

See also Ronald N. Giere, *Explaining Science. A Cognitive Approach* (University of Chicago Press, 1988), Chapter 4. Giere is, in this case, wrong. State space, as I will argue in Chapter 4, provides no causal information about the modelled processes.

46. > One's attitude towards modalities has a profound effect on one's whole theory of science. Actualists . . . must hold that the aim of science is to describe the actual history of the world. For [modal realists] . . . the aim is to describe the structure of physical possibility (or propensity) and necessity. The actual history is just that one possibility that happened to be realized . . . (Giere, *Constructive Realism*, p. 84)

47. Deleuze, *Logic of Sense*, p. 54.
48. Considering that Deleuze's analysis hinges on the difference between the differentiation and integration operators of the calculus, it will be necessary to remove one traditional objection to the very idea of giving an ontological dimension to these operators. This objection is that the output of the differentiation operator (instantaneous rates of change or infinitesimals) cannot be thought of as anything but mathematical fictions. Not to do so has led in the past to many sterile speculations and controversy. However, although a vector field is indeed composed of many of these instantaneous rates of change, what matters to us here are not the 'instants' themselves, taken one at a time, but the *topological invariants* which those instants display collectively, that is, the singularities of the field.
49. Stephen G. Eubank and J. Doyne Farmer, 'Introduction to Dynamical Systems', in *Introduction to Nonlinear Physics*, ed. Lui Lam (Springer-Verlag, New York, 1997), p. 76.
50. Abraham and Shaw, *Dynamics: The Geometry of Behavior*, pp. 7–11.
51. Attractors are indeed defined as a 'limit set' with an open inset (its basin). But the word 'limit' in the definition makes all the difference in the world, since it refers precisely to the tendencies of trajectories to approach the attractor in the limit. See *ibid*., p. 44.

52. > 'Intuitively, according to Russell, a system is deterministic exactly if its

> previous states determine its later states in the exact sense in which the arguments of a function determine its values. (Van Fraasen, *Laws and Symmetry*, p. 251)

See Van Fraasen's discussion of the relation between the modal category of physical necessity and deterministic laws in Chapters 3 and 4 of *Laws and Symmetry*.

53. Nicolis and Prigogine, *Exploring Complexity*, p. 14. (Emphasis in the original.)
54. For example, the way Deleuze approaches the question of necessity is by splitting the causal link: on one hand, processes of individuation are defined as sequences of causes (every effect will be the cause of yet another effect) while singularities become *pure incorporeal effects* of those series of causes; on the other hand, these pure effects are viewed as having a quasi-causal capacity to affect causal processes. By splitting causality this way, Deleuze manages to separate the determinism which links causes to causes, from strict necessity. See *Logic of Sense*, p. 169.

Deleuze uses the word 'determinism' as synonymous with 'necessity', and uses the word 'destiny' instead for the modified link between causes. I keep the word 'determinism' to avoid introducing neologisms, but emphasize the break with strict necessity. Another way of expressing Deleuze's conceptualization of this modality is from *Difference and Repetition*, p. 83,

> Destiny never consists in step-by-step deterministic relations between presents which succeed one another . . . Rather, it implies between successive presents *non-localizable connections*, actions at a distance, systems of replay, resonances and echoes . . . which transcend spatial locations and temporal successions.' (My emphasis)

The idea of 'non-localizable connections' is the key concept here and can be understood by reference to convection cells. While the causal interactions between the cell's components are localizable collisions (billiard-ball style causality), the source of coherence in the flow pattern (the periodic attractor) is, indeed, nowhere specifically in space or time. The attractor establishes connections (else there would be no coherence in the flow) but not localizable ones.

55. Willard Van Orman Quine, 'Reference and Modality', in *From a Logical Point of View* (Harper & Row, New York, 1965), p. 155. Even though most modal analyses deal with purely linguistic phenomena, such as counterfactual sentences, the moment one approaches such sentences as referring to the real world (technically, the moment we quantify over possible entities) we acquire an ontological commitment to the existence of essences. In other

words, we commit ourselves to affirm that objects possess some of their properties necessarily while others only contingently.

56. The first option (ensuring transworld identity through particular essences or hacceities) is exemplified by Alvin Plantinga, 'Transworld Identity or Worldbound Individuals?', in *Loux, The Possible and the Actual*, pp. 154–7.

    The second option (counterparts linked through general essences) is illustrated by David Lewis, 'Counterpart Theory and Quantified Modal Logic', in *The Possible and the Actual*, pp. 117–21.
57. Deleuze, *Difference and Repetition*, pp. 211–12. See also Deleuze, *Bergsonism*, p. 97. Deleuze does not, in fact, refer to the virtual as a physical modality, but the fact that he explicitly contrasts virtuality and possibility (following Bergson's lead) does indicate that he is thinking in modal terms.
58. I take this description of Aristotelian philosophy from Elliot Sober, *The Nature of Selection* (MIT Press, Cambridge, 1987), pp. 156–61.
59. Deleuze, *Difference and Repetition*, p. 29. To avoid falling prey to the dangers of representationalism (or as I call it typological thinking) Deleuze follows Michel Foucault's analysis of classical representation, which according to the latter forms an epistemological space with four dimensions or 'degrees of freedom': identity, resemblance, analogy and opposition, p. 262.

    For a discussion of this aspect of Foucault's thought from the point of view of an analytical philosopher see Gary Gutting, *Michel Foucault's Archaeology of Scientific Reason* (Cambridge University Press, 1993), Chapter 4.

    In what follows I simply take the idea that there are recurrent features in these classificatory practices (resemblance, identity, etc.) but not that these form a global entity called an 'episteme'. I do not believe such global entities or totalities exist as will become clear in the following chapters.

60. > 'The first formula posits resemblance as the condition of difference. It therefore undoubtedly demands the possibility of an identical concept for the two things that differ on condition that they are alike . . . According to the other formula, by contrast, resemblance, identity, analogy and opposition can no longer be considered anything but effects of a primary difference or a primary system of differences. (Deleuze, *Difference and Repetition*, p. 117)

    Deleuze, in fact, does not speak of 'constraints guiding a constructive project'. He rather affirms his desire for creating a *philosophy of difference*, and then denounces the categories of typological or representational thinking as obstacles to reaching that goal. The differences he has in mind are not the external *differences between things* that are part and parcel of classificatory practices, but productive differences perhaps best illustrated by *intensive* *differences*, differences in temperature, pressure, etc. within one and the same system, which are marked by thresholds of intensity determining phase transitions. See p. 222.

61. Ronald F. Fox, *Energy and the Evolution of Life* (W. H. Freeman, New York, 1988), p. 8.

> The mechanisms by which the chemical elements come into existence is *stellar nucleosynthesis*. The processes involved are an example of how *energy flow* produces complex states of matter from simpler constituents. A combination of gravitational energy and nuclear energy converts vast quantities of hydrogen gas, the simplest element, into the nuclei of other more complex elements. Nucleosynthesis involves nuclear reaction cycles and happens in stages that correlate strongly with changes in stellar structure. (Emphasis in the original)

62. Philosophers tend to imagine that a piece of bulk material is simply a collection of individual crystals arranged so perfectly that, for all practical purposes, the properties of the bulk sample are simply a sum of the properties of these crystals. In other words, they imagine we can *divide the bulk sample in extension* and, given the packing arrangement of the crystals, we will always end up with a similar if smaller sample. But in reality, we do not have perfectly regular crystal lattices (the irregularities playing a crucial role in the stability of the structure) and we cannot divide a bulk sample beyond a given size without losing some emergent properties:

> Like the biologist, the metallurgist is concerned with aggregates and assemblies in which repeated or extended *irregularities* in the arranged atoms become the basis of major structural features on a larger scale, eventually bridging the gap between the atom and things perceptible to human senses. (Cyril Stanley Smith, 'Structure, Substructure, and Superstructure', in *A Search for Structure* [MIT Press, Cambridge, 1982], p. 54; my emphasis)

See also, in the same volume, Smith, 'Grain Shapes and other Metallurgical Applications of Topology'. On the emergence of bulk properties at different critical scales, see Michael A. Duncan and Dennis H. Rouvray, *Microclusters* (Scientific American, December, 1989), p. 113.

## 2 THE ACTUALIZATION OF THE VIRTUAL IN SPACE

1. Michael T. Ghiselin, *Metaphysics and the Origin of Species* (State University of New York Press, Albany, 1997), p. 78.

2. A good history of this debate, explaining the role which Michael Ghiselin played in it, can be found in David L. Hull, *Science as a Process* (University of Chicago Press, Chicago, 1988), Chapter 4.
3. Ghiselin, *Metaphysics and the Origin of Species*, pp. 37–41.
4. It is unclear to what extent Deleuze subscribes to this idea of a flat ontology of singular individuals. Some parts of his theory (for example, his theory of time involving a nested set of larger and larger temporal scales) seem to demand such an ontology. Yet, elsewhere, he does seem to talk of totalities. Thus, while I view the realm of the social as a flat ontology (made of individual decision-makers, individual institutional organizations, individual cities, individual nation states) and thus would never speak of 'society as a whole' or 'culture as a whole', Deleuze does talk of 'society as a whole' and specifically, of a virtual multiplicity of society. See, for example, Gilles Deleuze, *Difference and Repetition* (Columbia University Press, New York, 1994), p. 186. There are also terminological problems that need to be noted given that Deleuze uses the term 'individual' in a very idiosyncratic way. In particular, he does *not* use 'actual entity' and 'individual' as synonyms as I do. For Deleuze the term 'individual' refers to an entity *in the process of actualization*, that is, before it acquires its final qualities and extensities. For example, a fully developed human being would be an actual entity, but the embryo as it is being unfolded and developed would be an individual. One would be an extensive being, the other an intensive one. (See, for example, pages 247 and 250.) I will use the word 'individual' in the sense in which it is used by Ghiselin to link it to anti-essentialist thought, but this should not cause much distortion to Deleuze.

   On the other hand, I do break with Deleuze's use of the term 'species' which does not seem to imply that species are also individuals, and hence, the product of an individuation process *distinct* from the one that gives rise to organic individuals during embryogenesis. He does not seem to keep the two levels of scale separate (as I think they should be) and speaks of 'species' and 'parts' as the organic expression of qualities and extensities respectively (p 251). Yet, he does acknowledge in passing the role of reproductive isolation in the individuation of species. He writes,

   > A kinetics of population adjoins, without resembling, the kinetics of the egg; a geographical process of isolation may be no less formative of species than internal genetic variations, and sometimes precedes the latter. (p. 217)

5. Ernst Mayr, quoted in Elliot Sober, *The Nature of Selection* (MIT Press, Cambridge, 1987), p. 156.

6. *Ibid.*, p. 159. Sober makes some corrections to Mayr's way of explaining the reversal of Aristotelian essentialism. He believes it is incorrect to compare averages and essences, as Mayr does in the extract, since averages may be taken to be real properties at the populational level. So the reversal is characterized in terms of the role of variation: while for Aristotelians homogeneity is the natural state and variation is what needs special explanation, for population thinkers it is variation which is natural, while homogeneity, when it exists, is what needs to be explained.
7. *Ibid.*, p. 160.
8. Gilles Deleuze and Félix Guattari, *A Thousand Plateaus* (University of Minnesota Press, Minneapolis, 1987), p. 48. (My emphasis)
9. Niles Eldredge, *Macro-Evolutionary Dynamics* (McGraw-Hill, New York, 1989), pp. 155–7.
10. J. D. Murray, *Mathematical Biology* (Springer-Verlag, Berlin 1989), pp. 1–4.
11. *Ibid.*, pp. 8–11.
12. In both organism and cellular populations, for example, we are concerned with rates of birth (rates of cell division), rates of death, as well as migration rates. These rates of change, in turn, define in both cases a dynamical process which displays threshold effects as well as asymptotic stable states. Divergent universality also implies that these organic phenomena may share dynamical features with inorganic ones. Some processes, like the formation of concentration patterns due to an interaction between the rate at which a chemical reaction proceeds and the rate at which the products of that reaction diffuse, occur in both embryological processes and non-biological chemical processes (like the famous Belousov–Zhabotinsky reaction), a fact which suggests that a virtual multiplicity can be divergently actualized in both organic and inorganic molecular populations. Indeed, the mathematical techniques and analytical methods which are used to model interactions between animal and plant populations (such as predator–prey systems) are directly applicable to *reaction kinetics*, that is, to the dynamical models of interacting populations of molecules, organic or inorganic. See *ibid.*, p. 63.
13. For a discussion of population-level qualities see Sober, *Nature of Selection*, p. 167.

