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DOCTORAL THESIS

Teaching Mathematics with Textbooks

A Classroom and Curricular Perspective

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A CLASSROOM AND CURRICULAR PERSPECTIVE

MONICA JOHANSSON

To my family

ABSTRACT

The thesis consists of four articles and a preamble that introduces the work and links the articles together. The overarching issue that guides this work concerns the textbooks and their use in mathematics teaching in Sweden. The intention of making four different studies was to be able to examine the textbooks and their use in the classroom from different perspectives.

All parts of the thesis share the same focus, namely the relationship between the textbook and the curriculum. In this case, the curriculum is seen in a broad sense. It involves *the intended*, *the implemented curriculum* and *the enacted curriculum*. The work is guided by the traditions in the curriculum field in Sweden. This means that the choice of educational content and contextualization of teaching is emphasized.

The mathematics textbook as an object is discussed from different points of view. Some important features and different conceptions of the textbook are highlighted, for example the authorization of a textbook and the role of the textbooks as links between the national guidelines and the teaching of mathematics in schools.

The empirical study of the use of textbooks in classrooms is made up of two parts, one is mainly quantitative and the other is qualitative. The quantitative part of the classroom study shows that the textbook influences not only what kind of tasks the students are working with during the lessons, but also the examples the teacher presents on the board, what kind of concepts of mathematics are introduced and how they are introduced. The organization of the lessons is also discussed. In considerable parts of the lessons, students are working on an individual basis solving tasks in the textbook.

From the qualitative part of the study, one could see that the teacher can get into difficulties *because* of too much reliance on the textbook. However, one could also recognize that there is room for maneuver and that the teacher sometimes uses this space and deviates from the book. It could for example happen when the teacher becomes aware of some mathematical aspects, which the textbook does not cover. It could also be the case that the teacher uses other resources than the textbook.

In all, the study shows the relative autonomy of the mathematics teacher in relation to the most common teaching tool in Swedish classrooms – the textbook.

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CONTENTS

LIST OF PAPERS INCLUDED IN THE THESIS

I. Johansson, M. (2005). The mathematics textbook: from artefact to instrument. *Nordic Studies in Mathematics Education, NOMAD, 10*(3-4), 43-64.

II. Johansson, M. (2005). *Mathematics textbooks: the link between the intended and the implemented curriculum?* Paper presented at the Eight International Conference of the Mathematics Education into the 21st Century Project: Reform, Revolution and Paradigm Shifts in Mathematics Education, Johor Bahru, Malaysia.

III. Johansson, M. (submitted). Textbooks as instruments: three teachers' way to organize their mathematics lessons.

IV. Johansson, M. (submitted). Mathematical meaning making and textbook tasks.

FOREWORD

The reader might wonder why this thesis focuses on textbooks and how they are used in the classroom. In order to understand why this became my area of interest one must perhaps know something about me and especially my experiences of mathematics in compulsory school. As far as I remember it, the textbook defined school mathematics. How good you were in mathematics was measured by how far you had reached in the book, i.e. on what page you were working on, and, of course, the results on the tests. I was good at mathematics, but I did not like the subject.

Later on, I learned that this experience is not unique; it is rather the culture in many mathematics classrooms in Sweden. It took several years after finishing school, during my studies of mathematics at university level, before I discovered that mathematics was a subject for me. I enjoy learning mathematics – and that was a surprise to me. My interest in the textbooks, i.e. the content of mathematics textbooks, started when I was helping my children with their homework. I especially remember one occasion when my daughter needed help with a task in the textbook, which she had brought home in order to 'catch up' with the other students in her class. The task was something like this: *Anna is eight years older than Per. If Per is x years old, how old is Anna.* According to the answer key at the end of the book, the answer should be *Anna is x+8 years old.* Ridiculous I thought, how could you expect that a child, who never had been in contact with variables in a formal sense before, can come up with such an answer? Furthermore, how can you expect that a child finds it reasonable to express an age in such a way? We all know that small children often say exact how old they are, for example 'five and a half' years old. I tried to help my daughter but she was not happy at all, she did not understand the purpose of the task. For me, this was frustrating and one could say that this made me start thinking about how important the content of a textbook is.

A first step in my research as a doctoral student was to study the link between the textbooks and the curriculum. Since many teachers seem to use their textbook as the main guide in their teaching, it was important to examine if this was appropriate, i.e. if textbooks really reflects the intentions in the national curriculum. I presented a licentiate thesis on this subject in December 2003. At that time, my understanding of the role of the textbook in the teaching of mathematics in schools had changed. From the beginning, I thought that I had found a 'key' for making a reform in the teaching of mathematics. If we just focus on the content, we could change the reality for many school children. I thought that to develop better material is crucial or at least very important. However, when reading about experiences from previous research on the area I realized that it was not that easy.

At that time, alarming reports were published and discussed. They concerned the declining level of attainment in mathematics in Sweden. International comparative studies, for example the TIMSS and the PISA studies, showed that Swedish students perform worse in comparison to other countries. An evaluation in the schools, made by the delegation of the National Agency of Education, which reported on the issue of students' joy to learn, painted a rather gloomy picture of the mathematics classrooms. In between the lines, one could understand some criticism of the teachers, especially for them letting the students do monotonous calculations in the textbooks. There were, however, no discussions about the quality of the actual textbooks in use. This could be interpreted as a wish to persuade teachers to get rid of their books, and thus make the students more interested in mathematics. For me, such a solution would be too simple.

The next step in my research efforts as a doctoral student was taken when I went into the classrooms to study the actual use of textbooks. For that purpose, an observation sheet was developed in cooperation with colleagues in the research group MaLiL (Matematik och Lärande i Luleå/Mathematics and Learning in Luleå). After observing four lessons, while at the same time marking the activity in the classrooms and the use of textbooks, I realized that more resources than just paper and pencil were needed (at least if I would like to capture further information than just time allocation in relation to the use of textbooks). What I did see, and what was difficult to capture with an observations sheet, was that two teacher can spend the same amount of time on textbooks but, nevertheless, teach mathematics differently. It is, for example, a quite different situation if the teacher, stressfully, tries to *answer* questions from the students, or if the teacher has time to *ask* them questions.

In order to capture these differences, I thought it would be necessary to make video-recordings of the lessons. But since there was no money for buying video-equipment and since the time was running, I was pleased when I got the opportunity to go to Uppsala and use the data material of the CULT-project. It is a rich and extensive material, which is well done and has a high quality. The observation sheet, which was developed for the use in ‘real time’, could be more fine-graded when it was possible to re-wind and look at sequences of the lessons again and again. Most beneficial at that time was the possibility to use the coding manual of the TIMSS Video Study as a support for the adjustments in the observation sheet. Instead of ending up with a more or less quantitative study, I could analyze the lessons in a more qualitative way.

Well, after doing a study like this, what is my opinion of textbooks and the use of textbooks today? At the beginning of this journey, I thought it was necessary to develop the textbooks. I still believe this is important, but it is not sufficient because the teachers are so central. The ‘key’, as I see it today, is that the teachers are feeling safe in their mathematical and didactical knowledge – then there is no need to rely upon a book. However, it is important to increase the awareness of the textbook as a critical ‘instrument’ in the classroom. On the one hand, we need to learn about its limitation, and on the other hand, we should think about its potential. It is not necessary to get rid of the book if it facilitates the daily work of the teacher and the teachers should not feel bad because they are making use of the good parts of the book.

Finally, a good friend of mine used to say that it is better to have a ‘good’ teacher with a ‘poor’ book than a ‘poor’ teacher with a ‘good’ book. This could be true but is difficult to prove.

INTRODUCTION AND AIMS

From a curricular perspective, one can see the textbook as the *potentially implemented curriculum* (M. Johansson, 2003; Johansson, 2005b). A textbook is often organized in such a way that it covers the topics that students should encounter during a particular school year. This means that it serves as some kind of agreement and support for the uniformity within the school system. In some reforms of schools, the textbooks could either have a prominent position, if the development of textbooks and other curriculum materials is seen as a possibility to change teaching, or they can be seen as obstacles. In both cases, the textbooks are regarded as influential factors.

Can one say that textbooks *influence* the teaching and learning? It is of course questionable whether a dead object like a book or a text really can lead people in to a certain direction in a pedagogical process. But the question would probably end up in a fruitless discussion. On the other hand, if one thinks about the influence of textbooks as something that is related to peoples' beliefs and values, the influence of textbooks will be based on a more or less conscious idea that the book is important (B. Englund, 1999). From the history of mathematics, we can actually notice their importance by considering the mathematicians that have had a prominent position – mainly because they have written books that become frequently used in education.[1] In that sense, textbooks contribute to the field of mathematics by preserving and transmitting skills and knowledge.

From a classroom perspective, one can see the textbooks as tools, or instruments, that facilitate the daily work of the teachers. They identify the topics and order them in a way students should explore them. They also attempt to specify how classroom lessons can be structured with suitable exercises and activities. For better or for worse, they provide an interpretation of mathematics to teachers, students and their parents. In Sweden, there is an ongoing discussion concerning too much reliance on textbooks in the teaching of mathematics in schools. An example of a contribution in this debate comes from a current evaluation of schools in 40 municipalities (out of 290). The inspectors found that the teaching of mathematics, more than any other school subject, relies on the use of textbooks.

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[1] Just think about the famous work of Euclid (born ca. 325 BC), *The Elements*, that became the centre of mathematical teaching for 2000 years (Johansson, 2005a).

> The evaluation shows the surprisingly dominant role of the textbooks in teaching ... especially from year 4-5 and onwards … Content as well as arrangement of teaching are to a high degree directed by the textbook. Mathematics is, for both students and teachers, simply what is written in the textbook (Lindqvist, Emanuelsson, Lindström, & Rönnberg, 2003, p. 39, author's translation).

This study is an attempt to further analyze the textbooks and their influence in mathematics classrooms, especially how the influence of textbooks can be discussed from a Swedish perspective. The study is guided by a theoretical perspective that origins from the field of curriculum research in Sweden. A fundamental assumption is that students are offered different possibilities to create and construct meaning depending on, for example, what content is chosen and what context the textbook offers. In other words, different choices can be made, more or less consciously, which have crucial implications for teaching and learning (T. Englund, 1997).

The overall objective in all parts of this thesis is to study textbooks as critical factors in the teaching of mathematics in schools. The objectives are:

1. To further incorporate content and context issues in research in mathematics education and contribute to the development of the field.
2. To deepening the understanding of what a textbook is and what kind of potentials and constraints it entails.
3. To examine the influence of textbooks in some Swedish classrooms in order to:
    - contribute to the picture of the enacted curriculum.
    - contribute to the discussion concerning teachers dependence on textbooks.

It is an explorative study that is supposed to be *descriptive-analytic* rather than *normative.* The purpose is to *problemize* rather than *criticize*.

THE ORGANIZATION OF THE TEXT

After this very brief introduction, a more precise description of the rationale and the theoretical framework will follow. The first subsection includes a short and not all-encompassing description of the conditions of teaching and learning in Sweden. The main purpose with this part is to guide the readers who are not familiar with schooling in Sweden. First an overview of the Swedish educational system and curriculum and then some aspects of textbooks and their role in the Swedish curriculum are discussed. As the main actors in a classroom are the teacher and the students, it could be appropriate to give an idea of what it means to be a teacher or a student in a Swedish school. Therefore, some features of this condition, which are regarded as relevant for this work, are considered.

This dissertation is presented in the subject *Matematik och lärande* in Sweden. Nationally and internationally, it is a contribution to didactics of mathematics. The second subsection gives a short and not thorough description of didactics of mathematics as a research field. A curricular perspective as the subtitle suggests, guides the study that is reported in this thesis. This means that a perspective, which comes from another field of research, the curriculum, influences the work. Therefore, definitional issues and curriculum research in Sweden will be discussed in the two subsections that follow.

After setting the scene in terms of rationale and theoretical framework, methodological issues, ethics, and scientific quality are discussed. Seven evaluation criteria: worthwhileness, coherence, competence, openness, ethics, credibility, and a general principle called ‘other qualities of good research reports’ form the basis for this discussion.

The thesis includes four papers. Two of these papers discuss the textbooks as objects, the other two concern the use of textbooks in classrooms. After this preamble, the papers are obtainable in full versions. However, in order to give an overview of the work, a summary of these papers and their main results is presented in the end of this part. In the concluding discussion, the main results from all parts of the work are considered, including implications for teaching and suggestions for further work.

RATIONALE AND THEORETICAL FRAMEWORK

TEACHING AND LEARNING MATHEMATICS IN SWEDEN

The school system and the curriculum

In Sweden, the public school includes both compulsory and noncompulsory schooling. *Preschool* is noncompulsory, and is aimed for children at the ages of 1-5 years. When the children turn 6, they will be offered a place in the *preschool class* and the schooling becomes free of charge (this remains throughout the whole public school system). The 9-year *compulsory school* is for children at the age of 7-16 years. Officially, there is no streaming or tracking; it is a 'school for all'. The *upper secondary school*, gymnasiet, is not compulsory but almost all students continue their studies at upper secondary level. Since mathematics is a core subject, the students are normally guaranteed at least 900 hours of mathematics education in compulsory school. To pass in mathematics is also requested for the three-year national programs at the gymnasiet.

In the Swedish language, there is no single corresponding term for 'curriculum' in its extensive meaning (Svingby, 1978). The National Agency of Schools in Sweden, *Skolverket*, publishes official documents that are parts of the *intended curriculum*. These are three documents, the Education Act, *Skollagen*, which includes the timetable, the curriculum, *läroplanen,* and the syllabus, *kursplanen* (Skolverket, 2001b)[2]. Sivesind et al. (2003) classified the läroplan in Sweden as *politically standardized*. This means that the focus is on principles and normative values that should permeate the schoolwork. The details of the intended curriculum are locally determined.

School mathematics is different from mathematics as a scientific discipline. Unlike the latter type of mathematics, particular forces related to the social responsibility of schooling determine school mathematics. This means that the subject is colored by methodological attitudes and philosophical believes (Jahnke, 1986). Bjerneby Häll (2002) states that school

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[2] These are the official translations from Swedish to English, retrieved from www.skolverket.se. Since the concept curriculum is recognized in a broad sense in this thesis, I will use the Swedish terms *läroplan* when referring to the political document that describes the overall contents and goals for the schools. The contents and goals for the school topics, for instance the course in mathematics, are called *kursplan*.

mathematics is first and foremost a part of the curriculum. It has been created and developed through social and historical conditions that are far from mathematics as a scientific discipline. For school mathematics, the assignments are to supply for conceivable needs of an individual, for the society at large and, at the same time, work for the future.

In the current läroplan (Lpo 94) and the kursplan for mathematics, the national educational objectives are explicated. How to achieve the goals is, however, not described. This means that there are no directions and instructions concerning teaching methods.

The kursplan for mathematics describes (Skolverket, 1998):

- The aim of mathematics and its role in education;
- Goals to aim for;
- The structure and nature of mathematics;
- Goals to attain, for students at the end of the fifth year and the ninth year;
- Assessment criteria.

An example of 'goals to attain' for students at the end of the ninth year is that they:

> should have developed their understanding of numbers to cover whole and rational numbers in fraction and decimal form … should be able to reproduce and describe important properties of some common geometrical objects (Skolverket, 2001a, p. 26).

The meaning of the notions 'developed their understanding', 'important properties' or 'common geometrical objects' are not explicated.

In comparison to its predecessors, which contain rules and recommended content, the current läroplan is describing goals and deals with the concepts *knowledge* in a more nuanced way. It has become less important to learn mathematical symbols and how to manipulate them and more important to understand the subject and how mathematics is used in the everyday life. This means that students are supposed to learn, not only mathematics but also about mathematics (Samuelsson, 2003). In the syllabus for mathematics in the compulsory school, this aim is expressed in the following way:

> Mathematics is an important part of our culture and the education should give pupils an insight into the subject's historical development, its importance and role in our society. The subject aims at developing the pupil's interest in mathematics, as well as creating opportunities for communicating in mathematical language and expressions. It should also give pupils the opportunity to discover aesthetic values in mathematical patterns, forms and relationships, as well as experience satisfaction and joy in understanding and solving problems (Skolverket, 2001a, p. 23).