14. > How does actualization occur in things themselves? . . . Beneath the actual qualities and extensities [of things themselves] . . . there are spatio-temporal dynamisms. These are the actualizing, differenciating agencies. They must be surveyed in every domain, even though they are ordinarily hidden by the constituted qualities and extensities. Embryology shows that the division of the egg is secondary in relation to more significant morphogenetic movements: the augmentation of free surfaces, stretching

of cellular layers, invagination by folding, regional displacement of groups. A whole kinematics of the egg appears which implies a dynamic. (Deleuze, *Difference and Repetition*, p. 214)

15. Gerald M. Edelman, *Topobiology. An Introduction to Molecular Embryology* (Basic Books, New York, 1988), pp. 22–4.

16. As a result of epithelial-mesenchymal transformation, two kinds of motion can arise that differ to some degree in scale. The first involves the obvious cell migration that can take place after conversion to mesenchyme, as well as its cessation following condensation of mesenchyme into rounded epithelial masses. The second . . . is the folding, invagination or evagination of whole tissue sheets to form various structures, including tubes. In both cases, new cellular environments are created, leading to the possibility that different inductive signals will be released. (*Ibid.*, p. 70)

17. *Ibid.*, p. 94.
18. *Ibid.*, pp. 80–1.
19. The phrase 'anexact yet rigorous' is used on several occasions by Deleuze to refer to a style of thought, but also to a characteristic of topological manifolds themselves. One occasion is the discussion of Bertrand Russell's concept of 'ordinal distances' which I will discuss later in the main text. See, Deleuze and Guattari, *A Thousand Plateaus*, p. 483. Another use of the phrase occurs while discussing Husserl's notion of 'vague and material essences', topological essences which are assimilated to singularities (events) and affects (p. 407).
20. Arthur T. Winfree, *When Time Breaks Down. The Three-Dimensional Dynamics of Electrochemical Waves and Cardiac Arrhythmias* (Princeton University Press, Princeton, 1987), p. 253. (My emphasis)
21. Stuart Kauffman, *The Origins of Order. Self-Organization and Selection in Evolution* (Oxford University Press, New York, 1993), p. 461.
22. *Ibid.*, p. 442.

23. The expected network connectivity features exhibit strong self-organization properties analogous to phase transitions in physics, as the number of regulatory connections, M, among N genes increases. If M is small relative to N, the scrambled genomic system consists of many small genetic circuits, each unconnected to the remainder. As the number of regulatory connections, M, increases past the number of genes, N, large connected circuits form. The crystallization of large circuits as M increases is analogous to a phase transition. (Stuart Kauffman, 'Self-Organization,

Selective Adaptation and its Limits', in *Evolution at a Crossroads*, eds. David. J. Depew and Bruce H. Weber [MIT Press, Cambridge, 1996], pp. 180)

24. In Deleuze's philosophy the connection between multiplicities, on one hand, and qualities and extensities, on the other, is more intimately defined, with differential relations corresponding to qualities and singularities to extensities.

> [A] multiplicity such as that of colour is constituted by the virtual coexistence of relations between genetic or differential elements of a particular order. These relations are actualized in qualitatively distinct colours, while their distinctive points are incarnated in distinct extensities, which correspond to those qualities . . . We have seen that every process of actualization was in this sense a double differenciation, qualitative and extensive. (Deleuze, *Difference and Repetition*, p. 245)

25. K. Eric Drexler, 'Biological and Nanomechanical Systems: Contrasts in Evolutionary Capacity', in *Artificial Life*, ed. Christopher G. Langton (Addison-Wesley, Redwood City, 1989), p. 510.
26. Deleuze, *Difference and Repetition*, p. 223.

> Intensity creates the extensities and the qualities in which it is explicated; these extensities and qualities are differenciated . . . Creation is always the production of lines and figures of differenciation. It is nevertheless true that intensity is explicated only in being canceled in this differenciated system that it creates. (p. 255)

27. Van Wylen, *Thermodynamics*, p. 16.
28. Bertrand Russell, *Principles of Mathematics* (W. W. Norton, New York), p. 104 (for remarks on pleasure) and p. 171 (for remarks on colour). Deleuze would not count pleasure as an intensive quantity part of mental *individuating processes*. He seems to view pleasure as an effect of the cancelling of intensive differences:

> Biophysical life implies a field of individuation in which differences in intensity are distributed here and there in the form of excitations. The quantitative and qualitative process of the resolution of such differences is what we call pleasure. (Deleuze, *Difference and Repetition*, p. 96)

29. Martin H. Krieger, *Doing Physics. How Physicists Take Hold of the World* (Indiana University Press, Bloomington and Indianapolis, 1992), p. 130.
30. Deleuze, *Difference and Repetition*, p. 222. (My emphasis)

In this extract, 'diversity' refers to the world of actual phenomena and their externally defined differences (that is, to difference as subordinated to resemblance) while intensive differences define the in-itself (nuomena) of the world, the positive and productive differences which create or generate phenomena.

31. Ilya Prigogine and Isabelle Stengers, *Order out of Chaos. Man's New Dialogue with Nature* (Bantam Books, New York, 1984), p. 135.
32. Deleuze explains the relation between intensive differences and genetic differences by saying that 'complex systems increasingly tend to interiorize their constituent differences', that is, their individuating factors (*Difference and Repetition*, p. 256). See also Deleuze's discussion of Darwinian differences on pp. 248–9.
33. When discussing the virtual and the intensive, Deleuze usually divides the subject into two areas, although the terminology varies. Sometimes he speaks of 'singularities and affects', other times of 'speeds and affects', yet in other places he speaks of 'events and attributes'. All these formulations are, I believe, equivalent. See further discussion and references in Chapter 3, footnote 46.
34. On this new class of formal spaces which complements state space, see Walter Fontana, 'Functional Self-Organization in Complex, Systems', in *1990 Lectures in Complex Systems*, eds. Lynn Nadel and Daniel Stein (Addison-Wesley, Redwood City, 1991); and, in the same volume, Stuart Kauffman, 'Random Grammars: A New Class of Models for Functional Integration and Transformation in the Biological, Neural and Social Sciences'.
35. > We know nothing about a body until we know what it can do, what its affects are, how they can or cannot enter into composition with other affects, with the affects of another body, either to destroy that body or to be destroyed by it, either to exchange actions and passions with it or to join with it in composing a more powerful body. (Deleuze and Guattari, *A Thousand Plateaus*, p. 257)
36. James J. Gibson, *The Ecological Approach to Visual Perception* (Houghton Mifflin Company, Boston, 1979), pp. 15–16.
37. *Ibid.*, p. 132.
38. Some of the recurrent assembly patterns that have been discovered (and which may turn out to be universal) are of the type that articulates heterogeneous elements. Stuart Kauffman has coined the term 'meshwork' to refer to this type of assemblage. See Stuart Kauffman, *Random Grammars*, p. 428.

I have made extensive use of Kauffman's meshworks, and of their opposite, hierarchies, as recurrent assembly patterns for the analysis of human history in Manuel DeLanda, *A Thousand Years of Nonlinear History* (Zone Books, New York, 1997). A similar distinction (or a special case, that of centralized and decentralized decision-making systems) as well as a related set of recurrent assembly patterns (clockworks, motors and networks) is discussed and applied to history in Manuel DeLanda, *War in the Age of Intelligent Machines* (Zone Books, New York, 1991).

39. It is no longer a question of imposing a form upon a matter but of elaborating an increasingly rich and consistent material, the better to tap *increasingly intense forces*. What makes a material increasingly rich is the same as what *holds heterogeneities together* without their ceasing to be heterogeneous. (Deleuze and Guattari, *A Thousand Plateaus*, p. 329; my emphasis)

40. Deleuze, *Difference and Repetition*, p. 223.

    There is an illusion tied to intensive quantities. This illusion, however, is not intensity itself, but rather the movement by which difference in intensity is canceled. Nor is it only *apparently* canceled. It is really canceled, but outside itself, in extensity and underneath quality. (p. 240; my emphasis)

41. It is now an easy matter to extend our discussion to *nonequilibrium states* . . . They can be *transient* . . . But they can also be permanent if we establish and maintain appropriate conditions, which we refer to as *constraints*. Thus, a temperature difference applied between two sections of a slab . . . will result in nonequilibrium situations in which the system is never allowed to identify itself with its environment. We should not conclude from these examples that nonequilibrium is an artificially imposed condition . . . we see nonequilibrium states in much of our natural environment – for example, the state of the biosphere which is subjected to an energy flux that arises from the balance of radiation between the sun and the earth. (Emphasis in the original; Gregoire Nicolis and Ilya Prigogine, *Exploring Complexity* [W. H. Freeman, New York 1989], p. 56)

42. *Ibid.*, p. 59.
43. *Ibid.*, p. 60.
44. David Acheson, *From Calculus to Chaos. An Introduction to Dynamics* (Oxford University Press, Oxford, 1997), pp. 54–6.
45. Deleuze and Guattari, *What is Philosophy?*, p. 140. (My emphasis)
46. Richard Hinchliffe, 'Toward a Homology of Process: Evolutionary Implica-

tions of Experimental Studies on the Generation of Skeletal Pattern in Avian Limb Development', in O*rganizational Constraints on the Dynamics of Evolution*, eds. J. Maynard Smith and G. Vida (Manchester University Press, Manchester 1990), p. 123. (Emphasis in the original)

The biologist Brian Goodwin, who has taken the broken symmetry approach to classification to its extreme, argues that these insights about specific organs may be generalized to explain the dynamical origin of all the morphological features behind our static classifications:

> There are several consequences of this view of morphogenesis. First, it is evident that morphology is generated in a hierarchical manner, from simple to complex, as bifurcations result in spatially ordered asymmetries and periodicities, and nonlinearities give rise to fine local detail. Since there is a limited set of simple broken symmetries and patterns that are possible (e.g., radial, bilateral, periodic), and since developing organisms must start off laying down these elements of spatial order, it follows that these basic forms will be most common among all species. On the other hand, the finer details of pattern will be most variable between species, since the pattern-generating process results in a combinatorial richness of terminal detail, and specific gene products in different species stabilize trajectories leading to one or another of these . . . The fact that virtually all the basic organismic body plans were discovered and established during an early evolutionary period, the Cambrian, is often remarked with surprise, but it is just what one would expect on the basis of the above argument. (Brian C. Goodwin, 'The Evolution of Generic Forms', in *Organizational Constraints on the Dynamics of Evolution*, eds. Maynard Smith and Vida, pp. 114–15)

See also Brian Goodwin, *How the Leopard Changed its Spots* (Simon & Schuster, New York 1996), Chapter 5.

47. When I introduced differential geometry in Chapter 1 I said that one of Gauss's achievements was to get rid of an embedding space by coordinatizing the manifold itself. This allowed him to define the equivalent of metric lengths (and other properties) in this differential space. This coordinatization is an example of what I mean when I say that a nonmetric space is metricized. Deleuze also refers to this operation in his discussion of the relation between metric (striated) and smooth spaces in Deleuze and Guattari, *A Thousand Plateaus*, p. 486.

48. > Consistency necessarily occurs between heterogeneities, not because it is the birth of a differentiation, but because heterogeneities that were formerly content to coexist or succeed one another become bound up

> with one another through the 'consolidation' of their coexistence or succession . . . What we term machinic is precisely this synthesis of heterogeneities as such. (*ibid*., p. 330)

Terms like 'self-consistent aggregate' and 'machinic assemblage' are used synonymously in this book.

49. Although there are a few mathematical functions which produce several outputs, the majority of them have *a single output*. That is, some functions map inputs and outputs (arguments and values) in a one-to-one fashion, others in a many-to-one fashion, and a few in a one-to-many form. See Russell, *Principles of Mathematics*, pp. 265–6. Deleuze's reciprocal determination, I believe, would imply a *many-to-many mapping*, and a mapping such as this would be useless as a function. On the other hand, this 'useless' mapping would capture the desired idea for a multiplicity, an organization of the 'many' as such, without the need for the 'one'.
50. Deleuze, *Difference and Repetition*, pp. 172–4.

    In 'What is Philosophy', the distinction between virtual multiplicities (there referred to as 'concepts'), and functions is made the basis of Deleuze's critique of science's inability to grasp the virtual. Unfortunately, the analysis there is obscured by his introduction of unfamiliar terms like 'functive'. See Deleuze and Guattari, *What is Philosophy?*, pp. 117–18.

    I think its is clearer to see his rejection of functions as models for the virtual in terms of the pre-individual nature of the virtual coupled to the fact that functions may be taken to represent individuation processes. This way it becomes clear why functions without this individuation aspect (without a distinction between dependent and independent variables) can indeed be made part of the virtual. Reference to 'formless functions' as a defining element of concrete universals (or as these are sometimes referred to, 'abstract machines') can be found in Deleuze and Guattari, *A Thousand Plateaus*, p. 141.
51. Deleuze, *Logic of Sense*, p. 52. Let me elaborate this point (events as pre-individual entities) by first considering the type of individuality of *actual events*. Compared to the individuality of an organism or a species (to mention only the two entities for which I have given individuation processes) an actual event has a more fleeting and changing individuation. Deleuze argues that events have the individuality of a *haecceity*, exemplified by the 'thisness' or unique singularity of a moment. As he says

> There is a mode of individuation very different from that of a person, subject, thing, or substance. We reserve the name *haecceity* for it. A season, a winter, a summer, an hour, a date have a perfect individuality lacking nothing, even though this individuality is different from that of a

> thing or a subject. They are haecceities in the sense that they consist entirely of relations of movement and rest between molecules or particles, capacities to affect and be affected. (Deleuze and Guattari, *A Thousand Plateaus*, p. 261)

Some of the heterogeneous assemblages I mentioned before, such as the assemblage of a walking animal, a piece of ground and a gravitational field, have this individuality. This is particularly clear if we do not picture an abstract case but think instead about a concrete event: this animal walking on this hot and humid summer day. ('This should be read without a pause: the animal-stalks-at-five-o'clock', p. 263). This event consists of affects, not only the affordances of animal, ground and field, but also the capacities of the other individuals involved, including degrees of heat and humidity. ('A degree of heat is a perfectly individuated warmth distinct from the substance or subject that receives it. A degree of heat can enter into composition with a degree of whiteness, or with another degree of heat, to form a third unique individuality . . .', p. 253) The event also consists of relations of rapidity and slowness: the ground affords the animal a solid surface only because relative to the speed or temporal scale of change of the animal, the ground changes too slowly. At geological time scales this piece of solid ground would indeed be much more fluid.

To apply this to ideal events. The singularities which populate the virtual are also haecceities, but the two defining features (speeds and affects) are distributed differently: a singularity is nothing but an accidental feature in a field of speeds (or velocity vectors) its individuality consisting entirely of its invariance, that is, its capacity of not being affected by certain transformations which affect the rest of the field.

52. The term 'condensation of singularities' to refer to the expansion of singularities into series, and the establishment of convergent and divergent relations between series, is used for example in Deleuze, *Difference and Repetition*, p. 190.
53. Multiplicities (or Ideas) are referred to as 'complexes of coexistence' in *ibid.*, p. 186.

    In other words, unlike the singularities which define an intensive process which may be actualized only one at a time (either because the bifurcations need to be crossed sequentially, or because only one among alternative attractors may be occupied) virtual singularities all coexist within their own special temporality. Within the intensive 'the Ideas, relations, variations in these relations [embedded levels] and distinctive points [singularities] are in a sense separated: instead of coexisting they enter states of simultaneity or succession' (p. 252).

54. The importance of order, from a purely mathematical standpoint, has been immeasurably increased by many modern developments. Dedekind, Cantor, and Peano have shown how to base all Arithmetic and Analysis upon series of a certain kind . . . Irrationals are defined . . . entirely by the help of order . . . Projective Geometry [has] shown how to give points, lines and planes an order independent of metrical considerations and of quantity; while descriptive Geometry proves that a very large part of Geometry demands only the possibility of serial arrangement. (Russell, *Principles of Mathematics*, p. 199)

55. *Ibid.*, pp. 157–9. Actually, Russell uses the more general term 'magnitude' to refer to these indivisible intensities, and 'distance' as a special case of a magnitude. A terminological confusion should be avoided here. Russell uses the term 'magnitude' to oppose that of 'quantity' (one involves only serial order, the other cardinal number). But when Deleuze comments on Russell's work (as well as Meinong's), he uses 'magnitude' as synonym with 'quantity' and opposes both to 'distance'. See Deleuze and Guattari, *A Thousand Plateaus*, p. 483.

This terminological conflict should not be a problem here since I will not be using the term 'magnitude', and I will always use the term 'ordinal distance' instead of just 'distance' to distinguish the latter from 'metric distances' or lengths. Although Russell introduces distances as intensive, that is, as indivisible in extension, he then devises a scheme which allows him to speak of distances as divisible (by reducing them, via a convention, to extensive 'stretches') and thus abandons any hope of linking the intensive and the extensive morphogenetically. (Russell, *Principles of Mathematics*, pp. 180–2)

56. Ordinal construction does not imply a supposed same unit but only . . . an irreducible notion of distance – the distances implicated in the depth of an intensive *spatium* (ordered distances). Identical unity is not presupposed by ordination; on the contrary, this belongs to cardinal number . . . We should not, therefore, believe that cardinal number results *analytically* from ordinal, or from the final terms of finite ordinal series . . . In fact, ordinal number becomes cardinal only by extension, to the extent that the distances [are] developed and equalized in an extensity established by natural number. We should therefore say that, from the outset, the concept of number is *synthetic*. (Deleuze, *Difference and Repetition*, p. 233; my emphasis)

Russell, on the other hand, establishes between magnitudes and numbers only a logical relation, that between the general and the particular: quantities are magnitudes which are *particularized* by spatio-temporal position (Russell, *Principles of Mathematics*, p. 167).

One way of bringing up the difference between Deleuze's and Russell's approaches to series and numbers, is by contrasting their analyses of the theory of irrational numbers of Dedekind. Arguing that there were gaps in the compact series of rational numbers, Dedekind introduced the notion of a 'cut', a way of segmenting a dense continuum into two, mutually excluding, parts. His idea was to define the concept of number in terms of such cuts performed on purely ordinal continua. Some of these discontinuities yield rational numbers, but others, he postulated, must yield irrationals. Russell, for whom the density of the rationals seems to be enough, objects to this merely postulated existence of irrational cuts, and equates irrationals with one of the classes of rationals created by the cut. This, in effect, explains one extensive concept (number) in terms of another, equally extensive one (class or set). Deleuze, on the contrary, sees in the concept of a cut a way to express the genesis of numerical quantity out of intensive non-numerical continua: 'In this sense, it is the cut which constitutes the next genus of number, the ideal cause of continuity or the pure element of quantitativity' (Deleuze, *Difference and Repetition*, p. 172).

57. Deleuze, *Logic of Sense*, p. 109.

58. Divergence and disjunction are, on the contrary, affirmed as such. But what does it mean to make divergence and disjunction the objects of affirmation? As a general rule two things are simultaneously affirmed only to the extent that their difference is denied . . . We speak, on the contrary, of an operation according to which two things . . . are affirmed through their difference . . . to affirm their distance as that which relates one to the other insofar as they are different . . . The idea of a positive distance as distance (and not as an annulled or overcome distance) appears to us essential . . . The idea of positive distance belongs to topology and the surface. (*Ibid.*, p. 172)

59. Convergent and divergent relations define the *modal status* of virtual relations. Following Leibniz, Deleuze calls these virtual relations *compossibility and incompossibility*:

> Two events are compossible when the series which are organized around their singularities extend in all directions [that is, converge]; they are incompossible when the series diverge in the vicinity of constitutive singularities. Convergence and divergence are entirely original relations which cover the rich domain of alogical compatibilities and incompatibilities. (*Ibid.*, p. 172)

The modal status of the virtual may be more easily grasped by contrasting it with other modal relations, such as the relations which modal logicians postulate to exist between possible worlds. The modern theory of possible worlds is also based on the ideas of Leibniz, but disregards these alogical capacities or affects. Briefly, the key relation between possible worlds is that of *accessibility*: one world is accessible from another possible one, if every situation possible in one is also possible in the other. Given this relation, possible worlds may be grouped together into families or equivalence classes. Whenever situations in one class are impossible in another one, that is, when there exist logical or physical *contradictions* between them, worlds belonging to one are inaccessible from those belonging to the other (Michael J. Loux, 'Introduction: Modality and Metaphysics', in *The Possible and the Actual*, pp. 20–8).

Deleuze would accept these ideas but argue that contradictions between possible worlds are a derivative phenomenon. In other words, that distributions of possible worlds, and their fully individuated contents, depend on deeper relations of compossibility and incompossibility between pre-individual multiplicities: where the series emanating from multiplicities converge, a family of accessible possible worlds would be defined; where they diverge, an inaccessible family of worlds would begin. See Gilles Deleuze, *The Fold. Leibniz and the Baroque* (University of Minnesota Press, Minneapolis, 1997), p. 60. See also Deleuze, *Difference and Repetition*, p. 48, where he adds 'the notion of incompossibility in no way reduces to that of contradiction and does not even imply real opposition: it implies only divergence . . .'

60. Deleuze, *Logic of Sense*, p. 5.
61. Speaking of the particular case of catastrophe theory, where the limitation to potential-driven systems with four degrees of freedom makes a full classification of attractors and bifurcations possible, Alexander Woodcock and Monte Davies write

> In any system governed by a potential and in which the system's behavior is determined by no more than four different factors, only seven qualitatively different types of discontinuity [bifurcation] are possible. In other words, while there is an infinite number of ways for such a system to change continuously (staying at or near equilibrium), there are only seven structurally stable ways for it to change discontinuously (passing through non-equilibrium states). Other ways are conceivable, but unstable; they are unlikely to happen more than once . . . The qualitative type of any stable discontinuity does *not* depend on the specific nature of the potential involved, *merely on its existence*. It does *not* depend on the specific conditions regulating behavior, merely on their number. It does

*not* depend on the specific quantitative cause-and-effect relationship between the conditions and the resulting behavior *merely on the empirical fact that such a relationship exists*. (Alexander Woodcock and Monte Davies, *Catastrophe Theory* (E. P. Dutton, New York, 1978), p. 42; my emphasis)

There are two important ideas expressed here. The first is related to the question of *universality*: as long as different equations or different physical systems share the same topological invariants (the same number of singularities, the same number of dimensions) the detailed nature of the equations or of the system (the specific type of intensive difference driving the process, or the specific quantities which define the process) does not make much difference in the specification of their long-term tendencies. The second idea relates to the question of immanence: the long-term (asymptotic) tendencies of a process may be independent of specific causes, but they do depend for their very existence on there being *some causal process or another*.

62. Deleuze, *Logic of Sense*, p. 169. (My emphasis) Deleuze adopts this approach from the Stoics who were the first to split the causal link: on one hand, processes of individuation are defined as sequences of causes (every effect will be the cause of yet another effect) while singularities become pure incorporeal effects of those series of causes; on the other hand, these pure effects are viewed as having a *quasi-causal* capacity to endow causal processes with coherent form. By splitting causality this way, Deleuze manages to separate the determinism (or destiny) which links causes to causes, from strict necessity.
63. *Ibid*., p. 147.
64. The image of echoes and resonances as that which links multiplicities recurs throughout Deleuze's work. See Chapter 3, footnote 53 for an explanation and examples.
65. Kenneth M. Sayre, *Cybernetics and the Philosophy of Mind* (Routledge and Kegan Paul, London, 1976), p. 23.
66. *Ibid*., pp. 26–30.
67. There is a close relation between communication theory and thermodynamics. Much as in the latter the equilibrium state (for an isolated system) is defined as the one characterized by maximum disorder (maximum entropy), the state achieved once differences in intensity have been cancelled, so in the former equilibrium corresponds to a situation where the differences within series have been cancelled, where all the events have become *equiprobable*. In such state no information may flow in the channel (*ibid*., pp. 38–43).

    Deleuze uses this connection between the intensive and the informational to define the relations between the series of ideal events. As I have said, he refers to an information channel as a 'signal', and to the information quanta as 'signs',

> Such systems, constituted by placing disparate elements or heterogeneous series in communication, are in a sense quite common. They are signal–sign systems. The signal is a structure in which differences in potential are distributed, assuring the communication of disparate components: the sign is what flashes across the boundary of two levels, between two communicating series. Indeed, it seems that all phenomena respond to these conditions inasmuch as they find their ground in a constitutive dissymmetry, difference, inequality. All physical systems are signals, all qualities are signs. (Deleuze, *Logic of Sense*, p. 261)

See also Deleuze, *Difference and Repetition*, pp. 20 and 222.

68. If we examine the singularities corresponding to the two important basic series we see that they are distinguished, in both cases, by their distribution. From one to the other, certain singular points disappear or are divided, or undergo a change of nature and function. The moment the two series resonate or communicate we pass from one distribution to another. (Deleuze, *Logic of Sense*, p. 53)

69. It is in difference that . . . phenomena flash their meaning like signs. The intense world of differences . . . is precisely the object of a superior empiricism. This empiricism teaches us a strange 'reason', that of the multiple, chaos, and difference. (Deleuze, *Difference and Repetition*, p. 57)

There is in addition a temporal dimension of the virtual, which I will discuss in the next chapter, which also defines this other empiricism.

> An Idea, in this sense, is neither one nor multiple, but a multiplicity constituted of differential elements, differential relations between those elements, and singularities corresponding to those relations . . . All three are projected in an ideal temporal dimension which is that of progressive determination. There is, therefore, *an empiricism of the Idea* . . . (p. 278; my emphasis)

On the concepts of multiplicity and quasi-causal operator (and related ideas, like 'perplication', 'complication', etc.) as empirico-ideal notions, see p. 284.

70. Stephanie Forrest, 'Emergent Computation: Self-organizing, Collective and Cooperative Phenomena in Natural and Artificial Computing Networks', in

*Emergent Computation*, ed. Stephanie Forrest (MIT Press, Cambridge, 1991), p. 2.

71. Sayre, *Cybernetics and the Philosophy of Mind*, p. 30.
72. When the two series of events are collapsed into one we get what is called a 'Markov process'. See *ibid*., p. 29.
73. David L. Goodstein, *States of Matter* (Dover, New York, 1985), pp. 468–86. See also Nicolis and Prigogine, *Exploring Complexity*, pp. 168–85.
74. These other characteristics are a 'critical slowing down' (relaxation times become longer as the singularity is approached) and 'sensitivity to size' (the dynamics of a system can take into account details about boundary conditions). However, the link between these phenomena and information processing and storage has been established only within the narrow field of 'cellular automata' models of computation. See Christopher G. Langton, 'Computation at the Edge of Chaos', in *Emergent Computation*, ed. Forrest, pp. 32–3.
75. Christopher G. Langton, 'Life at the Edge of Chaos', in *Artificial Life II*, eds. Christopher G. Langton, Charles Taylor, Doyne Farmer and Steen Rasmussen (Addison-Wesley, Redwood City, 1992), pp. 85–6.
76. Melanie Mitchell, James P. Crutchfield and Peter T. Hraber, 'Dynamics, Computation, and the "Edge of Chaos": A Reexamination', in *Complexity: Metaphors, Models, and Reality*, eds. George A. Cowan, David Pines and David Meltzer (Addison-Wesley, Redwood City, 1994), p. 510.