From a curricular perspective, considering that textbooks are a predominant source in mathematics classrooms in Sweden, one could expect that textbooks and their role in the teaching and learning of mathematics should have a key position in the läroplan/kursplan. However, tools for teaching, for example textbooks and other curriculum material are not discussed in the text. In the läroplan, it is stated that it is part of the responsibility of the school principal, the 'rektor', to form the working environment of the school so that the students have access to curriculum material of good quality (Skolverket, 1998).

Mathematics textbooks

Textbooks are "designed to provide an authoritative pedagogic version of an area of knowledge" (Stray, 1994, p. 2). They are special kinds of books, intended to be used in education, holding a unique and significant social function in relation to other texts since they "represent to each generation of students an officially sanctioned, authorized version of human knowledge and culture" (de Castell, Luke, & Luke, 1989, p. vii). The textbook is an artefact, and as such, it is human-made. Thus, there exists an author (or a group of authors) and a producer of the textbook, whom one can assume to have the intention to offer a well-made, carefully prepared pedagogical version of a school topic. Publishing is however an industry in most countries. The forces that drive the design and production of textbooks are therefore both pedagogical and economical.

A view of learning is, in some sense, inherent in each textbook. One could for example recognize the ideas of behaviorism in a book that focuses on getting the right answers on well-defined questions. From a constructivist and socio-cultural perspective, it would be more important to start from the students own experiences and create problems that nurture discussions and cooperation (Selander & Skjelbred, 2004). Textbooks are also colored by certain traditions but also the educational philosophy of the state. In Swedish mathematics textbooks it is, for example, extremely common that exercise sets are graded according to their level of difficulty. This could be seen as a result of the curriculum reform in the middle of the 20th century, which aimed at establishing the comprehensive school, and the emphasis on individualized teaching in the current läroplan. One difficulty that follows from the reforms concerns how to manage a non-homogeneous group of students so that each individual student can work according to his/her prerequisites and needs. If the exercises in the textbook are graded by level of difficulty,[3] it facilitates students' individual work. Hence, for better or for worse, the textbook could be seen as one solution to the problem. To some extent, this explains why much of the activity in many Swedish classrooms consists of 'silent calculation' in the textbook[4].

In Sweden, the textbooks and their use in teaching has been a concern of government authorities for a long time. The school committee from 1946, for example, argued that the education was dependent on the textbook to an unacceptably high degree. They meant that the reliance on textbooks was an obstacle to the development of a democratic school. Whether teaching material should be put under scrutiny to be approved or not, has also been a question for the educational authorities in Sweden. Inspections of textbooks were done on a regular basis from the 1930s' to 1992 (Marklund, 1987). The most recent committee in charge was the *Statens Institut för Läromedelsinformation*, which was a government authority, active in 1974-92. One of the evaluations of mathematics textbooks, *Matematikgranskning* (Areskoug & Grevholm, 1987), described the textbooks as being monotonous, characterless and uninteresting. At present, there is no governmental control or evaluation of textbooks in Sweden.

The absence of guiding principles in the current läroplan (see the previous section) and the absence of control or evaluation with regard to the textbooks could be interpreted as a very passive strategy on behalf of the educational authorities. It is however in accordance with the educational steering system, which is goal-based and offers a high degree of local

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[3] How this is done is examined by Brändström (2005) through a content analysis with a special focus, inhomogeneity of learners (open differentiation).

[4] The issue is especially relevant to Sweden where students and teachers seem to be extraordinarily dependent on textbooks. Textbooks seem to define 'school mathematics' as well as the 'learning path' for the majority of students, at least in lower and upper secondary school (Lindqvist et al., 2003).

responsibility. Thus, to evaluate and judge if a textbook mirrors the principles in the national curriculum, is a local issue[5].

To be a teacher

All parts of the thesis share the same focus, namely the relationship between the textbook and the curriculum. In this case, the curriculum is seen in a broad sense. It involves *the intended* and *the implemented curriculum* (see Article II and M. Johansson, 2003) and *the enacted curriculum*, which includes the relationship between the teacher and the textbook (see Article III and IV). The relationship between the teacher and the curriculum has, however, not been a part of the study. So, in a few paragraphs I will try to describe what I believe are the most crucial features, related to the focus of this study, about what it means to be a teacher in Sweden.

The current läroplan delegates a considerable part of the responsibility for the education to the teachers. They have to find ways to handle each and every student in their class so that they attain the goals that are stated in the läroplan/kursplan. Gustafsson (1999) argues that the responsibility is too big. The way the documents are written is built upon the expectation that the teachers are professionals and can interpret the situations in an adequate way. The teacher should be able to identify the conditions and what kind of possibilities they offer (Gustafsson, 1999). How the conditions for teaching have changed and teachers' professionalism is discussed in a paper of Carlgren (1999). She states that the latest school reforms mainly affect teachers' work *outside* the classroom. The teachers are supposed to perform a new kind of work, which they are less experienced in. Carlgren refers to the design aspect of teachers' work, namely to interpret and develop the meaning of the goals in the läroplan and transform this knowledge into the organization and selection of content.

U. Johansson (2003) states that even though a vaguely written läroplan, like the current one, provokes local discussions regarding goals and content, it increases the workload of the teachers. It takes time to create consensus. The local school plan, which is supposed to be drawn up and shape the pedagogical practice, is in some cases only a copy of the national

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[5] If the textbook really reflects the national curriculum is discussed in a study of three decades and three editions of the same textbook series. Johansson (2003; 2005b) found that the development of the curriculum is only partly reflected in the development of the textbook.

curriculum. Considering the large budget cuts of school funding, U. Johansson argues that "there is not much for teachers and the local authorities to decide on, besides how to save money" (U. Johansson, 2003, p. 582). Still, even though the teachers may feel constrained by the economical conditions, I believe that there is room for maneuver that must be emphasized. A teacher, or a group of teachers, can often decide which textbook to use, how to use it in the classroom, and how to organize the students. These decisions could have substantial impact as regards the content of the lessons (see for example Article III). Furthermore, in a micro-perspective, i.e. in the teacher-student interaction, the teacher could choose to take a particular course of action (see for example Article IV). Both types of decision making have implications for the pedagogical practice.

Documents that describe the objectives of education, such as the Swedish läroplan, have to be interpreted and implemented in the classroom by a teacher. However, since the classroom is a complex and unpredictable environment, the teacher often has to face situations that require prioritizing and fast decision-making (cf. Jackson, 1992). These priorities could be in conflict with the objective of the läroplan but could also be in conflict to their own ideas of teaching mathematics (cf. Bjerneby Häll, 2002, p. 31). Skott (2001) discusses the possible relationship between teachers' explicit priorities, *school mathematics images* (SMI)[6], and classroom interactions in a study of the emerging practices in a novice teacher's classroom. In the *critical incidents of practice*, the teacher's priorities seem to be inappropriate in terms of his 'reformist' intentions. Skott suggests that this should not be seen as a result of teacher inconsistency but rather a result of multiple and sometimes conflicting educational priorities. The priorities related to the teacher's SMIs could sometimes loose some of their practical significance and "be regulated or overshadowed by more general educational priorities such as building students' confidence or by practical concerns such as managing the classroom" (Skott, 2001, p. 21).

The teacher in the study of Skott, had the opportunity to introduce a new textbook with an approach that he regarded as consistent with his own priorities. The teacher declared that "the textbook made the educational decisions and the mathematical priorities of his day-to-day teaching as far as the aims, the contents and the tasks were concerned" (Skott, 2001, p. 10).

[6] The term school mathematics images (SMIs) is used to describe how teachers express their own personal interpretation of and priorities as regards mathematics, mathematics as a school subject, and the teaching and learning of mathematics in schools.

As a novice teacher, he felt like he had neither the time nor the energy to make these decisions.

To be a student

The most important actor in the classroom, the student (the teachers and the textbooks ground their existence on the students), is not in focus in this study. This is a deliberate decision because one of my colleagues in the research group in Luleå, Anna Brändström, shares the same interest in textbooks and their use in classroom. To cooperate and work complementary, but still have well-defined areas, has been possible since my focus is on the teachers while Anna focuses on the students. Nevertheless, just as in most research in mathematics education, the rationale of this work is to improve teaching, which indirectly should be beneficial for the students. Considering the students and school mathematics, there are at least three issues that are relevant in relation to the use of textbooks: a) the joy to learn, b) individualization, and c) students' influence (elevinflytande).

Considerable parts of teaching and learning mathematics seem to involve mechanical calculations page after page in a textbook. "And then it continues … it's so many pages", says Beata. She complains to the teacher, expresses her frustration over the tasks in the textbook, which she thinks are monotonous and boring (see Article III). Beata is a concrete example of a young person that seeks for meaning and 'wants to do funny stuff'. But how can we maintain the *joy to learn* if the students find the tasks uninteresting and meaningless? The joy to learn is an important issue that is discussed in connection to an evaluation of schools in 40 municipalities in Sweden (Lindqvist et al., 2003). "If it is difficult to understand the mathematics in the book, it is probably also difficult to, on your own, maintain the joy to learn" (p. 21, author translation) is one of many comments concerning the use of textbooks in the school in this report. The school inspectors observed that the students often are forced to learn from the textbook by themselves and that many students have difficulties to understand the tasks.

Perhaps one of the most crucial questions, at least from a Swedish perspective, concerns how textbooks are used in the *individualization* (or differentiation) of students. From the point of view that students should be challenged and stimulated throughout their learning of mathematics, Brändström (2005) analyzed tasks in mathematics textbooks in order to reveal their level of difficulty and how they are differentiated. She found that though textbooks tasks are offered on different levels, “the processes and required demands are too low” (p. 75) on all levels.

Students’ right to influence their own education (elevinflytande) is an important issue in Sweden. According to the läroplan, it is the teachers’ responsibility to make sure that the students have a real influence on content as well as teaching methods and ways of working. The students’ possibility to influence, however, is almost nonexistent if a teacher follows the textbook very closely. Englund (1999) argues that “Some obstacles for students’ influence on their education are, obviously, related to the textbook, its function and strong position” (p. 343, author’s translation).

The three issues, which I have raised in this section, concern the most important actor in the schools, the student. It is, however, out of the scope of this study to further examine these areas. However, I want to stress that the issues are extremely important for didactics in mathematics, especially in Sweden.

DIDACTICS OF MATHEMATICS

There is no agreed upon definition of didactics of mathematics (or research in mathematics education as the Anglo-Saxon ‘world’ would call it). One reason is that it is a rather new field. Internationally, one could say that it started to develop as a scientific discipline in the 1960s when many countries standardized their school system (Björkqvist, 2003; Grevholm, 2006). Another reason is that much of the activities within the field are supported by research in other areas, for example psychology and curriculum (cf. Niss, 2001).

The field could be described in the following way:

> Didactics of Mathematics is made up of the scientific activities of describing, analysing and better understanding people’s struggle for and with Mathematics (Strässer, 2005, p. 9).

By saying that this struggle *sometimes* is highly organized, for example in compulsory schools or university, Strässer does not exclude teaching and learning outside the school system.

Didactics as a research field has a dual mission, to be both *descriptive-analytical* and *normative* (Imsen, 1999; Niss, 2001). Niss (2001) compares the duality of the field with the duality in the research field of medicine. Just as medicine has to deal with issues such as reasonable and necessary treatment, health, and sickness, he argues that didactics of mathematics has to deal with questions like: *What is the case?* and *What should it be?* The analytical perspective within didactics is, according to Imsen (1999, p. 98, italics in original), "an important prerequisite to bring about *change and development in schools"*. Bengtsson (1997) claims that didactics, as a research field, should include three domains: *normative didactics*, *descriptive didactics*, and *meta-didactics*. The *normative didactics* concerns development and evaluation. It involves discussions about the educational goals, choice of content and methods, but should also include justifications and recommendations. The *descriptive didactics* should conduct empirical studies of the actual teaching. To distinguish between the normative and the descriptive didactics does not mean that descriptive studies have to be value-free. Bengtsson suggests that the range of these studies, in terms of questions, perspectives, and methods, could be extensive, including classroom studies as well as studies of the running of schools, quantitative as well as qualitative studies. The *meta-didactics*, for Bengtsson, concerns issues within the philosophy of didactics, which forms the basis for the normative and the descriptive didactics. For me, the representation that Niss suggests is more appropriate. He offers a picture in which the meta-activities of the field belong to the 'second floor' of a 'box'. "We can imagine the floor as transparent, which makes it possible to watch the first floor from the second floor (and the opposite)" (Niss, 2001, p. 28). In this perspective, one can see the meta-activity within the field as a reflective practice, something parallel to the activity on the first floor.

From a Swedish perspective, it is important to recognize that didactics, in relation to a specific school subject (ämnesdidaktik), did not develop out of the German tradition of Didaktik. To define didactics of mathematics in Sweden is complicated. There is no 'national tradition', like the German *Stoffdidaktik*, that dominates or has dominated the national scene (Bergsten, 2002). Within the field of Pedagogy[7], from the 1970s and onwards, there are two distinct research traditions. One direction has its roots in the frame factor theory, which was linked to the field of curriculum research. The other direction, the phenomenological approach[8], comes from

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[7] In the middle of the 20th century, the professors of education in Sweden were in general aligned within psychology. At that time, that often implied a behaviorist psychology of learning and psychometrics (Wallin, 1998).

[8] Carlgren and Kallós (1997) state that studies carried out within the tradition of 'phenomenography' were by many teachers and teacher educators perceived as close to their interest, offering new teaching methods.

the work of Ference Marton and his colleagues in Gothenburg in the 1980s (Bengtsson, 1997; Carlgren & Kallós, 1997). Several of these studies are related to the teaching and learning of mathematics.[9] Hence, one could say that didactics of mathematics has been carried out for decades, though in the field of Pedagogy. But as an independent research field, it has been introduced relatively late in comparison to the other Scandinavian countries (Grevholm, 2006).

The frame factor theory with its curriculum perspective and ämnesdidaktik with its phenomenographical perspective could have continued to develop independently. But in the middle of the 1980s, when didaktik was introduced in Swedish teacher training it became important to find an understanding of the meaning of the word[10]. At the same time, there have been increased interests to move towards the German didactical tradition (Gundem, 1992). Carlgren and Kallós give their view on the efforts to define didaktik and the debate that followed:

> We might add that we feel that German Didaktik is deeply rooted in German traditions. […] while at the same time the concept of curriculum is deeply embedded in traditions from USA and UK. In Sweden, however, the debate has been mainly confined to issues in teacher education and to the place and role of the academic discipline of education (or in Swedish *Pedagogik*) […] Thus the debate concerning curriculum theory and various forms of didactics in Sweden to an increasing degree also concerns the identity of Pedagogik as an academic discipline (Carlgren & Kallós, 1997, p. 420).

Strässer (2005) dates the birth of didactics of mathematics in Sweden to the 1990s but says that "the late start was somehow 'compensated' by a stormy development, additionally supported by the creation of a national graduate school in the research domain" (p.21)[11]. This was a period when didactics move into mathematics departments. The first PhD thesis that has a didactical content, at a department of mathematics in Sweden, is presented in

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[9] See the list of Swedish dissertations in the field of didactics of mathematics in (Strässer, 2005, p. 12)

[10] The term 'didaktik' starts to be used as late as in the 1980s. Björkqvist (2003) suggests that the term was introduced in a work of Ference Marton (*Fackdidaktik* from [0]1986). A university course by the name *matematikdidaktik* was organized in Linköping the year before that (Bergsten, 2002).

[11] See also (Leder, Brandell, & Grevholm, 2004)

Luleå by Dunkels (1996) and the first professorship was established at the very start of this century at the same university. So, at present, studies in didactics of mathematics are been carried out from departments of pedagogy as well as departments of mathematics.

What is the field occupied with? In an international survey of researchers' view of the field the last five years, Sfard (2005) found that most researchers in didactics of mathematics are using carefully recorded classroom interactions as their empirical data. The teachers are in focus in most studies, which Sfard comments the following way:

> The first thing I wish to say is that I am pleased to find out that the last few years have been *the era of the teacher* as the almost uncontested focus of researchers' attention. This is quite a change with respect to the last two decades of the 20th century which were almost exclusively *the era of the learner*. And we have certainly come a long way since *the era of the curriculum*, roughly corresponding to the 1960s and 1970s when the main players in the educational game were the developer and the textbook (Sfard, 2005, p. 409).