> The results presented here do not disprove the hypothesis that computational capability can be correlated with phase transitions in [cellular automata] rule space. Indeed, this general phenomena has already been noted for other dynamical systems . . . More generally, the computational capacity of evolving systems may very well require dynamical properties characteristic of phase transitions if they are to increase their complexity.

## 3 THE ACTUALIZATION OF THE VIRTUAL IN TIME

1. On the history of these conflicting conceptions of time and a philosophical discussion of the different ways in which the conflict has been approached in both physics and philosophy of science, see Lawrence Sklar, *Physics and Chance. Philosophical Issues in the Foundations of Statistical Mechanics* (Cambridge University Press, Cambridge, 1995), Chapter 10. And Robert B. Lindsay and Henry Margenau, *Foundations of Physics* (Ox Bow Press, Woodbridge, 1981), Chapter 5.
2. Joe Rosen, *Symmetry in Science* (Springer-Verlag, New York, 1995), p. 141. In addition to reversing the order of the temporal sequence, a time

'reflection' transformation changes the sign of any variable (such as velocity) that depends on the time variable. This introduces some subtle ideas that matter in a serious analysis of the symmetry properties of laws. See discussion of this point in Sklar, *Physics and Chance*, pp. 246–8.

3. Gregoire Nicolis and Ilya Prigogine, *Exploring Complexity* (W. H. Freeman, New York 1989), p. 52.
4. Eugene P. Wigner, 'Invariance in Physical Theory', in *Symmetries and Reflections*, eds. Walter Moore and Michael Scriven (Ox Bow Press, Woodbridge, 1979), p. 4.

   As the physicist Eugene Wigner remarks, if physical regularities had not displayed this minimal amount of invariance, we would probably never have discovered them at all simply because they would not appear to us as regularities. Invariance under transformations can also reveal subtle assumptions behind a law. For instance, to say that a law is invariant under spatial or temporal displacement implies that, as far as the regularities described by the law are concerned, *space and time are homogeneous*. Similarly, to say that a law is invariant under rotation in space is to say that the absolute orientation of the states of the process makes no difference in the process's behaviour, but it also means that we assume space to have uniform properties in all directions (technically, we assume it to be isotropic).
5. There are several strategies for explaining irreversibility away. Some physicists, for example, think the inherent directionality of the arrow of time, so evident in macroscopic processes, is merely a subjective effect (an effect of our ignorance of all the micro details). To others the directionality of time is not reducible to psychology but it is nevertheless denied the status of a true law, being merely a contingent statistical result. As the physicist John Wheeler puts it, the real molecular interactions are 'time-symmetric with only the statistics of large numbers giving it the appearance of asymmetry' (John A. Wheeler, 'Time Today', in *Physical Origins of Time Asymmetry*, eds. Jonathan J. Halliwell, Juan Perez-Mercader and Wojciech H. Zurek [Cambridge University Press, Cambridge, 1996], p. 1).

   In general, the authority of the old reversible time has been preserved and the time of classical thermodynamics has disappeared from the structure of the edifice of physics. As Wheeler puts it,

   > The expansion of the empire of time has elevated the concept, human born as it is, to platform upon platform upon platform of authority. Regularities of sun and season raised the first foundation. On top of it Newtonian dynamics erected a second and tighter platform; special relativity a third, terraced further in and up; and general relativity stands at the summit, the final level of authority. Not except out of the mouth of

> Einstein's 1915 and still standard theory of spacetime can one hear the generally agreed account of all that 'time' now means and measures. (p. 6)

6. Ilya Prigogine, *From Being to Becoming* (W. H. Freeman. New York, 1980), p. 19.
7. Arthur S. Iberall, *Towards a General Science of Viable Systems* (McGraw-Hill, New York 1972).

   Iberall's ontology is based on individuals which he calls 'atomisms' (a category of which atoms would be only one instance). He conceives of these in general as autonomous, nonlinear oscillators. Thanks to their nonlinearity these atomisms are shown capable of interactive ordering (via entrainment, for example) and capable of forming a continuum at a larger scale. These continua, in turn, are shown to undergo symmetry-breaking bifurcations which fragment them (or quantize them) to yield super-atomisms, that is, individuals at a larger spatio-temporal scale. Iberall shows in detail how this alternation of atomism and continuum can be used recursively to account for many features of physics, chemistry, biology and even sociology. He also shows, on the other hand, how much this picture breaks with those of classical and, more importantly, quantum physics, given that the latter does not give a morphogenetic account of quantization.
8. Winfree does not use the terms 'intensive' or 'nonmetric'. Yet, in the previous chapter I quoted Winfree's ideas about topological thinking when applied to biology and his ideas are indeed very close to those of Deleuze. Using my terminology, we can say that an *anexact yet rigorous* approach characterizes Winfree's research on the birth and death of oscillations, a process which also exhibits divergent universality or mechanism-independence. In his words,

   > As a result of these collective efforts, the reality of phaseless sets, phase singularities, time crystals, and so on became firmly established. Their physiological 'meaning' is less clear . . . But that deficiency is in a way the most interesting aspect of these findings: because their prediction was *in no way dependent on the mechanistic underpinnings* of circadian physiology, the same principles might find applications in other areas of physiology and biochemistry. These principles are not 'mathematical', in the familiar sense of 'mathematics' as 'moving symbols around on paper' or 'moving numbers around in computers'. They are, rather, [topological] *concepts about continuity* that could be used in diverse contexts *with sufficient rigor* to precisely infer biological or chemical events that had not been observed. (Arthur T. Winfree, *When Time Breaks Down. The Three-Dimensional Dynamics of Electrochemical Waves and Cardiac Arrhythmias* [Princeton University Press, Princeton, 1987], pp. 264–5; my emphasis)

9. Ian Stewart and Martin Golubitsky, *Fearful Symmetry* (Blackwell, Oxford, 1992), pp. 66–7.
10. Nicolis and Prigogine, *Exploring Complexity*, p. 21.
11. *Ibid.*, p. 103.
12. Iberall, *Towards a General Science of Viable Systems*, p. 153.
13. *Ibid.*, p. 161.
14. Gilles Deleuze, *Logic of Sense* (Columbia University Press, New York, 1990), p. 162. (My emphasis)
15. *Ibid.*, p. 62.
16. Gilles Deleuze, *Difference and Repetition* (Columbia University Press, New York, 1994), pp. 70–1. In these pages Deleuze, following Hume, does indeed present this contraction which synthesizes present time as a faculty of the mind: a *contractile power of contemplation* or *imagination* which *retains* a past and *anticipates* a future. But a few pages later (p. 73) he says that 'we are made of contracted water, earth, light and air – not merely prior to the recognition or representation of these, but prior to their being sensed'. Clearly, this remark makes no sense within a purely psychological interpretation, but it does if we think of this contraction as involving a metabolic cycle with a characteristic time scale. He goes on to ascribe to habits (or to the contraction of repetitive, habitual behaviour) a similar power of synthesis (p. 74) but again, this applies not only to the habits of human beings but to any repetitive, cyclic behaviour at all scales.

    > A soul should be attributed to the heart, to the muscles, nerves and cells, but a contemplative soul whose entire function is to contract a habit. This is no mystical or barbarous hypothesis. On the contrary, habit here manifests its full generality: it concerns not only the sensory-motor habits that we have (psychologically), but also, before these, the primary habits that we are; the thousands of passive syntheses of which we are *organically composed*. (My emphasis)

17. The philosopher who argued against the relativistic conclusions regarding the contraction of time in the twins' case is, of course, Henri Bergson. Bergson was wrong in assuming that the case for the two twins is symmetric, or as he put it, a pure 'effect of perspective' similar to that of two observers looking at each other at a distance and seeing each other shrunk in space. See for example his reply to criticisms by André Metz in Henri Bergson, 'Fictitious Times and Real Time', in *Bergson and the Evolution of Physics*, ed. P. A. Y. Gunter (University of Tennessee Press, Knoxville, 1969), pp. 169–71.

    This volume contains many of the pieces written about the debate including the exchange between Bergson and Einstein himself. If one focuses on Bergson's excessively psychological interpretation of relativity then one must grant that he lost this debate. On the other hand, if instead one sees him as arguing for the need of an account of metric time (which must emerge from a nonmetric, virtual time) then the outcome of the debate is less clear. This is Deleuze's own interpretation. He sees Bergson as criticizing Einstein for not having understood the difference between the actual and the virtual, the difference between metric and nonmetric multiplicities.

> Bergson thus brought to light two very different kinds of multiplicity, one qualitative and fusional, continuous, the other numerical and homogeneous, discrete . . . The confrontation between Bergson and Einstein on the topic of Relativity is incomprehensible if one fails to place it in the context of the basic theory of Riemannian multiplicities, as modified by Bergson. (Gilles Deleuze and Félix Guattari, *A Thousand Plateaus* [University of Minnesota Press, Minneapolis, 1987], p. 484)

See also Gilles Deleuze, *Bergsonism* (Zone Books, New York, 1988), Chapter 4.

18. Hans Reichenbach, *The Philosophy of Space and Time* (Dover, New York, 1958), p. 194. An examination of the relations between the theories of time in nonlinear and relativistic physics is beyond the scope of this book, but nevertheless two conclusions follow rather directly. One is that there is no incompatibility between the two and indeed the nonlinear theory may complement that of relativity by giving a morphogenetic account (via concepts like the Hopf bifurcation) of the emergence of the oscillators (clocks, electromagnetic vibrations) used in the exposition of relativity. On the other hand, once we realize that the metric of time is emergent, that is, that oscillators operating at different scales literally quantize time, the shrinkage of time at velocities near the speed of light becomes less counter intuitive: an emergent metric, as opposed to an intrinsic one, is easier to visualize as subject to intensive transformations that do not preserve certain of its properties invariant.
19. In his careful examination of foundational questions the philosopher Lawrence Sklar shows that besides the need to derive the time-asymmetric macroscopic behaviour of a thermodynamic system from the time-symmetric microscopic laws, there are two additional fundamental questions in the foundations of statistical mechanics: to show that the final equilibrium state of a system is indeed *an attractor* for its initial and all its other intermediate states, and that the *time scales* of approach to equilibrium in mathematical models reflect the time scales observed in the laboratory. Sklar argues that these two questions are *open problems* in equilibrium thermodynamics:

physicists have not yet rigorously demonstrated that equilibrium states attract, nor explained why the relaxation time exhibits a characteristic scale (Sklar, *Physics and Chance*, pp. 156–8, 189 and 216).

Sklar, however, neglects to mention that both of these open problems have indeed been given a more precise formulation, if not solved, in far-from-equilibrium thermodynamics. In this field one gets the asymptotic approach to a particular state as an integral part of one's model, while in conservative systems without attractors the asymptotic stability of the final equilibrium state needs a special explanation. A similar point applies to relaxation times. Unlike the conservative system case, in non-conservative systems we have an explanation, in terms of the 'area' covered by the basin of attraction, which is an integral part of the model. Sklar does discuss Prigogine's work to some extent, but not the specific points raised here (pp. 269–76).

20. Iberall discusses this issue in more technical terms (including terms like bulk viscosity and bulk modulus needed to define the relaxation time of internal modes) which are beyond the scope of this book to explain. Yet I believe his basic point is captured by my simplified example. See his discussion in *Towards a General Science of Viable Systems*, pp. 122–6.

21.

> . . . the interactions between bodies condition a sensibility, a proto-perceptibility and a proto-affectivity . . . What is called 'perception' is no longer a state of affairs but a state of the body as induced by another body, and affection is the passage of this state to another state as increase or decrease of potential-power through the action of other bodies . . . Even when they are nonliving, or rather inorganic, *things have a lived experience* because they are perceptions and affections. (Gilles Deleuze and Félix Guattari, *What is Philosophy?* [Columbia University Press, New York, 1994], p. 154; my emphasis)

Elsewhere he is even more explicit about this. We saw before that the actualization of the world relies on intensive processes of self-organization (such as convection cells or the migration and folding of embryonic cells). He refers to these phenomena as 'spatio-temporal dynamisms' and says

> Actualization takes place in three series: space, time and also consciousness. Every spatio-temporal dynamism is accompanied by the emergence of an elementary consciousness which itself traces directions, doubles movements and migrations, and it is born on the threshold of the condensed singularities of the body or object whose consciousness it is. (Deleuze, *Difference and Repetition*, p. 220)

22. Deleuze, *Logic of Sense*, p. 62.
23. Winfree, *When Time Breaks Down*, p. 22.
24. Leon Glass and Michael C. Mackey, *From Clocks to Chaos. The Rhythms of Life* (Princeton University Press, Princeton, 1988), p. 94. (This text contains a discussion of Winfree's work, and references to black holes.)
25. Winfree, *When Time Breaks Down*, p. 99.
26. *Ibid.*, Chapters 7 and 8.