The comment of Sfard, that we have "come a long way since the era of the curriculum", could be interpreted as a critique to the curriculum field. Nevertheless, I like to explain my opinion on this issue, especially since the subtitle of the thesis suggests that a curricular perspective can be used in didactics of mathematics. First, we have to remember that "the era of curriculum" was before the paradigm shift in the 1980s, when positivistic models and testing of hypotheses was abandoned in favor of research, which was more close to practice, or as Kilpatrick states "research in mathematics education was moving out of the library and laboratory and into the classroom and school[12]" (Bergsten, 2002). Secondly, we have to see that a classroom is a complex and multi-dimensional milieu. In order to understand teaching and learning of mathematics in the classroom, it could be most helpful to study different aspects of the activity as well as the context. Thus, what I suggest is that the concept curriculum, in its broad meaning, could and should be a part of what didactics of mathematics deals with.

[12] (Kilpatrick, 1992, p. 31) quote in Bergsten, 2002.

CURRICULUM: DEFINITIONAL ISSUES

The work presented in this thesis is guided by the traditions in the curriculum field in Sweden. In the following section, I will give a brief, and not all-encompassing, description of that field. I start with a discussion concerning definitional issues.

The traditional definition of the term curriculum, as a course of study, can be tracked back to the 17th century. It origins from the Latin word *curriculum*, which refers to running (Jackson, 1992). How the word came to be used in an educational context is not clear but one theory is that it started as an instrument of social efficiency, "an organizational structure imposed by authorities for the purpose of bringing order to the conduct of schooling" (p. 10). The term curriculum, as meaning simply a course of study, was established and routinely applied to the subjects studied in different levels of schooling by the middle of the 19th century. At present time, the term has a much broader meaning. Nevertheless, for many people, and some dictionaries[13] as well, the term curriculum is still considered in the narrow sense as a course of study (Jackson, 1992).

John Dewey initiated the need of an expanded definition of curriculum. He did not seek to redefine it in a formal sense but he challenges the conventional way to look upon the child and the curriculum (Jackson, 1992).

> Abandon the notion of subject matter as something fixed and ready-made in itself, outside the child's experience; cease thinking of the child's experience as also something hard and fast; see it as something fluent, embryonic, vital; and we realize that the child and the curriculum are simply two limits which define a single process (Dewey, 1990 [1956], p. 189).

In the 1960s, a new perspective emerges: "schools do harm and do it systematically to many, if not all, students" (Jackson, 1992, p. 8). So instead of focusing on the positive incidental learning (cf. Dewey) the development changed towards "a tendency to speak of there being two separate curricula in every school: one explicitly endorsed, the other not" (ibid.). The outcomes of the two are clearly different. The latter has sometimes been re-

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[13] The *Oxford English Dictionary* defines curriculum as "A course; *spec.* a regular course of study or training, as at a school or university" (*Oxford English Dictionary*. (1989). Retrieved 20 Dec, 2005, from www.oed.com).

ferred to as the *hidden* curriculum (e.g. Jackson, 1990 [1968]) and focuses on aspects of schooling that are overlooked or deliberately hidden.

So now we can talk about the *official* or *intended* curriculum, on the one hand, and the *hidden* or *unintended*, on the other. This way to distinguish the aspects of the curriculum focuses on the gap between the educational goals and what the schools actually accomplish. Further distinctions can be made, for the same reason, for example between the *enacted* curriculum, e.g. what appears in the teacher's guide or textbook, and the *delivered* curriculum, i.e. what is taught, and what the students understand. The latter is sometimes referred to as the *experienced* or *received* curriculum.

Curriculum can also be defined as a sequence of learning opportunities. When Schmidt et al. (2001) study the curriculum, as a part of the TIMSS, they need not only make the distinctions between different aspects of the curriculum but also choose artefacts and effects of curriculum that reflect these aspects. The official documents or content standards documents are taken as indicators of the *intended* curriculum and students textbooks are taken as indicators of the *potentially implemented* curriculum. The *implemented* curriculum is represented by content goals and duration of content coverage, stated by the teachers. The TIMSS achievement tests were taken as an indicator of the *attained* curriculum. Valverde et al. (2002) are also using data from TIMSS when they study textbooks. In their model of the curriculum (see Figure 1), the textbooks are regarded as artefacts that translate policy into pedagogy, the link between the intended and the implemented curriculum.

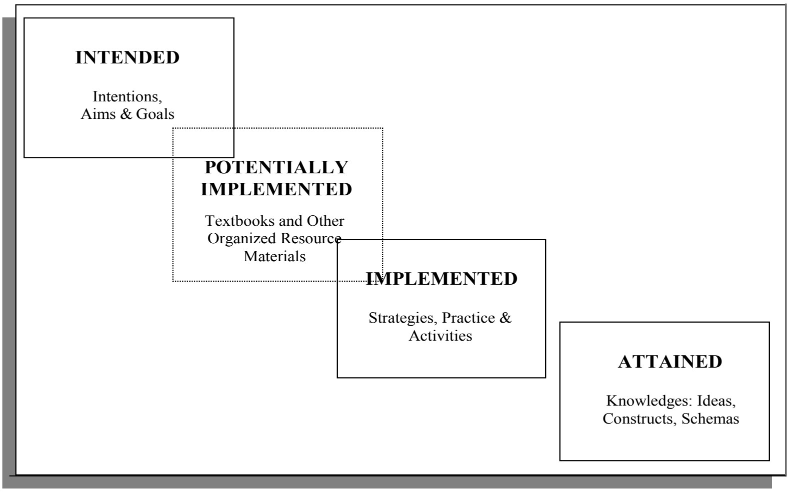

Figure 1: Textbooks and the tripartite model[14]

The hierarchical arrangement of aspects in this curriculum model can however make us believe that the curriculum is designed at one level, handed down, primarily via textbooks to the next level (the teachers), and received by the third (the students). But the curriculum, from my point of view, exists also in the interaction between intentions, practices and achievements. In Sweden, where teachers are involved in the formulation of the national curriculum, one could say that the 'reality' of the classrooms, thus also students' actions and achievements, influence the *intended* curriculum.

I will end the discussion concerning definitional issues with some comments by Jackson (1992). He seeks to make sense of the definitional shifts in the history of curriculum theory and questions whether any of the redefinitions is an improvement over the dictionary definition of the word curriculum. One conclusion he makes is that none of the new definitions has replaced the old; 'curriculum' is still 'a course of study' for many people. Another conclusion is that it has not brought us closer to a *true* definition of the curriculum. "There is no definition of curriculum that will endure for all time and that is foolish to search for one, that every definition serves the interest of the person or group putting it forward" (Jackson, 1992, p. 10). What would be lost if we restricted the use of the word curriculum to its dictionary definition? he asks. Well, a definition has a rhetorical function. All definitions are parts of arguments and are brought forward with the purpose to persuade us of the value of looking at something

[14] Valverde et al., 2002, p.13.

in a particular way. So if we abandon the assumption that there is a single, correct definition of the term, we can focus on what purpose each definition serves: "Why is it being put forward and who stands to gain what by adopting it? What would be the consequences of doing so?" (Jackson, 1992, p. 11).

CURRICULUM RESEARCH IN SWEDEN

The field of curriculum occupies groups of people with different views of what their tasks should be and even different views of the history of the field[15]. The nature and scope of curriculum studies goes from 'doing curriculum work' to the expansive view of the tasks of the field, namely 'understanding curriculum' (Westbury, 2005). To begin with, curriculum research in Sweden was carried out in the scientific tradition (Darling-Hammond & Snyder, 1992). It was in the spirit of positivism. The politicians defined the educational goals and asked the questions, and the researchers provided the answers (U. Johansson, 2003). Even if the research was political independent it was nevertheless related to political decision making (Lindblad, Linde, & Naeslund, 1999).

After the main reforms in the 1960s, the educational system is only modified during the two following decades[16]. The relationship between the educational researchers and the decision-makers changed during this period. From being one that carried out a specific study to answer a specific question the researcher became more of a consultant (Lindblad et al., 1999). The change also implied that the earlier dominance of logical empiricism ceased and that education psychology as well as US influences became less important (Carlgren & Kallós, 1997).

The work of Urban Dahllöf and his colleagues in the 1960s are recognized as the first curriculum studies in Sweden. Dahllöf formulated a theory, the frame factor theory, which initiated a shift of purpose from serving decision-makers to analyzing the consequences of decision making (Lundgren, 1998). The frame factor theory, in its early stage, focuses on how political decisions regarding teaching and education (e.g. time schedules, grouping, etc.) influence the pedagogical work. Using quantifiable

[15] See for example the discussion concerning the field of curriculum in the USA in the Journal of Curriculum Studies (Reynolds, 2003; Urban, 2003; Wraga & Hlebowitsh, 2003a, 2003b).
[16] There have been revisions of the läroplan and the kursplan for the compulsory school presented in 1969 (Lgr69) and 1980 (Lgr80).

factors of concrete nature it stresses the constraints caused by the organization of schooling. Results from these studies, for instance the theory about the steering-group[17], was especially relevant in debates of that period (Broady, 1999; Dahllöf, 1999; T. Englund, 1995, 1997; U. Johansson, 2003; Lindblad et al., 1999).

The frame factor theory opened up a new line of thinking about curriculum research. It had a great influence in the 70's in Sweden; it also found its way abroad, for instance into Germany and USA. But there were also some critics, partly because it disregarded the students and the teachers. It also reduced the responsibility of the teacher who indirectly was regarded as a 'marionette' and not a reflective practitioner (Broady, 1999; Lindblad et al., 1999). Hence, at this stage of the frame factor theory, a teacher activity such as 'scaffolding' could be regarded as a pattern of behavior that depends on external factors. Further criticism concerned the failure of the theory to consider the variations within given frames (Lindblad et al., 1999).

In the late 1970s and twenty years onwards, the influence of the educational researchers in the development of the comprehensive school was reduced and curriculum research declined (Carlgren & Kallós, 1997). The role of the researchers has changed in two steps. First when the organizational problems were solved in the 70s and then when the schools were changed from being rule-regulated to goal-regulated (U. Johansson, 2003). The characteristics of the frame factor theory changed as well. Two further dimensions were introduced when Ulf P. Lundgren defined the goal system and the rule system as frames. In what Lindblad et al. (1999) denote as the second phase of the development of the frame factor theory, new types of questions were asked. For example: Why are these frames being formed? Why these relationships between frames and processes and results?

In the 1990s, the frame factor theory was still in use in curriculum research in Sweden and some researchers started to include 'teachers thinking' as a 'frame factor' (cf. Gustafsson, 1999). At the third level in the development of the frame factor theory, questions of the choice of educational content and the contextualization of teaching are emphasized. A fundamental assumption is that the students will be offered different possibili-

[17] Dahllöf's hypothesis of the steering group concerns how the teaching pace is adjusted for the pupils between the 10 and 25 percentile.

ties to construct meanings depending on choices of, for example, content, textbook, and teacher. (T. Englund, 1995, 1997).

Combining the broad definition of the concept curriculum with the complex milieu of a classroom, it is not surprising that the curriculum field becomes distorted. In the study presented in this thesis I will even argue that curriculum not only can be seen as a research field in itself but also as an analytical perspective within other fields, in this case research in mathematic educations. The perspective is based on what Englund (1997) describes as the third stage of the frame factor theory. It emphasizes the choice of educational content and contextualization of teaching. The fundamental assumption is that different choices can be made, more or less consciously, which have crucial implications for teaching and learning. The student is offered different possibilities to create and construct meaning depending on, for example, what content is chosen and what context the textbook offers.

METHODOLOGICAL ISSUES, ETHICS, AND SCIENTIFIC QUALITY

Open criticism, which is guided by clearly articulated principles, is a natural and important aspect of research. Lester and Lambdin (1998) offer seven evaluation criteria to use for the quality control of research in mathematics education: worthwhileness, coherence, competence, openness, ethics, credibility, and a general principle that they call "other qualities of good research reports" (p. 422). Throughout this work, these issues have been taken into consideration.

Worthwhileness is the most important criterion for a good quality, according to Lester and Lambdin. But what does it mean that a study accomplishes the criterion of worthwhileness? Lester and Lambdin say that the study "must resonate with the issues and questions that are regarded as interesting and important to mathematics educators at a given point in time" (p. 420). They offer four key indicators of worthwhileness (ibid.):

(1) The study generates good research questions.

(2) The study contributes to the development of rich theories of mathematics teaching and learning.

(3) The study is clearly situated in the existing body of research on the question under investigation.

(4) The study informs or improves mathematics education practice.

Other researchers offer comparable principles. Sierpinska (1993) and Kilpatrick (1993) suggest *relevance* and *relatedness* as two of the eight criteria for the evaluation of quality of research in mathematics education. The term *relevance* could be subdivided in *pragmatic/theoretical* and *cognitive* relevance. "Something is *pragmatically relevant* in the domain of mathematics education if it has some positive impact on the practice of teaching; it is *cognitively relevant* if it broadens and deepens our understanding of the teaching and learning phenomena" (Sierpinska, 1993, p. 38, italics in original). Just as Lester and Lambdin, Sierpinska stresses that research in mathematics education should have the practice of teaching in view, i.e. accomplish the criterion of *relatedness*.

Worthwhileness has been a fundamental principle for all parts of the study. The worthwhileness, i.e. the importance and the implication of this work, will be discussed in the concluding discussion in this thesis.

Coherence encompasses validity and validity concerns the agreement between the research questions and the methodology used for answering the questions, i.e. *Does the method make it possible to answer the questions?* Coherence is also about consistency and logical connections. This means that it is important to carefully see that all parts of the work form a unity and that there are no contradictions. As regards data collection, in the classical sense, I am using data material that is gathered by the research team of the CULT-project in the empirical study of the classrooms (Article III and IV). As an 'outsider', considering that I took no part in the design or the practical work, I was very pleased that my research questions fitted in so well within the frames of the project. The obvious reason, as I see it, is that the richness and the high quality of the data open up excellent opportunities for decent and important research using many different perspectives. Even so, instead of asking the question if the method answers my questions, I had to think about *What are the questions (within my area of interest) that can be answered when using this material?* This means that different questions could be asked if I had chosen to make my own data collection, but it does not mean that the focus would be different.

Competence concerns precision and carefulness. Within the research team of the CULT-project there is a long-standing experience of educational research and several years of practice using technical equipment in relation to classroom studies[18]. As regards the methodological approach of the coding procedure, which clearly is my responsibility[19], the coding schema was developed in several steps, tested in a number of lessons, and discussed in seminars consisting of doctoral students and researchers in mathematics education before it was used in this study. Furthermore, as a reliability test of the coding procedure, one of the thirteen lessons was re-coded one year after it was coded for the first time. The reliability score, which is calculated by dividing the number of agreements by the number of agreements plus disagreement, is 95.8 % (see Article III).

Openness means that personal biases and assumptions should be made public, and that the research methods and techniques should be reported in such a way that the research community can scrutinize them. Kilpatrick refers to the criterion *Reproducibility* and states that:

> What we do in research and what we find must not remain private. It must be put on display so that it can be criticized, tested by others against their experience, and possibly refuted. It must be public (Kilpatrick, 1993, p. 29).

In all parts of this study, the research methods and techniques as well as the findings are carefully described. The coding procedure is, for example, described in detail in the paper *Textbooks as instruments: three teachers' way to organize their mathematics lessons* (Article III).

Ethics concerns confidentiality and accuracy in relation to the research subject but also that credits should be given to all types of contributors, the persons that have been involved in the project and those whose research has influenced the project. The scientific leader and the research team of the CULT-project have been acknowledged and ethical principles of the CULT-project have been followed during the work with the data material for this study. Whenever other person's research has influenced the study, references are made according to general principles for scientific texts.

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[18] For a comprehensive description of the methodological and technological design in the CULT-study, see Häggblom (2005). Information about the study can also be found on: http://www.ped.uu.se/kult/default.asp. The fieldwork and the data collection in the CULT-project is based on the research design set out for the Learner's Perspective Study (http://extranet.edfac.unimelb.edu.au/DSME/lps/).

[19] The coding manual of the TIMSS Video Study (Jacobs et al., 2003) is used a support for the design of the coding procedure.

Credibility is about to ground and justify the findings and the conclusion in the data. In order to make it possible to verify or refute the conclusions drawn, the arguments and interpretations are explicitly presented in the texts. Tables and diagrams are sometimes, when I found it appropriate, included to support these.

Other qualities of good research reports concerns valuable but intangible features like lucidity and clarity and that the research report should be well structured. As a reader of a research report, you probably prefer *conciseness* over *verbosity* and *directness* over *obscurity*. It has been my ambition to assist the reader throughout this text. It is of course up to the reader to judge how well I have accomplished. Within the criterion of 'other qualities' one can also place the feature *originality* (Kilpatrick, 1993), which has been a catchword for my work.

> Originality does not mean a lack of connection to previous research. It refers to the way in which evidence is marshaled and portrayed so as to cause the reader to think again. We are surprised when we read an original study. We did not expect the story to be told this way or to turn out that way. We have a new insight even if (and perhaps especially if) the situation is one we know well (Kilpatrick, 1993, p. 25).

That textbooks guide teaching of mathematics in Sweden, to a high degree, is a well-known phenomenon that has been confirmed by 'anecdotic' evidence. For this study, it has been my intention to look into the issue in a new way, to see the textbook and the use of the textbook in a multi-dimensional perspective. Furthermore, using curriculum theory, or rather frame factor theory, in order to reveal and discuss the textbooks and the use of textbooks as a frame of teaching and learning in mathematics classrooms is one way to use an 'old' approach in a new way.

SUMMARY OF THE PAPERS AND THEIR MAIN RESULTS

This thesis consists of four articles and a preamble that introduces the work and links the articles together. The overarching issue that guides this work concerns the textbooks and their use in mathematics teaching in Sweden.

The mathematics textbook: from artefact to instrument, entails a study of previous research in the area, in Sweden and internationally. The mathematics textbook is discussed from two points of view, as an object and as a tool for teaching. The first part involves content analysis of textbooks from different parts of the world and serves as a background for a general discussion about the textbook as an artefact. The second part concerns the actual use of the textbooks in the mathematics classroom. The aim of this paper, which can serve as an introduction and a background to the empirical study of the classrooms, is to highlight and problemize some important features and different conceptions of the textbooks. The discussion concerns, in particular, the authority and the authorization of textbooks. As a predominant source in many mathematics classrooms, textbooks have a unique status. Therefore, in order to understand the processes of teaching and learning mathematics, it is essential to increase the awareness of textbooks and how they are used.

From a curricular perspective, it is important to consider possible discrepancy between pedagogical objectives in the curriculum and the textbook, especially if the practitioners expect that the textbooks follow the guidelines. The role of the textbooks, as links between the national guidelines and the teaching of mathematics in schools, is discussed in: *Mathematics textbooks: the link between the intended and the implemented curriculum?* The paper is based on a study that is presented in a licentiate thesis (M. Johansson, 2003). In order to illustrate the role of textbooks as the potentially implemented curriculum, a content analysis of a textbook series was conducted. The development of the textbook series, a commonly used schoolbook in Sweden, is portrayed in the light of the curriculum development. The analysis shows that the there are very few instances in the textbooks where mathematics as a scientific discipline is discussed. So, if the teachers work very close to the textbooks, students might have less experiences concerning the role of mathematics in our society and the historical development of mathematics than the objectives in the national curriculum recommend. Moreover, the analysis of the textbooks indicates that the new edition (from 2001) is rather similar to the old editions (from 1979 and 1985). Special units with for instance problem solving and thematic work are added to the new edition so the number of pages is higher, but the number of exercises is, if we exclude these units, almost the same.

The empirical study of the use of textbooks in classrooms, consist of two parts, one is mainly quantitative and the other is qualitative. The article *Textbooks as instruments: three teachers' way to organize their mathematics lessons* reports the quantitative part of the study. The study shows that the textbook influences, not only what kind of tasks the students are working with during the lessons but also the examples the teacher presents on the board, what kind of concepts of mathematics are introduced and how they are introduced. The organization of the lessons is also discussed in this paper. In considerable parts of the lessons, students are working on an individual basis solving tasks in the textbook. Meanwhile, the teacher walks around, interacts with the students and gives individual assistance and support. The main results could be summarized as follows:

(a) Students are exclusively working with tasks in the textbook during the private work part of the lesson, which on average is more than half the time of a lesson.

(b) In the public part of the lesson, the examples and the tasks that the teachers present are mainly from the textbook. An exception is the teacher Mr. Larsson who uses his experiences as a Physics teacher in some of the examples on the board.

(c) The way that mathematics, as a scientific discipline, is presented is comparable with the approach in the textbook. A hundred of totally 119 occasions of *Mathematical generalizations or statements* are coded as comparable or the same as in the textbook. In principal, this means that hardly any other definitions, conventions, or rules than the textbook offers are presented to the students. It also means that the mathematical procedures, for example how to solve an equation, and how the structural features of mathematics are portrayed, are mainly the same as in the textbook.

(d) Two of the teachers, Mr. Andersson and Mr. Svensson, use their textbooks as the main sources for background and motivational discussions.

(e) Homework is not assigned on a regular basis. However, when the teachers do give assignments, students are supposed to work with tasks from the textbooks.

In *Mathematical meaning making and textbook tasks*, the interaction between the teacher and the students in some critical incidents in the classroom is analyzed and reported. The analysis reveals that the tasks and how they are constructed have an effect on the teacher-student interaction. In the 'standard' pattern of interaction, the teacher interacts with the students in a confident way, helping the students to solve tasks in the textbook. The situation changes however when there is a discrepancy between the answer in the textbook and what the teacher thinks is a correct solution. The teacher becomes ambiguous but, in this incident, he does not argue against the textbook. In another incident, the teacher shows that he can go outside the frame and deviate from the textbook. From a question that is raised by a student working on a specific task, the teacher establishes a general and public discussion. One conclusion from this study is that a textbook task, in an interrelationship with a teacher, can cause ambiguity as well as generate mathematical discussions.

The intention of making four different studies was to be able to examine the textbooks and their use in the classroom from different perspectives. If we combine the results and try to give a more or less comprehensive picture, we find that the textbook is a most influential factor. Hence, *the use of textbooks* is a very important 'frame' in the teaching of mathematics in Sweden. To be more precise: It is not the textbooks that are the frames but rather the use of them and how they are regarded from the perspective of a teacher or the educational authority. The word 'frame' does not, in this respect, mean something that hinders or prevents a certain activity. The word does not imply normative values; it should not be seen as something that is always negative. Nevertheless, from what I have learned, textbooks seem to rule the teaching of mathematics in many aspects. Or, to be provocative, I could say (and repeat) that mathematics in many classrooms in Sweden is simply what is written in the textbooks.

What I have noticed is that teachers, in many aspects, act as if the textbooks are superior. Textbooks influence not only *what* kind of tasks students are working with and the examples presented by the teachers but also how mathematics is portrayed in terms of the concepts and the features that are related to the subject. Thus, on the one hand we have the teachers who are using the textbooks as the guideline for teaching and on the other hand we have the educational authority, which expects the teaching to be based on what is written in the curriculum. The textbooks, however, are not playing the role as interpretation tools of the intended curriculum. There are many other aspects involved in the textbook production. I could even show that in some cases there are discrepancies between the national guidelines and the content of the textbooks. Textbooks do not guarantee that the läroplan and the kursplan are followed.

The 'frame', i.e. the use of textbooks, is not a static condition. Teachers are not forced to use the textbooks in a certain way. They do not even have to follow the guidelines from the authors. Hence, even if the textbook dominates the teaching, it does not decide all the details of a lesson. In the quantitative part of this study, where the three teachers' way to organize their teaching is explored, it was noticeable that two teachers can use the same book in a different way. For example, the difference could be a result of how they organized the lessons in terms of individual work and public discussions. While one of the teachers organizes the lesson by making a clear distinction between public and individual work, another teacher chooses to alter between individual and public work several times in a lesson – and these different ways to organize a lesson both rely on the same textbook.

From the qualitative part of the study, one could see that the teacher can get into difficulties *because* of too much reliance on the textbook. However, one could also recognize that there is room for maneuver and that the teacher sometimes uses this space and deviates from the book. It could for example happen when the teacher becomes aware of some mathematical aspects, which the textbook does not cover. It could also be the case that the teacher uses other resources than the textbook, for example an information sheet from the municipality, or his knowledge in Physics when he presents an example on the board.

Finally, we have to consider that textbooks rule the private individual learning - and private work is a common feature of mathematics classrooms in Sweden. It is time to raise the issue of the correlation of individualization and textbook influence. Even if the individualized teaching is out of the focus of this study, it is obvious that the role of the textbook for individual learning is a crucial question needing fundamental and detailed research.

CONCLUDING DISCUSSION

DISCUSSION OF RESULTS

In this part of a thesis, the author should consider the objectives and the results of the study and try to find out if s/he has accomplished the goals. For a research study in didactics of mathematics, it is also appropriate to discuss implications for teaching. First, in order to sum up, I would like to repeat the objectives. They were:

1) To further incorporate content and context issues in research in mathematics education and contribute to the development of the field.
2) To deepening the understanding of what a textbook is and what kind of potentials and constraints it entails.

3) To examine the influence of textbooks in some Swedish classrooms in order to:
    - contribute to the picture of the enacted curriculum.
    - contribute to the discussion concerning teachers dependence on textbooks.

The first objective concerns whether this study is noteworthy in the research field of mathematics education or not. One could for example question if the study contributes to new knowledge; it is an already well-known 'fact' that many teachers use textbooks in their teaching of mathematics. However, by showing that and also how the textbook influences the teaching of mathematics and at the same time deepening the understanding for this phenomenon, I contribute to further understanding on this issue. Furthermore, using a curricular perspective in didactics of mathematics should be seen as an attempt to bridge the gap between two Swedish traditions and a way to incorporate content and context matter.

I hope I have accomplished the second objective, to deepening the understanding of what a textbook is and what kind of potentials and constraints it entails. The intentions have been to show that textbooks:

a) are artefacts that preserve and transmit knowledge in the educational systems,
b) facilitate the daily work of the teachers,
c) can be seen as some kind of guarantee that the students have the necessary basic knowledge and training for the next level in the school system,
d) can be regarded as tools to accomplish uniformity and consistency within the school system, for example with respect to a reform,
e) are tools with constraints and weaknesses,
f) seem to reduce both freedom and responsibility of the teachers,

Furthermore, I would like to highlight the issue of authorization of a textbook. As regards the content, who is responsible: the author; the publisher; the educational authority; the mathematical society; the teachers; or maybe the society at large? The textbooks are most likely influenced by several aspects of the educational culture. The authorization of a textbook can be discussed from at least four perspectives. We can start from the governmental level and ask; how come that there is no approval system of textbooks in Sweden? A simple answer would be that it is not necessary since the teachers are responsible for the teaching of the students, thus also the contents of the lessons. As professionals and autonomous individuals, they can decide how to manage this. In the current, goal-driven, educational system, it would even be inappropriate to govern or limit the choice of textbooks.

From the perspective of the authors and the publishers, the authorization of textbooks is closely connected to economical interests. Textbook publishers are not obliged to follow the läroplan/kursplan in the current system. Thus, the main guiding principle is probably worthwhileness, for the editors defined by economical success. The expectations of how well a certain textbook or curriculum material is received by the buyer decide if it is worthwhile to produce. If the authors have ideas about a new and untraditional kind of material, they would probably need to establish a solid market before the ideas could be realized. So, in some sense, one could say that the development of textbooks follows the demands and the expectations of the teachers. Consequently, if the teachers choose the same type of textbooks, the selection and diversity of textbooks on the market will be narrow.

It seems like the responsibility always fall on the teachers. However, from the perspective of the teachers, one has to realize that the textbook facilitates the daily work. The inbuilt property of a textbook is that it offers a reduction of the working load. For the teachers, it could be a waste of time to invent and construct all the tasks that the students are supposed to work with. For individualized teaching, the textbook is especially helpful if the tasks are graded by level of difficulty. The reasons behind a textbook-guided teaching are not only practical. In Sweden, we have a deep-rooted tradition of using textbooks in school mathematics. Many students, parents, and even colleagues expect that a teacher would use a textbook in order to ensure that the students are offered all parts of mathematics that are necessary for the next level in the school system. For a teacher, who is not so confident in his/her mathematics, the textbook is a special support.

The last perspective, for the moment, concerns teacher education and in-service training. Since the responsibility in the current educational system lies on the teachers, it is most important to prepare the teacher students so that they can make wise decisions as regards what textbook to use and how to use it. Teachers need to know what kind of help the textbooks can offer and be aware of how they use the book. Hence, one should also, in for example in-service training, highlight this issue.

The last objective for the study presented in this thesis, number 3 in the list, is accomplished through the empirical study of three teachers' classrooms and the analysis of the data. The results show that the textbooks have a crucial impact on the teaching of mathematics. It is my hope that this study contributes to fruitful discussions about the role of the textbooks. If one considers the use of textbooks as a frame for teaching, it is easy to realize that it is the teachers who decide how strong the frame could be. In the qualitative part of the study, it is evident that the teacher can deviate from the textbook. The teachers need to evaluate the potential and limitations of the textbook and use the textbooks as a support in such a way that it corresponds to their pedagogical intentions. Teachers should not be slaves to the textbook but be its intelligent master, who profits from the potential of the book, but avoids its pitfalls.

SUGGESTIONS FOR FURTHER WORK

The interest in textbooks and their use in mathematics education is a growing field that catches the attention of more and more researchers. Workshops have been organized in the latest ICME congress as well as there will be one in the next PME-conference. The Nordic Graduate School for Mathematics Education (NoGSME) is running a workshop on this topic in May 2006. For a future research agenda, I have some suggestions of issues that could be worthwhile, at least from a Swedish perspective, to further examine.

I recommend that the content issue needs more attention. There are at least two studies that suggest that the Swedish textbooks do not fulfill the requirements of the läroplan/kursplan. A study of the textbooks as the potential implemented curriculum, reveal discrepancies between the educational aims and the content of a textbook series (M. Johansson, 2003). The study of Brändström (2005), focusing on textbooks task and their levels of difficulty, shows that the processes and required demands are too low. It would be interesting to further examine the content with the purpose to define and characterize quality aspects of the textbooks. One could for instance look into the way textbooks introduce a certain mathematical topic, how the topics are organized and how they are connected in the books. Other interesting areas concern teachers' guides and computer software. Several textbook series have already associated material to the books: How is this additional material connected to the textbook and to the teaching? Does the material offer additional, different ways to teach a certain topic?

It could be worthwhile to learn more about how teachers conceive textbooks and how they judge on their own use of textbooks. Another aspect of teachers' use of textbooks would be how they, themselves, see their margin of maneuver, the affordances and constraints, when using the textbooks. Do teachers feel that the textbook controls or restricts their teaching? Can the textbook serve as food for thoughts and new ideas?

Furthermore, it is also important to consider the students. What is there opinion on the textbooks and the tasks they offer? What are the consequences, for the students, if textbooks are guiding the teaching? It is also most important, especially in Sweden, to consider the individualized teaching and differentiation that is linked to the textbook.

For textbook studies, the methodological approach could be interviews and questionnaires. However, studying the actual use of textbooks in classrooms through observations is also needed. Another approach would be trying to know more about the role textbooks play in the everyday practice of teachers and students, for instance by asking them to write down a logbook.

What is also needed is a common framework that systematically examines the crucial aspects of the use of textbooks. Rezat (2006) introduces an approach, based on activity theory. Instead of using the didactical triangle, which includes the teacher, student and topic of study, Rezat suggests a tetrahedron-model. In this model, a fourth component is added, the textbook as a mediator. The artefact is put in the center of the activity system and the dichotomous role of the teacher, as a user and a mediator of the artefact is considered. At this stage, empirical work according to the framework is required to further develop this idea.

Finally, I would like to look into what teachers, who claim that they teach without a textbook, use as inspiration sources and what kind of tasks they offer to the students.

REFERENCES

Areskoug, M., & Grevholm, B. (1987). *Matematikgranskning* (No. 1987:3). Stockholm: Statens Institut för Läromedel.

Bengtsson, J. (1997). Didaktiska dimensioner: Möjligheter och gränser för en integrerad didaktik. *Pedagogisk forskning i Sverige, 2*(4), 241-261.