27. Throughout we will discover again and again, in a surprising diversity of contexts the same paradoxical entity: a motionless, timeless organizing center called a phase singularity. This is a place where an otherwise pervasive rhythm fades into ambiguity – like the South Pole, where the 24 hourly time zones converge and the Sun merely circles along the horizon. (*Ibid.*, p. 5)

    Our [topological] inferences seldom involved speculation about adaptive values, molecular mechanisms, or neural pathways. But they led us to ever sharper focus on experimental conditions in *which something strange was guaranteed to happen*: return of metamorphosing flies to the timeless condition of the newly fertilized egg, perpetual insomnia in mosquitoes, abrupt suspension of pacemaking in otherwise perfectly healthy and capable heart muscle, vortex centers of arrhythmia in electrically rhythmic tissue, chemically timeless rotors sequencing reactions around their perimeters, and chemical clocks made of shifting patterns of color topologically locked into three dimensional organizing centers. (*Ibid.*, p. 254; my emphasis)

28. Deleuze and Guattari, *A Thousand Plateaus*, p. 24. (My emphasis)

29. One of the more robust and striking predictions of the theory of mutual synchronization was that it should fail abruptly below a critical coupling strength. John Aldridge and E. Kendall Pye tried this experiment with yeast and found exactly that: when the cells get more than about twenty diameters apart, the amplitude of their collective rhythm falls abruptly. (Arthur T. Winfree, *Biological Clocks* [Scientific American Library, New York, 1987], p. 128)

30. Populations of crickets entrain each other to chirp coherently. Populations of fireflies come to coherence in flashing. Yeast cells display coherence in glycolytic oscillation. Populations of insects show coherence in their cycles of eclosion (emergence from the pupal to the adult form) . . . Populations of women living together may show phase entrainment of their ovulation cycles. Populations of secretory cells, such as the pituitary,

> pancreas, and other organs, release their hormones in coherent pulses. (Alan Garfinkel, 'The Slime Mold Dictyostelium as a Model of Self-Organization in Social Systems', in *Self-Organizing Systems. The Emergence of Order*, ed. F. Eugene Yates [Plenum Press, New York, 1987], p. 200)

31. M. Cohen, quoted in *ibid.*, p. 183.
32. Howard H. Pattee, 'Instabilities and Information in Biological Self-Organization', in *Self-Organizing Systems*, p. 334.
33. Stuart Kauffman, *The Origins of Order. Self-Organization and Selection in Evolution* (Oxford University Press, New York, 1993), p. 442. (My emphasis)
34. Rudolf A. Raff, *The Shape of Life. Genes, Development and the Evolution of Animal Form* (University of Chicago Press, Chicago, 1996), p. 260. Unlike terminal addition, which implies that early stages of the development of an embryo resemble (or recapitulate) early stages of species (or higher taxa) development, the type of *heterochrony* involved in parallel networks destroys any similarity between the two.
35. *Ibid.*, p. 255.
36. *Ibid.*, p. 337.

> Dissociation appears paradoxical as a creator of developmental novelty because nothing new is added. In the case of some heterochronic dissociations, such as neoteny in the axolotl, *a novel developmental pathway* and life history *have resulted from the loss of a feature* of the ancestral system. (My emphasis)

37. Deleuze and Guattari, *A Thousand Plateaus*, p. 48.
38. W. H. Zurek and W. C. Schieve, 'Nucleation Paradigm: Survival Thresholds in Population Dynamics', in *Self-Organization and Dissipative Structures: Applications in the Physical and Social Sciences*, eds. William C. Schieve and Peter M. Allen (University of Texas Press, Austin, 1982), pp. 203–22.
39. Stuart L. Pimm, *The Balance of Nature. Ecological Issues in the Conservation of Species and Communities*. (University of Chicago Press, Chicago, 1991), Chapters 2 and 3.
40. Kauffman, *The Origins of Order*, p. 256.
41. Rudolf A. Raff and Thomas C. Kauffman, *Embryos, Genes, and Evolution* (Indiana University Press, Bloomington, 1991), p. 40.

42. The fastest evolutionary rates fall in the last, and perhaps most interesting category, tachytely . . . Tachytely resembles the punctuation of Eldredge and Gould in that both rely on exceptional high rates of evolution. However while Eldredge and Gould focused on a speciation model . . .

> [Simpson] suggested that the primary concomitant of tachytely is a shift in a population from one major adaptive zone to another . . . Thus tachytely is possible during early radiations of new groups expanding into vacant adaptive zones. During the rapid radiation all lineages are relatively poorly adapted and not mutually competitive. The result . . . is the production of diverse lines that quickly become extinct as other lines consolidate their positions in the adaptive zone at the expense of their less-efficient cousins. (*Ibid*., p. 44)

43. Angela E. Douglas, *Symbiotic Interactions* (Oxford University Press, New York, 1994), pp. 7–9. This author emphasizes the emergence of novel metabolic capabilities related directly to the flow of biomass in food chains. As she says, 'Nutritional interactions are fundamental to most symbioses, because the metabolic capabilities most commonly acquired through symbiosis relate to nutrition' (p. 56, and see Chapter 7 for an evaluation of the ecological impact of symbiosis).
44. Werner Schwemmler, 'Symbiogenesis in Insects as a Model for Cell Differentiation, Morphogenesis, and Speciation', in *Symbiosis as a Source of Evolutionary Innovation*, eds. Lynn Margulis and Rene Fester (MIT Press, Cambridge, 1991), p. 195.
45. Deleuze places great emphasis on symbiosis as a means of becoming. Coevolution, as in the *aparallel evolution* of the wasp and the orchid it pollinates, is a well-known example. See Deleuze and Guattari, *A Thousand Plateaus*, p. 10. But more generally, the very definition of a heterogeneous assemblage as a 'rhizome' has its origin in symbiosis. Though his introductory example of rhizome is bulbs and tubers, that is, plants without an arborescent root system, he immediately acknowledges that 'plants with roots or radicles can be rhizomorphic in other respects altogether' (p. 6). This other respect may be illustrated by the formation of the so-called *rhizosphere*, the underground food web composed of the plant roots of different species together with the diverse micro-organisms that form symbiotic couplings with them and interface them to the flow of underground nutrients.
46. Throughout this book I have used his first formulation, singularities and affects, but he uses several others. Sometimes he says that in the virtual continuum (plane of consistency) bodies are characterized by speeds and affects (*ibid*., p. 260).

    Elsewhere, he says the virtual continuum (Aion) is 'the locus of incorporeal events and of attributes which are distinct from qualities' (Deleuze, *Logic of Sense*, p. 165). Here, 'events' refers to singularities, while 'attributes' are capacities to affect and be affected (to cut and to be cut, to use his example).

47. *Ibid.*, p. 255. Rapidity and slowness, however, should not be conceived as involving merely quantitative or extensive differences. Speed is an intensive property subject to critical thresholds, as in the case of fluids which, below a critical speed, have one pattern of flow (laminar) but which, beyond the threshold, display a completely different pattern (turbulence). See p. 371.
48. *Ibid.*, p. 258.
49. The term 'mechanisms of immanence' does not, to my knowledge, occur in Deleuze, but he expresses himself in similar ways.

> Many movements, *with a fragile and delicate mechanism*, intersect: that by means of which bodies, states of affairs, and mixtures, considered in their depth, succeed or fail in the production of ideal surfaces [plane of consistency]; and conversely, that by means of which the events of the surface are actualized in the present of bodies (in accordance with complex rules) by imprisoning their singularities within the limits of worlds, individuals and persons. (Deleuze, *Logic of Sense*, p. 167; my emphasis)

50. Connectance is, in fact, controlled in food webs. Good evidence suggests that the number of connections in food webs is adjusted such that each species maintains roughly a constant number of connections to other species, regardless of the number of species in the web . . . [as displayed in] data on more than 100 food webs – terrestrial, freshwater, and marine. A number of properties – such as length of food chains; connectance; ratios of top, intermediate and bottom species; and ratios of predators to prey – appear *stable and scale invariant*, both with respect to the numbers of species in the web and with respect to the aggregation of 'guilds' of similar species into single 'trophic species' or the aggregation of similar species into higher taxonomic units. (Kauffman, *The Origins of Order*, p. 263)

51. *Ibid.*, p. 219.
52. Although the famous Gaussian, or bell-shaped, distribution does represent an important emergent property of widely different populations (that is, there is something recurrent or universal about it) it is nevertheless an equilibrium distribution, and the populations exhibiting this bell shape are examples of *distributions in extensity*, fixed in their form and occupying a metric, divisible space (much as sedentary cultures do). At the virtual level, we must go beyond these distributions, we must make a different *use of chance*. Unlike traditional games of chance (roulette, dice) in which fixed rules force the aleatory factor to be retained only at certain points (the

spinning of the roulette, the throw of the dice) leaving the rest as a mechanical development of the consequences, at the level of the virtual we must allow the rules to change with every throw and inject chance at every point, to yield truly nonmetric (or nomadic) distributions. In Deleuze's words

> Each throw emits singular points . . . But the set of throws is included in the aleatory point [quasi-causal operator], a unique cast which is endlessly displaced throughout the series . . . These throws are successive in relation to one another, yet simultaneous in relation to this point which always changes the rule, or coordinates and ramifies the corresponding series as it insinuates chance over the entire length of the series . . . Each throw operates a distribution of singularities, a constellation. But instead of dividing a closed space between fixed results which correspond to hypotheses [as in traditional treatments of probability], the mobile results are distributed in the open space of the unique and undivided cast. This is a *nomadic* and non-sedentary *distribution*. (Deleuze, *Logic of Sense*, pp. 59–60; emphasis in the original)

53. Deleuze offers an alternative model for this task of the quasi-causal operator which is based on the idea of entrainment, or more specifically, the phenomenon of frequency entrainment. For two grandfather pendulum clocks to entrain, *weak signals must be transmitted* from one to the other to couple them (in some cases, these are weak vibrations in the wooden floor on which the clocks are placed). If the frequencies of the two clocks are close to each other they may resonate and the two clocks will lock into a single frequency. The resulting entrainment of the two oscillators represents a *much stronger linkage* (forced movement) between the two oscillators than the weak signals which originally coupled them. In Deleuze's words:

> A system must be constituted on the basis of two or more series, each series being defined by the differences between the terms which compose it. If we suppose that the series communicate under the impulse of a force of some kind [e.g. the quasi-causal operator], then it is apparent that this communication relates differences to other differences, constituting differences between differences within the system. These second degree differences play the role of 'differenciator' . . . This state of affairs is adequately expressed by certain physical concepts: *coupling* between heterogeneous systems, from which is derived an *internal resonance* within the system, and from which in turn is derived a *forced movement*, the amplitude of which exceeds that of the basic series themselves. (Deleuze, *Difference and Repetition*, p. 117)

Deleuze uses this 'resonance' model for the action of the quasi-causal operator in other places. For example,

> Concepts [multiplicities], which have only consistency or intensive ordinates outside of any coordinates, freely enter into relationships of *nondiscursive resonance* . . . Concepts are centers of vibrations, each in itself and everyone in relation to all others. This is why they all *resonate rather than cohere* or correspond to each other. (Deleuze and Guattari, *What is Philosophy?*, p. 23; my emphasis)

Clearly, if we interpreted the term 'concept' as 'semantic content of a term' (or in any other linguistic way) this paragraph would become meaningless. The term 'intensive ordinates' must be interpreted in terms of positive *ordinal* distances (which distinguishes it from any cardinal numerical coordinate) and not as referring to one of the members of the couple 'ordinates' and 'abscissas' which are simply the names of two coordinates.

54. Deleuze, *Logic of Sense*, p. 121. (My emphasis) This is about the specification of the conditions of a problem, but problems are, in Deleuze's ontology, nothing but virtual multiplicities. I discuss this relationship in Chapter 4.
55. Stewart and Golubitsky, *Fearful Symmetry*, pp. 14–16.
56. Lawrence Sklar, *Space, Time, and Space–Time* (University of California Press, Berkeley, 1977), pp. 251–86.
57. The reason why it is hard to find a physicist who would think of laws as entities in need of ontological analysis is that most of them have an instrumentalist or operationalist attitude toward theoretical entities. Ever since Newton refused to give mechanisms to explain the action of gravity and settled on describing *how* planets move, as opposed to explaining *why* they do so, many physicists have accepted a non-realist approach to laws, as well as unobservable entities in general. Thus, experimental laws (like Boyle's law) are defined as symbolic representations of laboratory regularities or *routines of experience*, while fundamental laws become basic hypotheses from which one can derive experimental laws, and the validity of which is not settled empirically but through the validity of their consequences. In neither case is the ontological status of the laws themselves an issue. See Lindsay and Margenau, *Foundations of Physics*, pp. 14–16 (for experimental laws) and pp. 22–6 (for fundamental principles).

While philosophers can take this stance and argue that, if all specific experimental laws may be derived from a set of fundamental ones, then the latter may be seen as a set of axioms and treated as eternal truths, as in Euclid's axiomatic treatment of geometry. But as the physicist Richard Feynman has argued, scientists cannot do this because they are aware that, unlike essences, fundamental laws may have several different forms. Newton's laws of motion, for example, may be expressed in three ways which are, mathematically, completely different: the original force form, the field form, and the variational form. These are taken to express one and the same law because they have the same mathematical consequences and thus we cannot tell them apart experimentally. But the existence of a variety of forms does eliminate the temptation to adopt a Greek axiomatic approach, forcing physicists to adopt, as Feynman puts it, a Babylonian approach. See Richard Feynman, *The Character of Physical Law* (MIT Press, Cambridge, 1995), pp. 50–3.

Perhaps the only clear statement one can get from physicists as to what fundamental laws are supposed to be comes from the application of group theory to the laws themselves. For example, the well-known invariance of Newton's laws under translations in space and time implies that

> given the same essential initial conditions, the result will be the same no matter when and where we realize these. This principle can be formulated . . . as the statement that the absolute position and the absolute time are never essential initial conditions . . . If the universe turned out to be grossly inhomogenous, the laws of nature in the fringes of the universe may be quite different from those we are studying . . . The postulate of invariance with respect of displacement in space and time disregards this possibility, and its application on the cosmological scale *virtually presupposes a homogeneous and stationary universe*. (Wigner, *Invariance in Physical Theory*, p. 4; my emphasis)

Clearly, this is a more sophisticated stance than naive essentialism, since this postulate of invariance (which may imply that basic laws are simultaneously valid everywhere, and have been so always) can, in turn, be treated as an approximate hypothesis. I return to the question of laws in Chapter 4.