Bergsten, C. (2002). Faces of Swedish research in mathematics education. In C. Bergsten, G. Dahland & B. Grevholm (Eds.), *Research and action in the mathematics classroom. Proceedings of the 2nd Swedish Mathematics Education Research Seminar* (pp. 21-36). Linköping: SMDF.

Bjerneby Häll, M. (2002). *Varför undervisning i matematik? Argument för matematik i grundskolan - i läroplaner, läroplansdebatt och hos blivande lärare.* Linköpings universitet, Linköping.

Björkqvist, O. (2003). *Matematikdidaktiken i Sverige - En lägesbeskrivning av forskningen och utvecklingsarbetet*. Stockholm: Kungliga Vetenskapsakademien.

Broady, D. (1999). Det svenska hos ramfaktorteorin. *Pedagogisk forskning i Sverige, 4*(1), 111-121.

Brändström, A. (2005). *Differentiated tasks in mathematics textbooks: an analysis of the levels of difficulty.* Luleå university of technology, Luleå.

Carlgren, I., & Kallós, D. (1997). Lessons from a comprehensive school system for curriculum theory and research: Sweden revisited after twenty years. *Journal of Curriculum Studies, 29*(4), 407-430.

Dahllöf, U. (1999). Det tidiga ramfaktorteoretiska tänkandet: en tillbakablick. *Pedagogisk forskning i Sverige, 4*(1), 5-29.

Darling-Hammond, L., & Snyder, J. (1992). Curriculum studies and the traditions of inquiry: the scientific tradition. In P. W. Jackson (Ed.), *Handbook of research on curriculum* (pp. 41-78). New York: Macmillan Publishing Company.

de Castell, S., Luke, A., & Luke, C. (Eds.). (1989). *Language, authority and criticism*. London: The Falmer Press.

Dewey, J. (1990 [1956]). *The school and the society: and, The child and the curriculum*. Chicago: The University of Chicago Press.

Dunkels, A. (1996). *Contribution to mathematical knowledge and its acquisition.* Luleå University of Technology, Luleå.

Englund, B. (1999). Lärobokskunskap, styrning och elevinflytande. *Pedagogisk forskning i Sverige, 4*(4), 327-348.

Englund, T. (1995). Läroplansteori och didaktik. *Didactica Minima, 33*, 36-44.

Englund, T. (1997). Towards a dynamic analysis of the content of schooling: narrow and broad didactics in Sweden. *Journal of Curriculum Studies, 29*(3), 267-287.

Grevholm, B. (2006). Matematikdidaktikens möjligheter i en forskningsbaserad lärarutbildning. In S. Ongstad (Ed.), *Fag og didaktikk i lærerutdanning*. Oslo: Universitetsforlaget.

Gundem, B. G. (1992). Notes on the development of Nordic didactics. *Journal of Curriculum Studies, 24*(1), 61-70.

Gustafsson, C. (1999). Ramfaktorer och pedagogiskt utvecklingsarbete. *Pedagogisk forskning i Sverige, 4*(1), 43-57.

Häggblom, J. (2005). *From analogue tapes to digital drives: On the development of methods and techniques for classroom interaction research.* (D-uppsats). Uppsala University, Uppsala.

Imsen, G. (1999). Reflection as a bridging concept between normative and descriptive approaches to didactics. *TNTEE Publications, 2*(1), 95-105.

Jackson, P. W. (1990 [1968]). *Life in classrooms*. New York: Teachers College Press.

Jackson, P. W. (1992). Conceptions of curriculum and curriculum specialists. In P. W. Jackson (Ed.), *Handbook of research on curriculum* (pp. 3-40). New York: Macmillan Pub. Co.

Jacobs, J., Garnier, H., Gallimore, R., Hollingsworth, H., Givvin, K. B., Rust, K., et al. (2003). *TIMSS 1999 Video Study. Technical report: Mathematics*. Washington, D.C: National Center for Education Statistics.

Jahnke, H. N. (1986). Origins of school mathematics in early nineteenth-century Germany. *Journal of Curriculum Studies, 18*(1), 85-94.

Johansson, M. (2003). *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum* (Licentiate thesis). Luleå: Department of Mathematics, Luleå University of Technology.

Johansson, M. (2005a). The mathematics textbook: from artefact to instrument. *Nordic Studies in Mathematics Education, NOMAD, 10*(3-4), 43-64.

Johansson, M. (2005b). *Mathematics textbooks - the link between the intended and the implemented curriculum?* Paper presented at the Eight International Conference of the Mathematics Education into the 21st Century Project: Reform, Revolution and Paradigm Shifts in Mathematics Education, Johor Bahru, Malaysia.

Johansson, M. (submitted-a). Mathematical meaning making and textbook tasks.

Johansson, M. (submitted-b). Textbooks as instruments: three teachers' way to organize their mathematics lessons.

Johansson, U. (2003). Frame factors, structures, and meaning making: shifting foci of curriculum research in Sweden. In W. F. Pinar (Ed.), *International handbook of curriculum research* (pp. 575-593). Mahwah, N.J.: L, Erlbaum Associates.

Kilpatrick, J. (1992). A history of research in mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 3-38). New York: Macmillan.

Kilpatrick, J. (1993). Beyond face value: assessing research in mathematics education. In G. Nissen & M. Blomhoej (Eds.), *Criteria for scientific quality and relevance in the didactics of mathematics*. Roskilde: Roskilde University, IMFUFA.

Leder, G., Brandell, G., & Grevholm, B. (2004). The Swedish Graduate School in mathematics education: Conception, birth and development of a new programme. *Nordic Studies in Mathematics Education, NOMAD, 9*(2), 165-182.

Lester, F. K., & Lambdin, D. V. (1998). The ship of Theseus and other metaphors for thinking about what we value in mathematics education research. In J. Kilpatrick & A. Sierpinska (Eds.), *Mathematics education as a research domain: A search for identity* (Vol. 2, pp. 415-425). Dordrecht: Kluwer.

Lindblad, S., Linde, G., & Naeslund, L. (1999). Ramfaktorteori och praktiskt förnuft. *Pedagogisk forskning i Sverige, 4*(1), 93-109.

Lindqvist, U., Emanuelsson, L., Lindström, J.-O., & Rönnberg, I. (2003). *Lusten att lära - med fokus på matematik* (No. 221). Stockholm: Statens skolverk.

Lundgren, U. P. (1998). The making of curriculum making: reflection on educational research and the use of educational research. In B. G. Gundem & S. Hopmann (Eds.), *Didaktik and/or curriculum: an international dialogue* (Vol. 41, pp. 149-162). New York: Peter Lang.

Marklund, S. (1987). *Skolsverige 1950-1975. Del 5: Läroplaner*. Stockholm: Utbildningsförlaget.

Niss, M. (2001). Den matematikdidaktiska forskningens karaktär och status. In B. Grevholm (Ed.), *Matematikdidaktik - ett nordiskt perspektiv* (pp. 21-47). Lund: Studentlitteratur.

Reynolds, W. M. (2003). Rejoinder: Debate, nostalgia, and ressentiment. *Journal of Curriculum Studies, 35*(4), 445-451.

Rezat, S. (2006). *A model of textbook use.* Paper presented at the PME 30, Mathematics in the centre, Prague, Czech Republic.

Samuelsson, J. (2003). *Nytt, på nytt sätt? En studie över datorn som förändringsagent av matematikundervisningens villkor, metoder och resultat i skolår 7-9.* Uppsala universitet, Uppsala.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H., Wiley, D. E., Cogan, L. S., et al. (2001). *Why schools matter : a cross-national comparison of curriculum and learning*. San Francisco: Jossey-Bass.

Selander, S., & Skjelbred, D. (2004). *Pedagogiske tekster: för kommunikasjon og læring*. Oslo: Universitetsforlaget.

Sfard, A. (2005). What could be more practical than good research? On mutual relations between research and practice of mathematics education. *Educational Studies in Mathematics, 58*, 393-413.

Sierpinska, A. (1993). Criteria for scientific quality and relevance in the didactics of mathematics. In G. Nissen & M. Blomhoej (Eds.), *Criteria for scientific quality and relevance in the didactics of mathematics*. Roskilde: Roskilde University, IMFUFA.

Sivesind, K., Bachmann, K., & Afsar, A. (2003). *Nordiska læreplaner*. Oslo: Læringssenteret.

Skolverket. (1998). *Lpo 94. Läroplan för det obligatoriska skolväsendet, förskoleklassen och fritidshemmet*. Stockholm: Fritzes.

Skolverket. (2001a). *Syllabuses for the compulsory school*. Stockholm: Fritzes.

Skolverket. (2001b). *Vad är styrdokumenten? De nationella styrdokumenten*. Retrieved September, 2003, from http://www.skolverket.se/styr/vad.shtml

Skott, J. (2001). The emerging practices of a novice teacher: the roles of his school mathematics images. *Journal of Mathematics Teacher Education, 4*, 3-28.

Stray, C. (1994). Paradigms regained: towards a historical sociology of the textbook. *Journal of Curriculum Studies, 26*(1), 1-29.

Strässer, R. (2005). *An overview of research on teaching and learning mathematics*. Stockholm: Vetenskapsrådet.

Svingby, G. (1978). *Läroplaner som styrmedel för svensk obligatorisk skola. Teoretisk analys och ett empiriskt bidrag.* Acta Universitatis Gothoburgensis, Göteborg.

Urban, W. J. (2003). Commentary: What makes a renaissance? *Journal of Curriculum Studies, 35*(4), 439-444.

Wallin, E. (1998). Changing paradigms of curriculum and/or didaktik? In B. G. Gundem & S. Hopmann (Eds.), *Didaktik and/or curriculum: an international dialogue* (Vol. 41, pp. 127-146). New York: Peter Lang.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the Book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht: Kluwer Academic Publishers.

Westbury, I. (2005). Reconsidering Schwab's "Practicals": a response to Peter Hlebowitsh's "Generational ideas in curriculum: a historical triangulation". *Curriculum Inquiry, 35*(1), 89-100.

Wraga, W. G., & Hlebowitsh, P. S. (2003a). Commentary: Conversation, collaboration, and community in the US curriculum field. *Journal of Curriculum Studies, 35*(4), 453-457.

Wraga, W. G., & Hlebowitsh, P. S. (2003b). Toward a renaissance in curriculum theory and development in the USA. *Journal of Curriculum Studies, 35*(4), 425-437.

APPENDIX

Paper I

The mathematics textbook

From artefact to instrument

Monica Johansson

This paper describes the mathematics textbook from two sides, the textbook as an artefact and the textbook as an instrument. It includes results from analysis of textbooks from different parts of the world as well as studies of the actual use of textbooks. It is not an attempt to give complete coverage of previous research in the area; it is rather an attempt to highlight some important issues.

The textbook as an artefact

On terminology

The ability to collect experiences and use them for our own purpose is significant for our culture and distinguishes us from the animals. *Artefacts*, intellectual (e.g. ideas, values, knowledge) as well as physical tools, are developed, refined and changed in a continuous process in the interactions between human beings and their environments (Säljö, 2000). Wartofsky (1979) defines artefacts as ”anything which human beings create by the transformation of nature and of themselves: thus, also language, forms of social organization and interaction, techniques of production, skills” (p. xiii).

Artefacts can be categorized as *primary*, *secondary* and *tertiary* (Wartofsky, 1979). ’Primary’ artefacts are those directly used in production, for instance axes and clubs. ’Secondary’ artifacts are externally embodied representations, created for the purpose of preserving and transmitting skills in the production and use of primary artefacts. Textbooks and other curriculum materials that are used in classrooms are examples of

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secondary artefacts. 'Tertiary' artefacts are "a class of artifacts [1] which can come to constitute a relatively autonomous 'world', in which the rules, conventions and outcomes no longer appear directly practical" (Wartofsky, 1979, p. 208).

Further examples of artefacts, which are used in mathematics classrooms, are hand-held calculators and computers. Whereas computer-aided design (CAD) programs are 'primary' artefacts, the learning-software and the computer game are 'secondary' and 'tertiary' artefacts, respectively (Strässer, 2004).

A glimpse into the history of mathematics textbooks

When discussing textbooks and their role in mathematics education, at the present time, we might forget about their importance for the development of mathematics. However, as secondary artefacts, textbooks contribute to the field of mathematics by preserving and transmitting skills and knowledge. We can notice their importance by considering the mathematicians that have a prominent position in the history of mathematics – mainly because they have written books that become frequently used in education. I can give two examples: Euclid of Alexandria (born ca. 325 BC) and Robert Recorde (1510-1558) from England.

The most famous work of Euclid is the treatise on mathematics, *The Elements*, which is a "compilation of knowledge that became the centre of mathematical teaching for 2000 years." (O'Connor and Robertson, 2002). B L van der Waerden considers the importance of the Elements in the Biography in *Encyclopaedia Britannica* (cited in O'Connor and Robertson, 2002):

> Almost from the time of its writing and lasting almost to the present, the Elements has exerted a continuous and major influence on human affairs. It was the primary source of geometric reasoning, theorems, and methods at least until the advent of non-Euclidian geometry in the 19th century. It is sometimes said that, next to the Bible, the 'Elements' may be the most translated, published, and studied of all the books produced in the Western world.

The other example of famous mathematicians (and textbooks authors) is Robert Recorde who lived in England in the 16th century, almost eighteen hundred years after Euclid. Their work and contribution to the field is not comparable to Euclid's work. Nevertheless, Recorde wrote many elementary textbooks that differ from the previous ones in that they were written in English, which was extremely uncommon at that time. In doing so, he made them available to a larger group of students, not only the few who could read Latin or Greek. Another special feature of his books is that basically all of them are written in the form of a dialogue between a master and scholar, for instance *The Grounde of Artes* that was a very successful commercial arithmetic book and *The Whetstone of Witte* on algebra published in 1543 and 1557 respectively. The only book of Recorde that is not written in the form of a dialogue between a master and a scholar is the *Pathwaie to Knowledge* (1551), which sometimes is considered as an abridged version of Euclid's *Elements*. Recorde had a clear idea of which order the mathematical topics should be taught and wrote his books in that order with the intention to offer a complete course of mathematical instruction. In addition, he tried to make the texts easy to understand by using clear and simple expressions (O'Connor and Robertson, 2002).

> Anyone who reads Recorde's works will be led to believe that, although no evidence survives, he must have taught for some time since he has a deep understanding of how to teach. Ideas are developed clearly step by step, with difficult points being left until the student has gained enough experience to understand them. (O'Connor and Robertson, 2002).

Textbooks (in general) were also important, as basic tools, for the progress of organized education in the western part of the world in the 17^{th} and the 18^{th} century (Selander, 1988).

The authority and the authors of textbooks

Textbooks are special kinds of books since they are intended to be used in education, "designed to provide an authoritative pedagogic version of an area of knowledge" (Stray, 1994, p. 2). Though 'normal' books might be used in teaching situations, "the pedagogic use is marginal to the intentions of the book's producers" (ibid.). Moreover, textbooks hold a unique and significant social function in relation to other texts available for a reader since they "represent to each generation of students an officially sanctioned, authorized version of human knowledge and culture" (de Castell, Luke and Luke, 1989, p. vii). An additional view of textbooks is offered by Johnsen (1993):

> A textbook is neither just subject content, nor pedagogy, nor literature, nor information, nor morals nor politics. It is the freebooter of public information, operating in the gray zone between community and home, science and propaganda, special subject and general education, adult and child (p. 330).

For whom is the textbook written? Well, students are in general regarded as the main readers of a textbook. The role of a teacher is to mediate the text and encourage students to read the book. Since the authors cannot intervene directly in the communication between teacher and students they usually write the textbook from the teacher's position (Kang and Kilpatrick, 1992).

The authority of the textbook comes "in virtue of its having been *authorized* by an administrative source" (Luke, de Castell, Fraser and Luke, 1989, p. 254). It could be the teacher or whoever decides which textbook to use, whose authority in turn is institutionally bound. The authorization involves the content as well as the physical object. Textbooks are, in many countries, school property that are cared for and venerated. The teachers often admonish students to be cautious, sometimes making the link between school property and private ownership with comments like: 'your parents pay tax dollars for these books!' Covering books with protective jackets is also an example of an annual ritual to show 'respect' for the text. Irrespective of that, textbooks are also disused by the students, scribbled in and dashed against walls or lockers, sometimes as a private act of criticism. But in the communicational system of the classroom, the students have a passive and *non-authoritative status* in relation to both text and teacher (Luke et al., 1989).