58. For if it is a question of knowing . . . 'why water changes its state of quality at 0° centigrade', the question is poorly stated insofar as 0° is considered as an ordinary point in the thermometer. But if it is considered, on the contrary, as a singular point, it is inseparable from the event occurring at that point, always being zero in relation to its realization on the line of ordinary points, *always forthcoming and already past*. (Deleuze, *Logic of Sense*, p. 80; my emphasis)

The exact same formulation recurs throughout Deleuze's work:

> Aion: the indefinite time of the event, the floating line that knows only speeds and continually divides that which transpires into an already-there that is at the same time not-yet-here, a simultaneous too-late and too-

> early, a something that is both going to happen and has just happened. (Deleuze and Guattari, *A Thousand Plateaus*, p. 262)
>
> The meanwhile, the event, is always a dead time; it is there where nothing takes place, an infinite awaiting that is already infinitely past, awaiting and reserve. (Deleuze and Guattari, *What is Philosophy?*, p. 158)

59. Deleuze, *Difference and Repetition*, p. 88. (My emphasis)

> The joint . . . is what ensures the subordination of time to those properly cardinal points through which pass the periodic movements which it measures [e.g. the nested set of cyclic presents] . . . By contrast, time out of joint means demented time . . . liberated from its overly simple circular figure, freed from the events that made up its content . . . in short, time presenting itself as an empty and pure form. Time itself unfolds . . . instead of things unfolding within it . . . It ceases to be cardinal and becomes ordinal, a pure *order* of time.

60. I have said before that each cyclic present is a contraction of past and future instants at a given temporal scale. Hence it is a veritable 'synthesis' of present time, a synthesis which Deleuze calls 'passive' because it involves no activity either by the world or by the subject.

> Passive synthesis or contraction is essentially asymmetrical: it goes from the past to the future in the present, thus from the particular to the general, thereby imparting direction to the arrow of time. (Deleuze, *Difference and Repetition*, p. 71)

61. The infinitely divisible event is always *both at once*. [future and past, active and passive] It is eternally that which has just happened and that which is about to happen, but never that which is happening . . . The event, being itself impassive, allows the active and the passive to be interchanged more easily, since it is *neither the one nor the other*, but rather their common result. (Deleuze, *Logic of Sense*, p. 8)

62. Deleuze, *Logic of Sense*, pp. 94–5.
63. *Ibid.*, p. 147.
64. *Ibid.*, p. 165.
65. *Ibid.*, p. 147.
66. Ralph H. Abraham, 'Dynamics and Self-Organization', in *Self-Organizing Systems. The Emergence of Order*, ed. F. Eugene Yates (Plenum Press, New York, 1987), p. 606.
67. On questions of simplicity and familiarity in the foundations of physics, see Lindsay and Margenau, *Foundations of Physics*, p. 18.

68. Deleuze, *Logic of Sense*, p. 166.
69. Ian Stewart, *Does God Play Dice? The Mathematics of Chaos* (Basil Blackwell, Oxford, 1989), pp. 114–21.
70. Deleuze and Guattari, *A Thousand Plateaus*, p. 251. (Emphasis in the original)
71. *Ibid.*, p. 9. The term 'line of flight', referring to the quasi-causal operator, is defined elsewhere (p. 488) as a fractal line. Precisely because the operator and the plane it constructs must cut and preserve N-dimensions for every multiplicity, Deleuze conceives of it as necessarily having a fractal number of dimensions, a number which is not a whole number but a fraction. For example, a flat piece of paper is a two-dimensional entity, but one folded into a ball has a dimension between two and three, that is, it is a fractal dimension. So does a one-dimensional string so folded that it begins to fill a plane. The operator itself would *not* be a transcendent agency operating in N+1 dimensions but on the contrary, it would work on N−1 dimensions (a *line* forming a plane, or an aleatory *point* circulating through one-dimensional series). On the fractal dimensionality of the plane, see also Deleuze and Guattari, *What is Philosophy?*, pp. 36–8.
72. Deleuze uses the term 'counter-actualization' for the extraction of ideal events from actual ones in Deleuze, *Logic of Sense*, pp. 150–2. He does not use the term 'pre-actualization' but this term does capture the meaning of the other task the quasi-cause must perform.

> In general, as we have seen, a singularity may be grasped in two ways: in its existence and distribution [in the vector field], but also in its nature, in conformity with which it extends and spreads itself out in a determined direction over a line of ordinary points. This second aspect already represents a certain stabilization and *a beginning of the actualization of singularities*. (*Ibid.*, p. 109; my emphasis)

73. 
> . . . the instant extracts singular points twice projected – once into the future and once into the past – forming by this double equation the constitutive elements of the pure event (in the manner of a pod which releases its spores). (*Ibid.*, p. 166)

74. 
> [When a multiplicity] is grasped in its relation to the quasi-cause which produces it and distributes it at the surface, it inherits, participates in, and even envelops and possesses the force of this ideational cause. We have seen that this [quasi-]cause is nothing outside its effect, that it haunts this effect, and that it maintains with the effect an immanent relation which turns the product, the moment that it is produced, into something productive. (Deleuze, *Logic of Sense*, p. 95)

This extract is about 'sense' not 'a multiplicity' but the two terms are closely related.

75. Once communication between heterogeneous series is established, all sorts of consequences follow within the system. Something passes between the borders, events explode, phenomena flash, like thunder and lightning . . . what is this agent, this force which ensures communication? Thunderbolts explode between different intensities, but they are preceded by an invisible, imperceptible, *dark precursor*, which determines their path in advance but in reverse, as though intagliated. (Deleuze, *Difference and Repetition*, pp. 118–19; emphasis in the original)

76. Deleuze does not speak of nonlinear, nonequilibrium areas of the world, but he does distinguish special processes (such as the spontaneous formation of metastable surfaces) from those characterizing full equilibrium structures. Only the former have the power to give rise to the virtual.

> When we say that bodies and their mixtures produce [the virtual], it is not by virtue of an individuation which would presuppose it. Individuation in bodies, the measure in their mixtures . . . presupposes . . . the pre-individual and impersonal neutral field within which it unfolds. It is therefore in a different way that [the virtual] is produced by bodies. The question is now about bodies taken in their undifferentiated depth and in their measureless pulsation. This depth acts in an original way, *by means of its power to organize surfaces and to envelop itself within surfaces*. (Deleuze, *Logic of Sense*, p. 124; emphasis in the original)

I have replaced references to 'sense' in this extract by 'the virtual'. (The term 'sense' is closely related to 'virtual multiplicity', but refers to the relation between virtuality and language, a relation I do not explore at all in this book.) The capacity of matter to form surfaces, even surfaces at equilibrium, constitutes the most primitive form of self-organization. The surfaces of liquid or solid bodies are, indeed, special or singular zones of those bodies, very different from the ordinary bulk material that they envelop. The bulk of a liquid body, a lake or ocean, for instance, consists of a population of molecules on which forces of attraction are exerted in all directions. At the surface of this body, on the other hand, there exists a changing sub-population on which forces are exerted inward but not outward. This gives those surface molecules special properties not displayed by the bulk. In particular, they will possess a certain amount of free energy (energy available for doing work) which accounts for the surface's spontaneous tendency to contract or minimize its extension (a 'surface tension' which explains why droplets of water spontaneously acquire a round shape). See Neil Kensington Adam, *The Physics and Chemistry of Surfaces* (Dover, New York, 1968), pp. 1–7.

Even at equilibrium, the surfaces of individuated bodies are capable of spontaneously giving rise to asymmetrical distributions of events, a distribution which is the signature of the quasi-causal operator. This is particularly clear in the case of electrical phenomena occurring at the surface of contact between different phases of matter.

> When two conducting phases are in contact, a difference of electrical potential is generally established between them. The establishment of this 'phase boundary potential' is intimately associated with the formation of an 'electrical double layer', at the surface, i.e. *an unsymmetrical distribution of electrically charged particles near the phase boundary*, with an excess of positive charges towards the phase which assumes a positive potential and of negative charges towards the phase assuming negative potential. (p. 300; my emphasis)

Here is Deleuze's version of the same ideas,

> Everything happens at the surface in a crystal which develops only on the edges. Undoubtedly, an organism is not developed in the same manner . . . But membranes are no less important, for they *carry potentials and regenerate polarities*. They place internal and external spaces into contact without regard to distance. The internal and the external, depth and height, have biological significance only through this *topological surface of contact*. Thus, even biologically it is necessary to understand that 'the deepest is the skin'. The skin has at its disposal a vital and properly superficial potential energy. And just as [virtual] events do not occupy the surface but rather frequent it, *superficial energy is not localized at the surface but rather bound to its formation and reformation*. (Deleuze, *Logic of Sense*, p. 103; my emphasis)

77. The term 'line of flight' is used in two ways, one to refer to relative, the other to absolute movements towards the virtual. A relative line of flight refers to actual assemblages, like those I described above when discussing embryogenesis and ecosystems, defined by affects and relations of speed and slowness.

> Comparative rates of flow in these lines produce phenomena of relative slowness or viscosity, or on the contrary, of acceleration and rupture. All this, lines and measurable speeds, constitute an assemblage. (Deleuze and Guattari, *A Thousand Plateaus*, p. 4)

I said that in these assemblages relative *accelerations* (neoteny, symbiosis) allow an *escape* from rigid morphologies, the term 'relative line of flight' referring to these phenomena, among others. An absolute line of flight is a further acceleration or boosting of these relative escapes which allows them to leave the extensive and intensive altogether.

> These relative movements should not be confused with the possibility of . . . an absolute line of flight . . . The former are stratic or interstratic [that is, concerned with extensities or intensities], whereas the latter concern the plane of consistency . . . There is no doubt that mad particles leave minimal trace of their passage through the strata as they accelerate, escaping spatio-temporal and even existential coordinates as they tend towards . . . the state of unformed matter of the plane of consistency. (pp. 55–66)

And it is these absolute lines that create the heterogeneous virtual continuum. 'Moreover, the plane of consistency does not preexist . . . the lines of flight that draw it and cause it rise to the surface, the becomings that compose it' (p. 270).

78. Philosophy is a constructivism, but constructivism has two qualitatively different complementary aspects: the creation of concepts and the laying out of a plane . . . Concepts are absolute surfaces or volumes, formless and fragmentary, whereas the plane is the formless, unlimited absolute, neither surface nor volume but always fractal . . . Concepts are events but the plane is the horizon of events, the reservoir or reserve of purely conceptual events . . . (Deleuze and Guattari, *What is Philosophy?*, p. 36)

Here the term 'concept' does *not* refer to 'concepts of the understanding', that is, to semantic or representational entities, but to virtual multiplicities: 'Every concept . . . is a multiplicity although not every multiplicity is conceptual' (p. 15). Without this definition reference to concepts as surfaces or volumes (that is, as manifolds) would be meaningless. That virtual multiplicities cannot be conceived as intellectual concepts is clear from the following extract, where the term 'Idea' gives a better rendering of what 'concept' means:

> If the Idea eliminates variability, this is in favour of what must be called variety [a synonym of manifold] or multiplicity. The Idea as concrete universal stands opposed to concepts of the understanding. (Deleuze, *Difference and Repetition*, p. 173)

79. Deleuze and Guattari, *What is Philosophy?*, p. 126.
80. Deleuze, *Logic of Sense*, p. 148.

## 4 VIRTUALITY AND THE LAWS OF PHYSICS

1. The rejection of totalities and the definition of social ontology as composed entirely of individuals operating at different scales needs to be defended in detail. I am aware that the way I present it here is rough and hardly compelling. Moreover, a convincing case for this point of view needs of necessity to have a historical dimension, that is, it needs to give the details of specific individuation processes, for institutions, cities and nation states. I have applied this ontology in the context of a historical analysis of Western history in Manuel DeLanda, *A Thousand Years of Nonlinear History* (Zone Books, New York 1997).
2. There are many approaches to the question of the disunity of science. Some particularly useful are John Dupree, *The Disorder of Things. Metaphysical Foundations of the Disunity of Science* (Harvard University Press, Cambridge, 1995); Jerry Fodor, 'Special Sciences, or The Disunity of Science as a Working Hypothesis', in *The Philosophy of Science*, eds. Richard Boyd, Philip Gasper and J. D. Trout (MIT Press, Cambridge, 1993); Peter Galison, 'Introduction: The Context of Disunity', in *The Disunity of Science*, eds. Peter Galison and David J. Stump (Stanford University Press, Stanford, 1996); Andrew Pickering, *The Mangle of Practice. Time, Agency, and Science* (University of Chicago Press, Chicago, 1995).
3. Ironically, some contemporary sociologists of science who are highly critical of the philosophers's approach make the mistake thinking that a novel approach to the study of science demands the elimination of causal relations. See H. M. Collins, *Changing Order* (University of Chicago Press, Chicago, 1992), pp. 6–8.