The authority of the textbook is more or less unrelated to the author. This is especially true for students. For them, the authors of the textbook are often anonymous and textbooks are in general identified with courses and teachers. Teachers unconsciously reinforce this when they refer to textbooks as 'the math book' or 'the blue book' rather than to their actual authors (Luke et al., 1989). This irrelevance of authorship, Luke et al. says, detach the text from the author and makes the school text "more closely associated with a corpus of 'indisputable' disciplinary or lesson content than with a potentially fallible author" (p. 258).

But the textbook, as mentioned earlier, is an artefact. And it is human-made in a specific sense; there exists an author (or a group of authors) and a producer of the textbook. One can assume that the purpose of textbook authors and publishers is to offer a well-made, carefully prepared pedagogical version of a school topic. On the other hand, publishing is an industry in most countries, so "gaining a large share of the market is perhaps the most important goal that drives the design and production of textbooks." (Chávez, 2003, p. 1). Hence, in countries where the market is free and teachers can select their own textbook, it is more likely that the economical interests have more influence than the pedagogical. However, our knowledge about textbooks authors and publishers are limited. Portraits of textbooks authors are rare (great textbooks authors in former days e.g. Euclid and Recorde excluded) and publishers are even more anonymous (Johnsen, 1993). As an unspecified group, they are however criticized from time to time. For example in a text from the beginning of the 80's where Howson, Keitel and Kilpatrick (1981) considered the business in the USA in the following way:

> Commercial textbook publishers in the USA have set up elaborate networks for the promotion and distribution of their products. Teachers in the USA depend heavily on textbooks. This dependence has roots that go back to the introduction of free public education into a frontier society: the textbook was used to compensate for a shortage of well-educated teachers. As a result, textbooks are marketed much like automobiles: publishers' representatives find out what the consumer wants and then the publishers compete to offer it to him in the most attractive package possible. Novelty is at a premium; yet the changes offered are frequently cosmetic and rarely radical. The greatest changes in US textbooks in the last three decades have come from outside the commercial arena – many of them in response to the stimulus provided by federal-funded projects – but in each case the forces of the marketplace acted quickly to dampen the change. (p. 62).

A conclusion must be that an author's contribution to a textbook is somewhat limited. But the interests of the publisher are only one of many influential factors. Even the authors' attitudes and norms, which mark a 'normal' book, are constrained since the author himself is a product of attitudes in a given society at a given time, certainly conscious of the formal requirements regarding the given school subject. Johnsen (1993) highlights this issue and says that "It is possible to reduce the significance of an individual authors' contribution to textbooks for reasons which may in themselves provide some explanation for the low status of these writers." (p. 244).

Perhaps as a consequence of their anonymity, textbooks authors are seldom categorized as a separate occupational group. There are however trade unions for this category of writers in Norway and Sweden, which can be a sign of professional awareness. In 1987, the Association of Norwegian Non-Fiction Writers[2] made use of their list of members and sent out a questionnaire to those who write general non-fiction books and/or textbooks in order to learn more about the authors. Most of the 731 who replied had full-time jobs in other fields. For example, thirteen percent of the authors were primary and secondary school teachers, three of four authors where men and their average age were between forty and fifty-five (Johnsen, 1993). It is hard to find documentations about the situation in Sweden, but it seems that most of the authors of mathematics textbooks are, or have been, teachers.

About mathematics textbooks

The following section contains some results from previous research on textbooks. These studies show that textbooks around the world have some common features. However, they also conclude that there exists no 'universal' book. But to begin with, I will bring forward some arguments why content analysis of textbooks is important.

Why content analysis?

The presence of textbooks in classrooms is not a clear indicator of how instruction is done since textbooks, as such, give no information on how they are used or how they influence what takes place in the classroom. So, why make an effort to find out about the content of a textbook? Is there any meaning of doing content analysis? I believe it is. Previous research suggests that:

(a) Mathematical topics in textbooks are most likely presented by the teachers (Freeman and Porter, 1989; Reys, Reys, Lapan, Holliday and Wasman, 2003);

(b) Mathematical topics not included in textbooks are most likely not presented by the teachers (Freeman and Porter, 1989; Reys et al., 2003);

(c) Teachers' pedagogical strategies are often influenced by the instructional approach of the material (Reys et al., 2003);

(d) Teachers' sequence of instruction are often parallels to that of the textbook (Freeman and Porter, 1989).

(e) Teachers report that textbooks are a primary information source in deciding how to present content (Schmidt et al., 2001).

In textbooks we can find various methods to present the same mathematical topic. A topic can be emphasized in one book and almost overlooked in another. Connections between topics can be illustrated in different ways and the topics can be organized differently. In fact, textbooks reveal underlying beliefs of what mathematics is and how it can be learned. An analysis of textbooks can identify (national or international) similarities and differences between textbooks. The findings can, subsequently, serve as one possible explanation of dissimilar learning experiences among students from different classrooms or countries (Valverde, Bianchi, Wolfe, Schmidt and Houang, 2002). Curriculum developers, teachers and others that are involved in mathematics education can hopefully learn from these studies. In addition, the results from these studies can also be helpful for the development and improvement of textbooks.

Finally, textbooks analysis can also be a record of past learning experiences and reveal historical changes in mathematics education. However, we must keep in mind that:

> Texts are often all that remain as records of past classrooms. It is tempting to make inferences about the actual curriculum and the pedagogic intentions of past authors on the basis of the texts alone. However, at the very least, the principles which governed the selection of material are clearly unavailable (Love and Pimm, 1996, p. 373).

Nevertheless, since the content and the structure of a textbook is influenced by the educational culture of a specific country (Pepin and Haggarty, 2001), a look into the historical development of textbooks can reveal interesting trends. One can for instance examine how textbooks changes with regards to mathematics content, text and physical features, from one time to another. The focus in such studies can be more or less narrow. For example, it can be the presence or absence of the proof of the general rule for finding the derivative in textbooks for upper secondary schools in Sweden, which was one of the focuses of Bremler (2003), or how well a specific textbook series is adjusted to changes in the national curriculum (Johansson, 2003).

Some content of textbooks

The presence or absence of mathematical topics and characteristics of the text and tasks are relevant issues in a content analysis of mathematics textbooks. Questions concerning the topics are for example: What are the topics in textbooks for grade eight? In what grade are linear equations introduced to students? These types of questions are maybe time-consuming but comparatively easy to find answers to. The text and the tasks in a textbook is however not that uncomplicated to characterize. First of all, there are several ways to break up a textbook into units. A unit to analyze can be a textbook (or a book-series) intended for a whole school year or a single exercise (or sentence) and everything in between. Second, it is necessary to use or develop some type of taxonomy or classification schemata.

Mathematics textbooks can also be analyzed regards to a specific mathematical topic. The focus in the study of Harries and Sutherland (1999) is for example how textbooks from five different countries[3] introduce multiplication and division to students at the age of 6 to 8. One of their findings is that the English textbook first presents multiplication as repeated addition of equivalent sets without using the standard notation of the ’x’ sign. The teacher’s guide implicitly suggests that pupils should first understand an idea before the mathematical notation can be introduced. In the Hungarian and the French textbook, multiplication and division are introduced a year later than in England (age 7-8) and the students are very quickly introduced to the formal multiplication notation. Harries and Sutherland also notice that the strength of the textbooks from Singapore, Hungary and France "is the use and consistency of use of appropriate representations." (Harries and Sutherland, 1999, p. 63). In that respect, they are rather critical to the English texts.

Some characteristics of textbooks

The characteristics of mathematics textbooks are reported in several, mainly comparative, studies from all over the world. Sometimes the focus is very broad and sometimes it is rather narrow. Probably the most comprehensive study of textbooks and curriculum material is conducted as a part of the *Third International Mathematics and Science Study*, TIMSS. Textbooks are for this study regarded as one of the representatives for the intended curriculum that could *explain* resulting differences (Robitaille et al., 1993). A considerable amount of material was collected from about 50 participating countries[4]. Three parameters are used to characterize each section of every textbook, syllabus and curriculum guide: subject matter content; performance expectations; and perspectives or context.

The data from the textbooks' analysis part of TIMSS curriculum study shows that textbooks around the world differ greatly in size, length and other structural features. There are also differences regarding the types of chapters and units they contain as well as lay out. In addition, a deeper examination reveals notable differences in sequencing and complexity (Valverde et al., 2002, p. 21).

One of the most obvious physical characteristics of textbooks is the number of pages. In the TIMSS's sample of textbooks[5], the average number of pages is about 125 pages for mathematics textbooks intended for fourth grade. The textbooks for eight grade have 225 pages on average. Extreme numbers of pages are rare. Only about 10 percent of mathematics textbooks have less than one hundred pages. Germany, the Russian Federation and Hong Kong and nine other countries had this type of textbooks. About 10 percent of mathematics textbooks consist of more than five hundred pages and US is certainly an exceptional case. With only one exception, a textbooks intended for nine-year-olds with 484 pages, all of their textbooks have more than 500 pages. To cover all of the content, the amount of time for doing this would be severely limited. But if not all content is covered, there must be some basis for choosing what to be enclosed and what to be exposed (Valverde et al., 2002).

Except pure text, many textbooks contain pictures with different functions. There are for example images for primarily illustrative purposes or photographs that suggest a context. But there are also images to be worked on mathematically, for instance measure length or angle (cf. Grevholm, Nilsson and Bratt, 1988; Love and Pimm, 1996). The sample of textbooks in TIMSS vary substantially in their use of visual aids such as photographs, pictures, tables, and graphs (Valverde et al., 2002).

As regards the physical features of the textbooks, Valverde et al. (2002) note that some textbooks correspond to the description:

– emphasizes exercises and problem solving to a high degree;
– have a small number of strands;
– are of moderate size,

Other textbooks can be described in the following way:

– have many pages;
– cover a moderate array of topics with less focus;
– are fragmented;
– are mostly made up of exercises (Valverde et al., 2002).

The majority of the mathematics textbooks in the TIMSS curriculum study correspond to these two descriptions (43,5% to the first and 47,3% to the second one). In general, the countries have textbooks with different structures but some country patterns are revealed. For instance, all Cypriot mathematics textbooks in the TIMSS study correspond to the first of these two descriptions and almost all US mathematics textbooks correspond to the second (Valverde et al., 2002).

The text in a textbook is often organized in an 'exposition–examples–exercises' model (Love and Pimm, 1996). In the exposition part, one can find sequences of tasks that are intended to support students' concept formation through a kind of 'guided discovery'. However, the text is usually moving to a particular destination and "students are often impatient with the exposition and skip to the 'essential' results" (p. 387). The exposition part is usually followed by examples, which offer a prototype for students to copy for the next part, the exercises. The exercise sets are often 'graded' and progresses from easier to more difficult or consist of parallel sets with different levels of difficulty. Such features have inbuilt assumptions about ability and learning. For example that progressing in small, well-defined steps is the best way to achieve learning (Love and Pimm, 1996).

In Swedish mathematics textbooks it is extremely common that exercise sets are graded according to their level of difficulty. Which level each individual student should work with is usually determined by a diagnosis in the textbook. For better or for worse, this facilitates individual work by the students but is also a way to differentiate them. How this is done is examined by Brändström (in preparation) through a content analysis with a special focus, inhomogeneity of learners (open differentiation).

An exception from the 'exposition–examples–exercises' model is the 'activities–cours–exercises' model, which can be found in textbooks from France (Pepin and Haggarty, 2001). The 'activities' part intends to introduce a notion to the students through small investigations. The 'cours' describes what needs to be taught, both in words and in worked examples. According to Pepin and Haggarty, this model seems to fit with Piaget's notions of constructivism.

The textbook as a teacher

Some mathematics textbooks contain only problems and exercises, other consist of two separate parts with theory in one part and problems and exercises in the other. Both kinds require support from a teacher who will play a central role in mediating the text to the students (Love and Pimm, 1996). There are also textbooks that have theoretical notes (remarks, clarifications, generalizations, etc.) interspersed with problem, exercises and other assignment. Van Dormolen (1986) notices that a book like this seems to be a teacher in itself. He asks if the author had wanted to write a teacher-proof text and if the teacher just sits back and let students work with the book. Van Dormolen continues his discussion questioning whether it is really possible to write teacher-proof texts. Books of such kind must, on the one hand, teach new concepts that might imply lot of texts, and on the other hand serve as a reference book that demands a short, concise text.

Another relevant question to ask in this respect is if mathematical problem solving is bookable[6]. In other words, is it possible to transform the dynamic process of problem solving into a linear, static text in a textbook? If the text contains a solution to the problem, one obvious dilemma is how to hinder the students from looking at it. Other difficulties concerns how to 'book' blind alleys, incorrect solutions, reformulations of the problem, hints, etcetera (Kang and Kilpatrick, 1992).

A more global question is if textbooks, themselves, can contribute to mathematics learning. The issue is especially relevant to Sweden where students and teachers seem to be extraordinarily dependent on textbooks. In an evaluation of schools in 40 municipalities (out of 290) it is reported that from year 4-5 and onwards, the teaching of mathematics is in principal based on the use of textbooks. Content as well as preparation and organization of the lesson is very much dictated by textbooks.

Hence, they seem to define 'school mathematics' as well as the 'learning path' for the majority of students, at least in lower and upper secondary school (Skolverket, 2003).

Some textbooks are even designed with the idea that students can learn mathematics directly from the text. Since student readers have to be able to work through the texts without help from a teacher, there are some special demands for such books. To eliminate the risk of causing difficulty to an isolated student, thus prevent him or her for asking (relevant) questions, the mathematical demands of a task need to be on a lower level than if a teacher is supposed to be around (Love and Pimm, 1996).

But if student-readers should handle texts with little or no assistance from a teacher, it is important that the text contains *clear* instructions. On the one hand, the instruction should guide the student so s/he can decide what to do. On the other hand, the decision should correspond with what the author or the teacher has in mind. Van Dormolen (1986) shows an example when the task is unclear for the students:

Compute

6 x -3 =

5 x -3 =

4 x -3 =

3 x -3 =

2 x -3 =

1 x -3 =

0 x -3 =

-1 x -3 =

-2 x -3 =

What do you notice?

> It was the teacher's purpose that the students should discover that the product of two negative numbers is a positive number. Everybody thought, however, that they should recognise the pattern, a task that they did not find difficult at all. This exampel shows how it may seem clear to students what they have to do, but the actual activity generated is not what the author had in mind. One can find many similar examples in mathematics books. (van Dormolen, 1986, p. 159).

If we assume that textbooks can contribute to mathematics learning by themselves, is one type of textbook better than the others? Several studies are trying to find answers to that question. Many of them stem from the USA, probably because of their development of new materials that intends to match up to the *Standards*. However, to answer these questions without taking into account variables such as quality of teaching is not sufficient. Nevertheless, there are indications that students who used a certain (reform-based) material perform better on achievement tests than their counterparts who used traditional textbooks (see for instance Reys et al., 2003) and many researcher are convinced that the choice of textbook matters. ”It is clear that in these cases, the nature of the mathematical tasks presented to students, their richness and quality, does depend on the textbook used by the teacher.” (Chávez, 2003, p. 159).

The textbook as an instrument

On terminology

A user of a textbook, whether it is a teacher or a student, develops an individual system to read the book, an individual *utilization schema*. But the emergent schemes are results from a collective process that both users and designers of textbooks contribute to so they have a 'social' dimension as well. Rabardel and Samurçay (2001) describe the process of learning with artefacts as an *instrumental genesis process*.

The subject's instrument is not a 'given', but is elaborated by the subject during the instrumental genesis process. This process concerns both the artifact and social utilization schemes (p. 15, section 2).

The *instrumental genesis process* has two dimensions:

> *Instrumentalization* – the emergence and evolution of the components (e.g. selection of functionally pertinent parts of the artefact)
>
> *Instrumentation* – the appropriation of social utilization schemes, the emergence and development of private schemes (Rabardel and Samurçay, 2001).

Figure 1 illustrates these two dimensions with a bidirectional arrow. The *instrumentalization* is the arrow from *artefact* to *utilizations schemes*. The *instrumentation* is illustrated by an arrow from *utilizations schemes* to *artefact* and demonstrates how a subject (for instance a person) develops a private schema for the specific artefact. For example, a scissor, which is mainly constructed for the purpose to cut paper, can be most useful for taking out thumbtacks from a bulletin board.

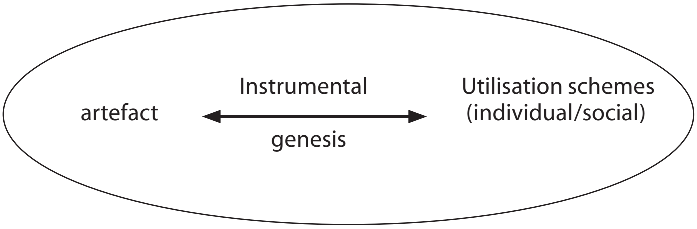

Figure 1: *Schematic representation of an 'instrument'* [7]

In educational research, the 'instrumental approach' is commonly used for studies of teaching and learning with computers or other technical devices. However, considering that textbooks are artefacts that are most important in many classrooms, especially in Sweden, it is meaningful to extend the theoretical framework to also include research on textbooks. Because when a teacher or a student uses a textbook, it becomes an instrument.

The use of textbooks by teachers

In general, textbooks should be *mediated* by teachers, not *replace* them. One reason is that a text will never be able to communicate directly to the students with strategies that a teacher can use, for instance voice tones and commentaries (Love and Pimm, 1996). Even though the book is an instrument for both teachers and students, it is the teacher who primarily influences how the students use their textbook. The discussions in this section are based on results from previous research that consider the actual use of textbooks. Under the assumption that it is the teacher who decides which textbook to use and how to use it, the focus in this part is solely on 'utilization schemes' of teachers.

The variation among teachers' use of textbooks is considerable (Sosniak and Stodolsky, 1993; Stodolsky, 1989; van Dormolen, 1986). Why? One reason is that teachers, even if they work with identical material, have a different understanding of the ideas that the material attempts to communicate (Lloyd, 1999). Another reason is that teachers' use of textbooks is related to their beliefs about mathematics and teaching (Chávez, 2003). The teachers can use their textbook rather consciously, for instance as a source of mathematical and representational ideas from which s/he can adapt and invent tasks (Remillard, 2000). But textbooks can also be used without awareness (cf. Sosniak and Stodolsky, 1993).

Teachers use textbooks in different kind of activities. Pepin and Haggarty (2001) found that textbooks are used ”for teaching in order to lay down rules and conditions; for explaining the logical processes and going through worked examples; and for the provision of exercises to practice.” (p. 169). The use of textbooks for exercises is emphasized by teachers in the three countries, France, Germany and England, but the theoretical parts and the worked examples in the textbooks are conceived differently. French teachers present the rules in a different way than the book and German teachers use different worked examples than the textbook (Pepin and Haggarty, 2001).

Do teachers depend upon textbook to a large extent? Well, there is a good deal of evidence that many teacher do (Chávez, 2003; Love and Pimm, 1996). Surprisingly not since the material facilitates teachers' work; the topics are hopefully ordered in the best way and notations are most likely consistent. Moreover, they do not need to be concerned whether a student will have met the necessary pre-requisites for a new topic.

Teachers can defend their decision to follow the textbook closely by arguing that it prevents them from skipping important topics or teaching topics out of an appropriate sequence. However, even those teachers who follow the book very closely do not necessarily cover it as completely as is commonly believed (Freeman and Porter, 1989).

Freeman et al. (1983) categorize teachers' use of textbooks into four distinct styles: textbook-bound, selective omission, focus on the basics, and management by objectives. Teachers that are 'textbook-bound' progress almost page–by–page through the entire book over the course of the year. If the teacher is textbook-bound but skips some chapters, for instance geometry because s/he thinks that the topic is unimportant for students to learn, then the teacher has a 'selective omission' style of use. Certain general topics are emphasized by teachers who adopt the 'focus on basic style', for instance the four fundamental rules for arithmetic. Lessons in the textbooks that are not directly related to basic mathematical concepts and skills are therefore omitted. In some schools, teachers are required to work in a system that is designed to ensure that all students reach a minimal level of competencies in mathematics. The 'management by objective' style of use corresponds to teachers who define all textbook assignments by the system (Freeman et al., 1983).

In a study of four elementary school teachers, Freeman and Porter (1989) find that the greater the extent to which the teacher rely on the text the stronger is the relationship between topic emphasis in textbooks and topic emphasis in instruction. Nevertheless, even the teacher who followed her textbook very closely failed to use 40 % of the lesson in her textbook.

One can consider whether individual teachers are consistent in their style of textbook use across the subjects they teach. In other words, does the subject matter? Sosniak and Stodolsky (1993) found that single teachers vary considerably in their use of textbooks from one subject to another. For example, the distribution of instructional time with textbooks, for one of the teachers in the study [8] (Carol), was 98 % in mathematics but only 17 % in social studies. Carol reports that she did read the teacher's guide for the mathematics text when the book was new to her, but after one year – she only skims it. "Where reading textbook materials serve Carol as food for thought in the course of building her own reading program, mathematics textbook materials free Carol from having to do much thinking at all about her mathematics program." (Sosniak and Stodolsky, 1993, p. 260). Teachers are rather autonomous, and how they use textbooks for instruction is, according to Sosniak and Stodolsky, not strongly affected by district policy or the nature of the subjects themselves. Sometimes the teachers make heavy use of the textbook materials with careful attention to the full nature and the theoretical perspective they are based on. Sometime they are less aware of the ideas embodied in the materials. Their degree and type of thoughtfulness and purposefulness differs – with varying consequences for the students.

That teachers vary in their use of mathematic textbooks is confirmed in an earlier study of Stodolsky (1989). Generally, topics in instruction correspond to topics in the textbooks and vice versa [9], but it was less agreement between suggested activities in the teacher's editions and actual classroom practice. Manipulative activities and suggestions for enrichment were in particular seemed as dispensable to all of the teachers.

There are several aspects of instructions that textbooks rarely determine, for instance, time allocation and standards of students' achievement. The individual teacher can often decide:

a) which textbooks to use

b) which sections of the textbook to use

c) when and where the textbook is to be used

d) what topics to teach

e) the sequencing of topics in the textbook

f) how much time to spend on each topic

g) the way in which pupils engage with the text

h) the level and type of teacher intervention between pupil and text

So, do teachers, in general, feel that textbooks control their teaching? Well, not the teachers in the study of Pepin and Haggarty (2001) at least. They asserted that they were in charge of their teaching and student learning. A similar opinion was expressed by the teachers in the study of Sosniak and Stodolsky (1993). They did not perceive textbooks to be good or bad, strong or weak, in categorical terms. For these teachers, they were regarded as resources with strengths and weaknesses that teachers themselves had to decide upon.

> Textbooks apparently served as neither a blueprint nor a source of great frustration for the teachers. Instead, texts were merely materials available – tools, props, curricular embodiments (Sosniak and Stodolsky, 1993, p. 270).

Teachers deviate from textbooks for different reasons. One reason is if the suggested teaching method in a textbook does not corresponds to the way the teacher perceives that the subject should be taught. The individual teacher, on the base of his/her own learning experience, can then decide to not use the book (Sosniak and Stodolsky, 1993). Further reasons are that the teacher may judge that the textbook is inappropriate because the language is too difficult or because the exercises are inadequate (van Dormolen, 1986). It could also be the situation that the material is unfamiliar to the teacher and not provide enough pedagogical guidance (Remillard, 2000, p. 345).

Clear pedagogical guidance may however not necessary mean that a curriculum material is used in accordance to the intentions of the authors. The teachers in Lloyd's study, who used a curriculum material that emphasizes exploration and cooperation, saw similar benefits of this way of working. They interpreted however the suggested activities in terms of those benefits differently. One of the two teachers viewed the problems in the program as 'challenging and open to student interpretation – at times *too* open'. Whereas the other teacher claimed that the problems were 'overly structured and did not permit students to solve problems in their own ways or explore concepts sufficiently' and that 'students are led through the problem' – comments that bear remarkable similarity to the other teachers' remarks about the traditional curriculum's exercises (Lloyd, 1999).

So teachers conceive textbooks and other curriculum materials differently, but can they learn from them? Can textbooks change the way teachers teach mathematics? What can we say about the 'instrumental genesis process'? In a study of teachers reading of a reform-based curriculum material, Remillard (2000) found that Catherine, one of the two teachers in the study, made a considerable shift. After only one year with the new material, she changes her view of problem solving and increases her awareness of its importance and also of her own and her students' abilities to solve problems. But can the textbooks alone form the curriculum? Remillard doubts that they can since different teachers read texts in different ways.

> The variation in the two teachers' reading of the text ... illustrates the powerful role that teachers play in mediating the textbook's contribution to the enacted curriculum. Consequently, it is unlikely that textbooks can shape the curriculum directly (Remillard, 2000, p. 344).

Chávez (2003) notes another interesting phenomenon; teachers who use reform-based material (or more precise – NSF-funded curricula) ”seem to adhere more closely to their textbooks. They are also more likely to read and use the teacher guide” (Chávez, 2003).

Conclusions and discussion

The history of mathematics shows that textbooks are artefacts that preserve and transmit knowledge in the educational systems. But their role in this system is not easily determined; they can be regarded as both important tools *and* obstacles for the development. On the one hand, they facilitate the daily work for the teachers and serve as some kind of guarantee that the students have the necessary basic knowledge and training for the next level in the school system. On the other hand, they seem to reduce both freedom and responsibility of the teachers.

The authority of textbooks is an important issue that needs further discussion. In comparison to other school subjects such as History and Social studies, it is perhaps not so important if mathematics text is regarded as a corpus of 'indisputable' lesson content (cf. Luke et al., 1999) given that it is expected that the book is accurate with regards to mathematics. It is however important to stress that the book presents a selected part of the subject area. It paints a certain picture of mathematics, its role in the society and in the history. But on what basis is this made and what are the consequences?

What appears in textbooks is influenced by several aspects of the educational culture. That is a plausible reason why textbooks from different part of the world are different. However, under the assumption that the textbook is the predominant source in the teaching and learning of mathematics, this means that students are faced with different opportunities to learn (cf. Valverde et al. 2002). But who is responsible for the content of a textbook: the author; the publisher; the educational authority; the mathematical society; the teachers; or maybe the society at large?

In many countries, a textbook needs an approval by an educational authority before it reaches the market. In Sweden, there are no restrains (except economical of course). Which textbook to choose is often a decision of a teacher or a group of teacher. Thus the demands and the expectations of the teachers will influence the development of textbooks. If the teachers' choices of textbooks are more or less unanimous the selection and diversity of textbooks on the market will be more or less limited. Hence, the market offers books that most teachers want so the teachers have to choose among them. An alternative way for the authors/publishers to reach more purchaser is to make textbooks that are "all–in–one", i.e. designed in order to adhere to different styles of teaching. The fact that many textbooks have a large number of pages, which covers more than the class can handle during a school year, can be an indication of that.

Regarding the textbooks as an instrument, this text focus solely on 'utilization schemes' of teachers. My intention was however to put equal attention on the two main persons in the classroom: teachers and students. But I had to learn that very few studies involve the students' use of textbooks. Thus, there is a special need for more knowledge – and research – on the use of textbooks by students. That teacher depend on textbooks to a large extent is one conclusion from previous studies. But at the same time the variation among teachers' use of textbook seem to be considerable. It is also evident that teacher could interpret the vision and aim of a textbook differently and deviate from it for different reasons. This is important to have in mind when we think about and discuss the role of textbooks in classrooms, and especially their position in a reform of the curriculum.

For the understanding of the processes of teaching and learning mathematics, I believe it is important to increase the awareness of textbooks and how they are used. As a predominant source in many mathematics classrooms, textbooks have a unique status. They often determine what is school mathematics and also what is mathematics, for both teachers and students (cf. Skolverket, 2003; Valverde et al., 2002). However, textbooks and their role in the teaching and the learning of mathematics should be discussed in a thoughtful way. In order to achieve this goal, we need to analyze the situation in classrooms. Not only the *content* of textbooks and *how much* textbooks are used in relation to other activities should be analyzed, but also *how* and *why* textbooks are used. Studies of the textbooks and the use of them in classrooms would also offer further knowledge about the practice of mathematics teaching. This can for example be done through a combination of: observations in classrooms studying the actual use of textbooks by students and teachers; teachers' interviews asking for the role textbooks play in their professional life as teachers; an analysis of teachers' logbooks and content analysis of the textbooks in use.

References

Bremler, N. (2003). *Matteboken som redskap och aktör. En studie av hur derivatan introduceras i svenska läroböcker.* Stockholm: Lärarhögskolan.

Brändström, A. (in preparation). *Differentiated learning in mathematics textbooks*. Luleå University of Technology.

Chávez, Ó. (2003). *From the textbook to the enacted curriculum: textbooks use in the middle school mathematics classroom* (Ph. D. Thesis). University of Missouri – Columbia. Retrived January 12, 2006 from, http://www.missouri.edu/~chavezo/dissertation/index.shtml

de Castell, S., Luke, A., & Luke, C. (Eds.). (1989). *Language, authority and criticism*. London: Falmer Press.

Freeman, D. J., Belli, G. M., Porter, A. C., Floden, R. E., Schmidt, W. H., & Schwille, J. R. (1983). The influence of different styles of textbooks use on instructional validity of standardized tests. *Journal of Educational Measurement*, 20 (3), 259-270.

Freeman, D. J., & Porter, A. C. (1989). Do textbooks dictate the content of mathematics instruction in elementary schools? *American Educational Research Journal*, 26 (3), 403-421.

Grevholm, B., Nilsson, M., & Bratt, H. (1988). Läroböcker i matematik. In *Skolböcker 3. Den (o)möjliga läroboken. Rapport från Läromedelsöverssynen*. (Ds 1988:24). Stockholm: Allmänna Förlaget.

Harries, T., & Sutherland, R. (1999). Primary school mathematics textbooks: an international comparison. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools*. Buckingham: Open University Press.

Howson, G., Keitel, C., & Kilpatrick, J. (1981). *Curriculum development in mathematics*. Cambridge: University Press.

Johansson, M. (2003). *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum* (Licentiate thesis). Department of Mathematics, Luleå University of Technology.

Johnsen, E. B. (1993). *Textbooks in the kaleidoscope: a critical survey of literature and research on educational texts.* Oslo: Scandinavian University Press.

Kang, W., & Kilpatrick, J. (1992). Didactic transposition in mathematics textbooks. *For the Learning of Mathematics,* 12 (1), 2-7.

Lloyd, G. M. (1999). Two teachers' conceptions of a reform-oriented curriculum: implications for mathematics teacher development. *Journal of Mathematics Teacher Education*, 2 (3), 227-252.

Love, E., & Pimm, D. (1996). 'This is so': a text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International handbook of mathematics education* (Vol. 1, pp. 371-409). Dordrecht: Kluwer.

Luke, C., de Castell, S., Fraser, S., & Luke, A. (1989). Beyond criticism: the authority of the school textbook. In S. de Castell, A. Luke & C. Luke (Eds.), *Language, authority and criticism: readings on the school textbook*. London: The Falmer Press.

O'Connor, J. J., & Robertson, E. F. (2002, february). *The MacTutor history of mathematics archive.* Retrieved June, 2004, from http://www-groups.dcs.st-and.ac.uk/~history/

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms: a way to understand teaching and learning cultures. *Zentralblatt fuer Didaktik der Mathematik*, 33 (5), 158-175.

Rabardel, P., & Samurçay, R. (2001, March). *From artifact to instrument – mediated learning.* Paper presented at the New Challenges to Research on Learning. International symposium organized by the Center for Activity Theory and Developmental Work Research, University of Helsinki.

Remillard, J. T. (2000). Can curriculum materials support teachers' learning: two fourth-grade teachers' use of a new mathematics text. *The Elementary School Journal*, 100 (4), 331-350.

Reys, R., Reys, B., Lapan, R., Holliday, G., & Wasman, D. (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education*, 34 (1), 74-95.