   It is hard to tell whether Collins thinks causes do not exist, thus siding with Hume, or whether he thinks we should suspend belief in them as a methodological manoeuver to highlight the 'social' aspects of scientific fields. The latter interpretation would avoid my criticism (that he is siding with the oldest and most conservative philosophy of science) but it would still be open to criticism in a different way: bringing 'society' as a totality into the analysis.
4. Ian Hacking, *Representing and Intervening* (Cambridge University Press, Cambridge, 1992), p. 46. (My emphasis) In contemporary philosophy the revival of causality as a productive or genetic relationship, one to be studied empirically not merely conceptually, was foreshadowed by the philosopher Mario Bunge in 1959, although the degree to which he has influenced current authors is hard to evaluate. His key book in this respect is *Causality and Modern Science* (Dover, New York, 1979). Here I adopt many of Bunge's views on productivity and depart only in the terminology. He uses the term

'determination' for the general relation (including linear, nonlinear and statistical causality) reserving the term 'causality' for linear causality, so as not to depart from tradition. I myself prefer to speak of causal relations in general, taking the linear case as an untypical case, since the point of my discussion is to break with tradition in these matters.

5. The entire group of new philosophers that have taken the 'causal turn' are unanimous in their rejection of the deductive-nomological model of explanation (as well as related models which replace deduction by induction, and exceptionless laws by statistical laws) for its emphasis on logico-linguistic form at the expense of causal-productive processes. See Bunge, *Causality and Modern Science*, pp. 290–1; Nancy Cartwright, *How the Laws of Physics Lie* (Clarendon Press, Oxford, 1983), pp. 132–3; Wesley C. Salmon, *Scientific Explanation and the Causal Structure of the World* (Princeton University Press, Princeton, 1984), pp. 26–32; Dupree, *The Disorder of Things*, pp. 178–9.

   Deleuze sometimes echoes the philosophical mischaracterization represented by the nomological-deductive model when he asserts that the object of science is 'functions that are presented as propositions in discursive systems' (Deleuze and Guattari, *What is Philosophy?*, p. 118).

   Although in his early work Deleuze is very careful to differentiate between mathematical functions which are close to linguistic statements (such as algebraic functions) from those that are not (differential functions), in his last work where the differences between science and philosophy are most dramatically stated, he lapses into a less careful statement of the question. Elsewhere (p. 128) he adds that '[T]he fact that science is discursive in no way means that it is deductive', but gives as an example of non-deductive activity the use of computers in the study of nonlinear functions. I believe the non-deductive aspect needs to be stressed much more and extended to modelling practices much older than computer-based experimentation. I have already argued that Deleuze's main point, *the insufficiency of functions to capture the virtual*, can be made without subordinating mathematical models to propositions, that is, by showing that functions define individuation processes in such a way as to stress the direction towards the actual.

6. Ronald N. Giere, *Explaining Science. A Cognitive Approach* (University of Chicago Press, Chicago, 1988), p. 82. (My emphasis)

7. Commenting on a particular case of derivation, that of the model of the simple pendulum in one dimension from the two-dimensional case, Giere says

   > The move from the mass-on-a-spring example to the simple pendulum seems to me a clear case of what Kuhn called 'direct modeling'. The

> two examples are not just special cases of a general relationship. One manages to reduce the pendulum, a two-dimensional system, to the one-dimensional case only by means of a judicious approximation that restricts the pendulum to small angles of swing. In particular, the step from the original application of Newton's laws to the two-dimensional pendulum to the one-dimensional version is not a matter of purely mathematical, or logical, *deduction*. 'Approximation' is a valid rule of deduction only in physicists' jokes about mathematicians. (*ibid*., p. 71; see also pp. 76–80)

8. Ilya Prigogine, *From Being to Becoming* (W. H. Freeman. New York, 1980), p. 19.
9. Cartwright, *How the Laws of Physics Lie*, pp. 54–5.
10. 'This fits better with my picture of a nature best described by a vast array of phenomenological [or causal] laws tailored to specific situations, than with one governed in an orderly way from first principles,' (*ibid*., p. 66).

    On Giere's view see Giere, *Explaining Science*, p. 85, and pp. 90–1 on his views on Cartwright's work.
11. Cartwright, *How the Laws of Physics Lie*, p. 107.
12. Deborah G. Mayo, *Error and the Growth of Experimental Knowledge* (University of Chicago Press, Chicago, 1996), p. 128.
13. Cartwright, *How the Laws of Physics Lie*, pp. 96–7.
14. Morris Kline, *Mathematics and the Physical World* (Dover, New York, 1981), p. 440. (My emphasis)
15. Morris Kline, *Mathematical Thought from Ancient to Modern Times*. Vol. 2 (Oxford University Press, New York, 1972), p. 580. More generally, on the history of variational techniques see Chapters 24 and 30.
16. 

> Given appropriate variational principles each with an associated multiple integral and scalar integrand, we can produce all the important partial differential equations in physics: the wave equation, the diffusion equation, Poisson's equation, Shrodinger's equation, and each of Maxwell's equations . . . Such thinking bears fruit. General relativity and quantum mechanics both originated from variational principles. (Don. S. Lemons, *Perfect Form. Variational Principles, Methods and Applications in Elementary Physics* [Princeton University Press, Princeton, 1997], p. 111)

17. *Ibid*., pp. 17–27. In a passage where Deleuze contrasts the propositional approach to the problematic one (or what amounts to the same thing, an approach to thought in terms of its conditions as opposed to its productive genesis), he compares the Kantian conception of the concept of 'shortest distance' (as a representational schema) to the conception made possible by the calculus of variations. The term 'shortest', as he says,

> may be understood in two ways: from the point of view of conditioning, as a schema of the imagination which determines space in accordance with the concept (the straight line defined as that which in all parts may be superimposed upon itself) – in this case the difference remains external, incarnated in a rule of construction . . . Alternatively, from the genetic point of view, the shortest may be understood as an Idea [multiplicity] which . . . interiorizes the difference between straight and curved, and expresses this internal difference in the form of a reciprocal determination [differential relations] and in the *minimal conditions of an integral*. (Deleuze, *Difference and Repetition*, p. 174)

18. Leonard Euler, quoted in Stephen P. Timoshenko, *History of Strength of Materials* (Dover, New York, 1983), p. 31. (My emphasis)

19. > Far from being concerned with solutions, truth and falsehood primarily affect problems. A solution always has the truth it deserves according to the problem to which it is a response, and a problem always has the solution it deserves in proportion to its own truth and falsity – in other words, in proportion to its sense. (Deleuze, *Difference and Repetition*, p. 159)

    In what follows I will not speak of 'true problems' but of 'correct' or 'well-posed problems' but this constitutes, I believe, only a harmless terminological departure from Deleuze.

20. Kline, *Mathematics and the Physical World*, p. 441. Within this tradition, the unifying power of Hamilton's principle was almost inevitably interpreted as consisting in the generality of its truth, and axiomatic versions of classical mechanics were produced in the nineteenth century (by Heinrich Hertz, for example) to marry the unifying power of variational principles with the concept of general truth. See Robert B. Lindsay and Henry Margenau, *Foundations of Physics* (Ox Bow Press, Woodbridge, 1981), pp. 118–20.

    In a Deleuzian ontology eliminating essentialism from physics involves replacing clear and distinct truths (axioms and theorems) by problems, that is, replacing deductively connected linguistic propositions in the Euclidean geometry mould by problems defined by singularities (events) and affects.

    > Greek geometry has a general tendency on the one hand to limit problems to the benefit of theorems, on the other to subordinate problems to theorems themselves. The reason is that theorems seem to express and develop the properties of simple *essences* whereas problems concern only *events and affections* . . . As a result, however, the *genetic* point of view is forcibly relegated to an inferior rank: proof is given that something cannot

> be rather than that it is and why it is (hence the frequency in Euclid of negative, indirect and [reductio ad absurdum] arguments . . .). Nor do the essential aspects of the situation change with the shift to an algebraic and analytic point of view. Problems are now traced from algebraic equations . . . However just as in geometry we imagine the problem solved, so in algebra we operate upon unknown quantities as if they were known: this is how we pursue the hard work of reducing problems to the form of propositions capable of serving as cases of solution. We see this clearly in Descartes. The Cartesian method (the search for the clear and distinct) is a method for solving supposedly given problems, not a method of invention appropriate to the constitution of problems or the understanding of questions. (Deleuze, *Difference and Repetition*, p. 160.)

21. Deleuze, *Difference and Repetition*, p. 189.
    'For Problems–Ideas are by nature unconscious: they are *extra-propositional and sub-representative*, and do not resemble the propositions which represent the affirmations to which they give rise' (p. 267; my emphasis).
22. Ian Hacking, *Representing and Intervening*, p. 41. (Emphasis in the original) In addition to ignoring causes and downplaying explanations, positivist philosophy holds a 'verificationist' theory of meaning (if the truth of a statement cannot be tested the statement is meaningless), a belief that verification involves comparison with raw data (data from the senses) and a disbelief in theoretical (or unobservable) entities. Hacking later on also expresses some doubts about the role of explanations (pp. 52–5) but this is, I believe, limited to their role as arguments for realism. Hacking is well known for his championing of causal interventions in experimental reality as criteria for realism, or for belief in unobservable entities.
23. My focus on Why questions is not meant to link these matters to a specific syntactic form, and is simply a matter of ease of exposition. Clearly, such questions may be paraphrased in other ways: the request for a causal explanation expressed by the question 'Why did event X occur?' may be expressed by 'How was event X produced?' or something like that. Though Deleuze does not refer to Why questions he does differentiate between questions with simple propositions as answers (which subordinate the question to a search for essences) from those more properly problematic.

> Rationalism wanted to tie the fate of Ideas [multiplicities] to abstract and dead essences; and to the extent that the problematic form of Ideas was recognized, it even wanted that form tied to the question of essences – in other words, to the 'What is X?' . . . It should be noticed how few philosophers have placed their trust in the question 'What is X?' in order to have Ideas. Certainly not Aristotle. Once the dialectic [the art of

> posing problems] brews up its matter instead of being applied to propaedeutic ends, the questions 'How much', 'How', 'In what cases' and 'Who' abound . . . These questions are those of the accident, the event, the multiplicity. (Deleuze, *Difference and Repetition*, p. 188)

A more important omission in my discussion is that it does not include Deleuze's distinction between problems and questions. Problems are the epistemological counterpart to virtual multiplicities, while questions (which involve an imperative, a request or demand for an explanation, for example) are the sources of problems or the counterpart of the quasi-causal operator. There are also epistemological counterparts to the intensive and the actual,

> We distinguished four instances: imperative or ontological questions; dialectical problems or the themes that emerge from them; symbolic fields of solvability in which these problems are 'scientifically' expressed in accordance to their conditions; the solutions given in these fields when the problems are incarnated in the actuality of cases. (p. 200)

24. Alan Garfinkel, *Forms of Explanation* (Yale University Press, New Haven, 1981), p. 21. Other philosophers have developed similar approaches to Why questions and their relation to the distributions of the relevant and the irrelevant. See, for example, Salmon, *Scientific Explanation and the Causal Structure of the World*, pp. 1–6. See also Salmon's discussion of Van Frassen's approach to Why questions and contrast spaces (pp. 102–6) which, unlike Garfinkel's, is completely linguistic.
25. Alan Garfinkel, *Forms of Explanation*, p. 40.
26. *Ibid.*, p. 64. Garfinkel takes this characterization of state space from René Thom, creator of catastrophe theory and of the concept of structural stability. Here the term 'critical point' may refer to both the unstable separatrix that defines (as a repellor) the border of a basin of attraction, or to a bifurcation which defines the point of structural instability at which one distribution of attractors changes into another.
27. Deleuze, *Difference and Repetition*, p. 159.
28. Alan Garfinkel, *Forms of Explanation*, pp. 53–8.
29. *Ibid.*, pp. 58–62.
30. *Ibid.*, p. 168.
31. Robert M. May, 'Chaos and the Dynamics of Biological Populations', in *Dynamical Chaos*, ed. M. V. Berry (London Royal Society, 1987), pp. 31–2. May's focus in this essay is chaotic attractors, but he does mention periodic attractors. (The latter are less controversial in population studies than the former.) I avoid discussion of 'chaos' in the main text due to the excessive hype surrounding the subject, but more importantly, because ontologically

the key notion is that of 'attractor' not the particular chaotic case. That is, the key is quasi-causality itself not any one of its particular forms.

32. Deleuze, *Difference and Repetition*, p. 212.
33. *Ibid.*, p. 211.
34. Deleuze views the solving of a virtual problem by individuation processes as an 'explanation' or rather, an 'explication'. This term is used to refer to the cancelling out of intensive differences during a process of individuation, the hiding of intensity under the extensities and qualities it gives rise to.

> It is not surprising that, strictly speaking, difference should be 'inexplicable'. Difference is explicated, but in systems in which it tends to be canceled; this means only that difference is essentially implicated, that its being is implication . . . Intensity is developed and explicated by means of an extension which relates it to the extensity in which it appears outside itself and hidden beneath quality. (Deleuze, *Difference and Repetition*, p. 228)

Some scientists today (Chris Langton, for instance) are beginning to view some processes of morphogenesis as involving the solution to *computational problems*.

> A material near its critical transition point between the liquid and the gas states, must, in effect, come to a global decision about whether it must settle down to a liquid or to a gas. This sounds almost anthropomorphic, but the results reported here suggest that we must think about such systems as effectively *computing* their way to a minimum energy state. (Christopher G. Langton, 'Life at the Edge of Chaos', in *Artificial Life II*, eds. Christopher G. Langton, Charles Taylor, Doyne Farmer and Steen Rasmussen (Addison-Wesley, Redwood City, 1992), p. 82.

35. This, in fact, occurs in a different context. Deleuze never makes this point relative to theoretical and experimental physics, but I believe his idea can be extended in that direction. The actual extract reads,

> Not only do linguistic variables of expression enter into relations of formal opposition or distinction favorable for the extraction of constants; non-linguistic variables of content do also. As Hjelmslev notes, an expression is divided, for example, into phonic units in the same way a content is divided into social, zoological, or physical units . . . The network of binarities, or arborescences, is applicable to both sides. *There is, however, no analytic resemblance, correspondence or conformity between the two planes. But their independence does not preclude isomorphism* . . . (Deleuze and Guattari, *A Thousand Plateaus*, p. 108; my emphasis)

36. Bunge, *Causality and Modern Science*, p. 175. (My emphasis)

37. In a linear system the ultimate effect of the combined action of two different causes is merely the superposition [e.g. addition] of the effects of each cause taken individually. But in a nonlinear system adding a small cause to one that is already present can induce dramatic effects that have no common measure with the amplitude of the cause. (Gregoire Nicolis and Ilya Prigogine, *Exploring Complexity* [W. H. Freeman, New York 1989], p. 59)

38. Bunge, *Causality and Modern Science*, p. 127.
39. *Ibid.*, p. 49.
40. This is Wesley Salmon's characterization of statistical causality, meant to replace previous versions stated in terms of *high probability*. These older versions, due to the absoluteness of the probability value (near = 1), are simply weakenings of necessity (the case with probability = 1) whereas enhanced probability is not. The latter demands that we know the prior probabilities (the probability of occurrence of an event without the presence of the cause) as well as the posterior probabilities. Whether or not the value of the enhanced probability is near = 1 is not an issue in Salmon's version, hence it really breaks with necessity not just weakens it. See Salmon, *Scientific Explanation and the Causal Structure of the World*, pp. 30–4.
41. *Ibid.*, p. 203.
42. Bunge, *Causality and Modern Science*, Chapter 6.
43. *Ibid.*, Chapter 8.
44. Deleuze and Guattari, *A Thousand Plateaus*, p. 408. (Emphasis in the original)
45. Ian Hacking, *Representing and Intervening*, p. 158. (My emphasis) Hacking explicitly compares experimentalists and artisans, both suffering a relatively lower social status due to their involvement with an active materiality, one that does not obey simple theoretical laws or allow external forms to be imposed on it as a command (p. 151). In classical mechanics perhaps the best examples of these two scientific castes are the theorists Isaac Newton or Robert Boyle, on one hand, and the experimentalist Robert Hooke, on the other. As one scientist puts it, 'unlike Newton, Hooke was intensely interested in what went on in kitchens, dockyards, and buildings – the mundane mechanical arenas of life . . . Nor did Hooke despise craftsmen, and he probably got the inspiration for at least some of his ideas from his friend the great London clockmaker Thomas Tompion . . .' (James Edward Gordon, *The Science of Structures and Materials* [Scientific American Library, 1988], p. 18).
46. 'Phenomena accumulate. For example, Willis Lamb is trying to do optics without photons. Lamb may kill off the photons [i.e. create a new theory or

a new paradigm for optics] but the photoelectric effect will still be there' (Hacking, *Representing and Intervening*, p. 56. Also see pp. 155–62).

47. *Ibid.*, pp. 83–4.
48. *Ibid.*, p. 265. (Emphasis in the original)
49. Pickering, *The Mangle of Practice*, p. 70.
50. Deleuze, in fact, does not refer to learning in a laboratory context, but his idea of learning as involving an intensive assemblage or a *problematic field* is clearly applicable to the case of experimental physics. Here's how Deleuze expresses this idea,

> For learning evolves entirely in the comprehension of problems as such . . . Learning to swim or learning a foreign language means composing the singular points of one's own body or of one's own language with those of another shape or element which tears us apart but also propels us into a hitherto unknown or unheard-of world of problems. (*Difference and Repetition*, p. 192)

And he adds that this composition of one's singularities and affects with those of water (in the case of swimming) or with those characterizing the sounds and patterns of a language, forms a problematic field (p. 165). A 'problematic field' refers to a heterogeneous assemblage since, as he says, 'learning is the . . . structure which unites difference to difference, dissimilarity to dissimilarity, without mediating between them' (p. 166).

51. *Ibid.*, p. 164.
52. Hacking, *Representing and Intervening*, p. 209.
53. Bunge, *Causality and Modern Science*, p. 71. (Emphasis in the original)
54. Deleuze, *Difference and Repetition*, p. 25.
55. *Ibid.*, p. 177.
56. Generalizing, we can say that a dynamical theory is approximately true just if the modeling geometric structure approximates (in suitable respects) to the structure to be modeled: a basic case is where trajectories in the model closely track trajectories encoding physically real behaviors (or, at least, track them for long enough). (Peter Smith, *Explaining Chaos* [Cambridge University Press, Cambridge, 1998], p. 72)
57. Arthur S. Iberall, *Towards a General Science of Viable Systems* (McGraw-Hill, New York, 1972), p. 7. (My emphasis)
58. Nelson Goodman, 'Seven Strictures on Similarity', in *Problem and Projects* (Bobbs-Merrill, Indianapolis, 1972), p. 445. Goodman's attack on the notion of similarity was as caustic as it was influential. Similarity, he said 'ever ready to solve philosophical problems and overcome obstacles, is a pre-

tender, an impostor, a quack. It has, indeed, its place and its uses, but is more often found where it does not belong, professing powers it does not possess' (p. 437). Today's generation of realist philosophers who have resuscitated this notion have learned Goodman's lesson that *any two things are similar* in some respect or another, and that therefore whenever valid judgments of similarity are made the *relevant respects* in which things may be said to be alike must be specified (p. 444). But this, of course, simply changes the task to one of specifying distributions of the relevant and the irrelevant, and *that* is just what a problematic approach is supposed to do. At this point the usual reply by defenders of similarity is to fall back on subjectivism and say that questions of relevance and irrelevance are *interest-relative*.

But far from settling the issue, to relativize relevance to subjective interests is fatal to realism. If there's one lesson to be learned from recent sociology of science it's that, as a matter of empirical fact, the interests of scientists cannot be viewed as being purely epistemological, born from some essential rationality or a driving curiosity. If we are to relativize relevance to interests then we should bring the full repertoire of interests here, including not only selfish professional and institutional interests but also those that may be derived from a scientist's membership in class or gender hierarchies, for example. The rampant relativism that this manoeuver has sometimes given rise to should be a cautionary lesson for any defender of realism. Alan Garfinkel sometimes expresses himself as if the choice of contrast space, that is, the choice of how to pose a problem, is relative to human interests and values, as in the different values held by the priest and the thief in his example. But questions of *explanatory stability* seem to point to an objectivity of the distributions of the relevant and the irrelevant. Whatever relativity there may be in explanations it is an objective one, depending on the existence of individuals with their own emergent causal capacities at many levels of scale. Human values would enter the picture in the choice of one or another of these levels of scale as the level of interest, but a correct explanation, as Garfinkel says, 'will seek its own level' (Garfinkel, *Forms of Explanation*, p. 59).

59. Gilles Deleuze, *Difference and Repetition*, p. 163.

There is nothing in the ordinary meaning of the words 'universal' and 'singular' that marks the philosophical distinction Deleuze is attempting to draw here. In fact, analytical philosophers use the words 'general' and 'universal' almost interchangeably, and the terms 'particular' and 'singular' as closely related. In *Difference and Repetition* universality and singularity are both properties of objective problems, the former defining their ontological status as virtual entities (capable of divergent actualization) the latter the status of that which defines their conditions (distributions of the relevant and the irrelevant). The very first page of this book states 'Generality, as generality of the particular, thus stands opposed to repetition as universality of the singular' (p. 1). Yet, Deleuze is not consistent in his usage, and elsewhere he says that the 'splendid sterility or neutrality [of multiplicities] . . . is indifferent to the universal and the singular, to the general and the particular, to the personal and the collective' (Gilles Deleuze, *Logic of Sense* [Columbia University Press, New York, 1990], p. 35).

60. Dialectic is the art of problems and questions . . . However, dialectic loses its peculiar power when it remains content to trace problems from propositions: thus begins the history of the long perversion which places it under the power of the negative. Aristotle writes: The difference between a problem and a proposition is a difference in the turn of phrase. (Deleuze, *Difference and Repetition*, p. 158)

61. Ian Stewart and Martin Golubitsky, *Fearful Symmetry* (Blackwell, Oxford, 1992), p. 42. (Emphasis in the original)
62. Deleuze, *Difference and Repetition*, p. 162.
63. The impact of group theory on physics is revealed not only by the fact that the change from classical to relativistic physics can be described in group theoretic terms (Einstein replaced the old Galilean group of transformations by another one, the Poincaré group) but also by the fact that the switch to relativistic mechanics involved a change of cognitive strategy in which invariances under transformations became more important than the physical laws themselves. As the physicist Eugene Wigner puts it,

   [Einstein's] papers on special relativity . . . mark the reversal of a trend: until then the principles of invariance were derived from the laws of motion . . . It is natural for us now to derive the laws of nature and to test their validity by means of the laws of invariance, rather than to derive the laws of invariance from what we believe to be the laws of nature. The general theory of relativity is the next milestone in the history of invariance . . . It is the first attempt to derive a law of nature by selecting the simplest invariant equation . . . (Eugene P. Wigner, 'Invariance in Physical Theory', in *Symmetries and Reflections*, eds. Walter Moore and Michael Scriven [Ox Bow Press, Woodbridge, 1979], p. 7)

64. Morris Kline, *Mathematical Thought from Ancient to Modern Times*, Vol. 2 (Oxford University Press, New York, 1972), p. 759. This idea can be explained by analogy with the use of transformation groups to classify geometrical figures. When one says that a cube remains invariant under a

group of rotations (e.g. the set containing 0, 90, 180 and 270 degree rotations) one means that, after performing one such transformation the cube's appearance remains unchanged: an observer who did not witness the transformation would not be able to tell that a change has in fact occurred. In a similar way, when Galois found a group of permutations that left algebraic relations invariant he found *a measure of our ignorance of the solutions*, since we cannot distinguish them from one another after they have been so transformed.

65. Deleuze, *Difference and Repetition*, pp. 180–1.
66. *Ibid.*, pp. 179–80. That Deleuze views the progressive specification of a problem as a kind of symmetry-breaking cascade (a term he never uses, preferring Galois's idea of an 'adjunction of fields') is clear from this extract:

> On the contrary, 'solvability' must depend upon an internal characteristic: it must be determined by the conditions of the problem, engendered in and by the problem along with the real solutions. Without this reversal, the famous Copernican Revolution amounts to nothing. Moreover, there is no revolution so long as we remain tied to Euclidian geometry: we must move to . . . a Riemannian-like differential geometry which *tends to give rise to discontinuity on the basis of continuity*, or to ground solutions on the conditions of the problem. (p. 162; my emphasis)

67. Ian Stewart, *Does God Play Dice? The Mathematics of Chaos* (Basil Blackwell, Oxford, 1989), pp. 38–9.
68. Nonlinear equations, due to factors like the occurrence of higher powers of the dependent variable, do not obey superposition. On the differences between the linear and the nonlinear, and on the (rare) conditions for the exact solvability of nonlinear equations (autonomy and separability), see David Acheson, *From Calculus to Chaos: An Introduction to Dynamics* (Oxford University Press, New York 1997), Chapter 3.

    On the superposition principle as criterion to distinguish these two types see David K. Campbell, 'Nonlinear Science. From Paradigms to Practicalities', in *From Cardinals to Chaos*, ed. Necia Grant Cooper (Cambridge University Press, New York, 1989), p. 219.
69. Stewart, *Does God Play Dice?*, p. 83. (Emphasis in the original)
70. June Barrow-Green, *Poincaré and the Three Body Problem* (American Mathematical Society, 1997), pp. 32–8.

    On the history of this approach prior to the work by Poincaré see Kline, *Mathematical Thought from Ancient to Modern Times*, pp. 721–5.
71. > We always find the two aspects of the illusion: the natural illusion which involves tracing problems from supposedly preexistent propositions,

> logical opinions, geometrical theorems, algebraic equations, physical hypotheses or transcendental judgments; and the philosophical illusion which involves evaluating problems according to their 'solvability' – in other words, according to the extrinsic and variable form of the possibility of their finding a solution. (Deleuze, *Difference and Repetition*, p. 161)

72. Bunge, *Causality and Modern Science*, pp. 203–4. (Emphasis in the original)

> The fact that nonlinear theories are rare is not so much a peculiarity of nature as a sign of the infancy of our science. Nonlinearity involves large mathematical difficulties; beside being mathematically clumsy, it affects the very symbolic representation of physical entities. Thus forces that add nonlinearly (as gravitational forces do) cannot be exactly represented by vectors since the addition of the latter conforms to the superposition 'principle'. From the moment it was realized that the laws of ferromagnetism are nonlinear, it has been more or less clearly suspected that all physical phenomena may turn out to be at least weakly nonlinear, linearity being only an approximation which is excellent in some cases but only rough in others. (p. 168; emphasis in the original)

73. Deleuze, *Difference and Repetition*, p. 189.

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