Robitaille, D. F., Schmidt, W. H., Raizen, S. A., McKnight, C. C., Britton, E. D., & Nicol, C. (1993). *Curriculum frameworks for mathematics and science* (TIMSS Monograph No.1). Vancouver: Pacific Educational Press.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H., Wiley, D. E., Cogan, L. S., et al. (2001). *Why schools matter: a cross-national comparison of curriculum and learning.* San Francisco: Jossey-Bass.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims: a cross-national investigation of curricular intentions in school mathematics* (Vol. 1). Dordrecht: Kluwer.

Selander, S. (1988). *Lärobokskunskap. Pedagogisk textanalys med exempel från läroböcker i historia* 1841-1985. Lund: Studentlitteratur.

Skolverket. (2003). *Lusten att lära – med fokus på matematik* (Report no. 221). Stockholm: Statens skolverk.

Sosniak, L. A., & Stodolsky, S. S. (1993). Teachers and textbooks: materials use in four fourth-grade classrooms. *The Elementary School Journal*, 93 (3), 249-275.

Stodolsky, S. S. (1989). Is Teaching Really by the Book. In P. W. Jackson & S. Haroutunian-Gordon (Eds.), *From Socrates to software: the teacher as a text and the text as a teacher* (pp. 159-184). The University of Chicago Press.

Stray, C. (1994). Paradigms regained: towards a historical sociology of the textbook. *Journal of Curriculum Studies*, 26 (1), 1-29.

Strässer, R. (2004). Artefacts – Instruments – Computers. In C. Bergsten & B. Grevholm (Eds.), *Proceedings of MADIF 4. The 4th Swedish Education Research Seminar*. Linköping: SMDF, Swedish Society for Research in Mathematics Education..

Säljö, R. (2000). *Lärande i praktiken: ett sociokulturellt perspektiv*. Stockholm: Prisma.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the Book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht: Kluwer.

van Dormolen, J. (1986). Textual analysis. In B. Christiansen, G. Howson & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 141-171). Dordrecht: D. Reidel.

Wartofsky, M. W. (1979). *Models: representation and the scientific understanding*. Dordrecht: D. Reidel.

Notes

1 With an exception of quotations, I have chosen to use the word *artefact* in this paper, which is chiefly the British variant of *artifact*.

2 Currently *Norwegian Non-fiction Writers' and Translators' Association*

3 Harries and Sutherland compare textbooks from England, France, Hungary, Singapore and the USA.

4 The resulting sample included for instance 241 curriculum guides and 318 mathematics textbooks (Schmidt, McKnight, Valverde, Houang and Wiley, 1997).

5 The TIMSS curriculum study contain 318 mathematics textbooks from the participant countries, selected for the purpose to represent material that no less than half the students in each TIMSS population are likely to have in their classes.

6 Kilpatrick [1980] asked the question, 'Is problem solving bookable?' at the 58^{th} annual meeting of the National Council of Teachers of Mathematics in Seattle

7 The picture originates from Strässer (2004)

8 Four teachers teaching in fourth grade participated in this study.

9 Six elementary school teachers took part in this study

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Sammanfattning

Denna artikel beskriver läroboken utifrån två aspekter, läroboken som artefakt och läroboken som instrument. Resultat från studier av läroböcker från olika delar av världen samt studier av hur läroböcker används i klassrummet behandlas. Syftet med artikeln är inte att ge en heltäckande bild av tidigare forskning inom området utan snarare att lyfta fram några viktiga frågeställningar.

Paper II

Mathematics textbooks – the link between the intended and the implemented curriculum?

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Textbooks are a predominant source in mathematics classrooms in Sweden as well as in many other countries. Consequently, they often determine what school mathematics is and also what mathematics is for students and teachers. They can also have a prominent position in reform of mathematics curriculum since the development of textbooks and other curriculum materials can be seen as a quick and easy way to change teaching. But do they reflect the intended curriculum in all aspects? This paper reports from a study of textbooks as a possible link between educational goals and classroom activities – the potentially implemented curriculum. The aim is to contribute to the discussion about the role of textbooks in mathematics education.

Introduction

Textbooks are a most important feature of the teaching of mathematics because of their close relation to classroom instruction. The textbooks identify the topics and order them in a way students should explore them. They also attempt to specify how classroom lessons can be structured with suitable exercises and activities. Hence, textbooks are designed for the purpose to help teachers to organize their teaching.

> There is a good deal of evidence that many teachers like the security and freedom from responsibility that a text series provides. […] when using a text series, teachers need not involve

> themselves in ordering the topics, in ensuring that notation is consistent, nor in concerning themselves whether a student will have met the necessary pre-requisites for a new topic (Love & Pimm, 1996, p. 384).

Some mathematics textbooks contain only problems and exercises. These kind of books require support from a teacher who will play a central role in mediating the text to the students (Love & Pimm, 1996). There are also textbooks that have a mix of theoretical notes, problems, exercises and other assignment. Such a book "seems to be a teacher in itself" (van Dormolen, 1986, p. 141). But is it possible to write a teacher-proof text? A more global question is if textbooks, themselves, can contribute to mathematics learning. The issue is especially relevant to Sweden where students and teachers seem to be very dependent on textbooks. Content as well as preparation and organisation of the lesson is very much dictated by textbooks. They define 'school mathematics' as well as the 'learning path' for the majority of students, at least in lower and upper secondary school (Skolverket, 2003). The situation in Sweden is however not unique. Previous research on textbooks and teachers' use of textbooks shows, among other things, that:

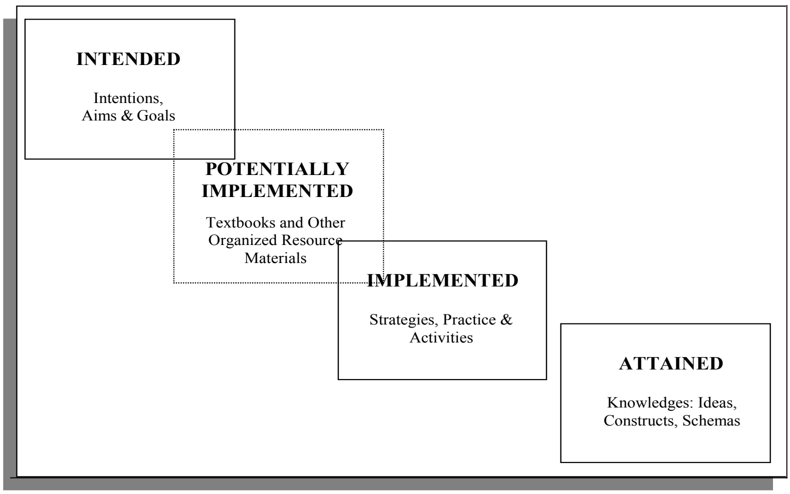
(a) Mathematical topics in textbooks are most likely presented by the teachers (Freeman & Porter, 1989; Reys, Reys, Lapan, Holliday, & Wasman, 2003);
(b) Mathematical topics not included in textbooks are most likely not presented by the teachers (Freeman & Porter, 1989; Reys et al., 2003);
(c) Teachers' pedagogical strategies are often influenced by the instructional approach of the material (Reys et al., 2003);
(d) Teachers' sequence of instruction are often parallel to that of the textbook (Freeman & Porter, 1989).
(e) Teachers report that textbooks are a primary information source in deciding how to present content (Schmidt et al., 2001)

With these results as a background, I believe that an increased awareness of textbooks and how they are used is crucial for understanding the process of teaching and learning mathematics. If one considers a reform of the mathematics curriculum it is therefore important to understand the role of textbooks. In this paper, I will briefly present a study of textbooks that I conducted in 2003. The development of a textbooks series, a commonly used schoolbook in Sweden, is portrayed in the light of the curriculum development (Johansson, 2003).

The curriculum model

In part, textbooks provide indications of students' opportunities to learn. The study of textbooks was therefore important in the research design of the *Third International Mathematics and Science Study*, TIMSS. In the curriculum model, textbooks are regarded as the potentially implemented curriculum, the link between aims and reality (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002).

Figure 1: Textbooks and the tripartite model (Valverde et al., 2002, p.13)



In this model (figure 1), the *intended* curriculum is at the educational system level. It is seen in national policies and official documents which reflect societal visions, educational planning, and official or political sanctioning for educational objectives. Intention and objectives at the level of the teacher and the classroom activity are considered as the *implemented* curriculum. The *potentially implemented* curriculum, which is represented by textbooks and other organized resource material, is regarded as a link between these two levels (Robitaille et al., 1993; Schmidt et al., 1997).

The conceptual framework for the TIMSS Curriculum Study is based on the view of the textbooks as mediators between general intentions and classroom instruction. But what is the relationship between textbooks and the intended curriculum? Are textbooks, in general, appropriate tools for translating guidelines that are stated by educational authorities into activities in classrooms?

A case study of the development of a Swedish textbook series

In Sweden, the objectives of teaching and learning mathematics in compulsory school are expressed and explicitly stated by the National Agency of Education in a national curriculum (the Swedish term is *läroplan*). During the last thirty years, the curriculum has been revised two times, 1980 and 1994. For the purpose to examine the link between the intended curriculum and textbooks, I made a content analysis of a textbook series. The development of the textbook series, a commonly used schoolbook in Sweden, was evaluated in light of the curriculum development. The aim was to examine to what extent a reform of the curriculum influences the development of mathematics textbooks. The study is published in full in the licentiate thesis *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum* (Johansson, 2003).

Three editions of the textbook series, which have been on the market since the beginning of the 70s', are chosen. The editions that are published in 1979 and 1985 consist of two books each, one for the general course (1979a and 1985a) and one for the more advanced course (1979b and 1985b). The third edition from 2001 consists of one book. There are two reasons why I chose this particular textbook: a) even though almost thirty years passed between the first and latest edition, the group of authors is the same all over time; and b) this was one of the two textbook series selected for the TIMSS curriculum study, which implies that it is a commonly used textbook in Sweden.

The textbooks are intended to cover the topic for a school year (year 7) and are designed in a way that facilitates individual work by the students, which is a common feature of Swedish textbooks. The chapters have sets of worked examples, exercises, word problems, and theoretical parts. The books also have sections with review and answers to all exercises. Besides that, the new edition has special units at the end of each chapter with, for instance, suggestions for group work and thematic work.

The three curricula that the textbook editions correspond to are from 1969, 1980 and 1994 respectively. The curriculum from 1994 is also the current one. They are quite different in terms of text and volume. During this period of revisions the text has changed from being very descriptive (in 1969) to very general (in 1994) and the number of pages has decreased from over two hundred to less than thirty. However, they all have a section where the objectives (different for each curriculum) of teaching mathematics are stated. One main difference between the curriculum from 1994 and its predecessors is that it emphasises the role of mathematics in our society as well as the historical development of mathematics. The idea that students should learn about the importance of mathematics is evident in the description of the objectives for mathematics as well as in the assessment criteria (Skolverket, 2001).

In the analysis of the textbook series, I found that there is minor agreement between the objectives of mathematics, explicitly stated in the national curriculum, and the content of the textbooks. For example, in the analysis of the most recent edition of the textbook series, I found that it presents very little information about the role of mathematics in our society and only one short story that could belong to the history of mathematics. In a free translation, the story goes like this:

> In the twelfth century before Christ, the Egyptians divided the day and the night into twelve hours each. This implied that the length of an hour varied at different times of the year. The system was abandoned in the fourteenth century after Christ. A couple of hundred years before Christ was born, Greek astronomers introduced the partitioning into 60 minutes and 60 seconds. The number 60 came from the Babylonian numerical system (Undvall et al., 2001, p. 236, my free translation).

When and *why* is mathematics useful? The textbooks chosen for this analysis have, as many other textbooks, blocks of text. In a few of them you can find arguments concerning *when* and/or *why* a specific mathematical topic is useful. For eight of the topics in the textbooks there are such arguments. The distribution of text blocks with arguments connected to the topics is presented in the table below.

Table 1: The number of text blocks with arguments associated to a topic

Textbook Topic	1979a	1979b	1985a	1985b	2001
Rough estimate	2	1	2	1	1
Rounding					1
Time	2	2	2		1
Diagrams					1
Statistics					1
History					1
Hand-held calculators	1	1	1	1	
Equations				1	
Total:	5	4	5	3	6

The arguments consist of short sentences and are mainly connected to every-day-life, for example: *When you are buying things in a store – a rough estimate is helpful if you want to find out how much the costs are.* More examples of these arguments can be found in the table below.

Table 2: Examples of arguments related to specific topics:

Topic	Example
Rough estimate	When you are buying things in a store – a rough estimate is helpful if you want to find out how much the costs are.
Rounding	Stores utilize rounding. If the total sum is 14.47 you must pay 14.50 because there are only whole and half crowns.
Time	If you want to know how long a trip will take – then you must know how to compute a difference in time.
Diagrams	The newspapers and the TV often use diagrams to illustrate facts and connections. Diagrams can also be used to illustrate a trip.
Statistics	Collected data can be more understandable if you compute the mean and the median.
History	A story about the historical development concerning mathematics.
Hand-held calculators	Hand-held calculators are used for solving practical problems in every-day life.
Equations	Solving equations is relevant mainly in physics and chemistry.

The analysis of the textbooks shows that arguments and explanations related to mathematical topics are few but also meager. So, if the teachers work very close to the textbooks, students might have less experiences concerning the role of mathematics in our society and the historical development of mathematics than the objectives in the national curriculum recommend.

Moreover, the analysis of the textbooks indicates that the new edition (from 2001) is rather comparable to the old editions (from 1979 and 1985). Special units with for instance problem solving and thematic work are added to the new edition so the number of pages is higher, but the number of exercises is, if we exclude these units, almost the same.

> This can imply that students are not working through the whole book and it has to be decided which part of the book they should leave out. This decision can be made by: (a) the teacher; (b) the individual student; (c) the student together with the teacher; or (d) the teachers of a school as a collective group. So even if the new edition of the textbook series investigated in this study is more varied with respect to suggestions for students' activities, it is easy to ignore the parts of the book dedicated to problem solving and other enrichments. Teachers could use the new book and teach in the same way as with the old one. Students can basically work with the same type of exercises as the students did in the beginning of the 80's (Johansson, 2003, p. 84).

Discussion

From the case study, one can clearly see that textbooks do not always and in a close way follow the guidelines of the intended curriculum. This implies that it is important to consider the textbooks when planning for a reform of the mathematics curriculum. But we cannot learn about the role of textbooks in mathematics education without taking their use into account. It is therefore important to gain more knowledge about the use of mathematics textbooks in classrooms. Not only *how much* textbooks are used in relation to other activities should be analyzed but also *how* and *why* they are used. Finally, the main elements in the classroom, the teachers and the students, must have the opportunity to reflect upon the characteristics of textbooks and how they use them.

References

Freeman, D. J., & Porter, A. C. (1989). Do textbooks dictate the content of mathematics instruction in elementary schools? *American Educational Research Journal, 26*(3), 403-421.

Johansson, M. (2003). *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum* (Licentiate thesis). Luleå: Department of Mathematics, Luleå University of Technology.

Love, E., & Pimm, D. (1996). 'This is so': a text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International handbook of mathematics education* (Vol. 1, pp. 371-409). Dordrecht: Kluwer.

Reys, R., Reys, B., Lapan, R., Holliday, G., & Wasman, D. (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education, 34*(1), 74-95.

Robitaille, D. F., Schmidt, W. H., Raizen, S. A., McKnight, C. C., Britton, E. D., & Nicol, C. (1993). *Curriculum frameworks for mathematics and science* (Vol. TIMSS Monograph No.1). Vancouver: Pacific Educational Press.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H., Wiley, D. E., Cogan, L. S., et al. (2001). *Why schools matter : a cross-national comparison of curriculum and learning*. San Francisco: Jossey-Bass.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims : a cross-national investigation of curricular intentions in school mathematics* (Vol. 1). Dordrecht: Kluwer.

Skolverket. (2001). *Syllabuses for the compulsory school*. Stockholm: Fritzes.

Skolverket. (2003). *Lusten att lära - med fokus på matematik* (No. 221). Stockholm: Statens skolverk.

Undvall, L., Olofsson, K.-G., & Forsberg, S. (1979a). *Matematikboken ak 7*. Stockholm: Esselte Studium.

Undvall, L., Olofsson, K.-G., & Forsberg, S. (1979b). *Matematikboken sk 7*. Stockholm: Esselte Studium.

Undvall, L., Olofsson, K.-G., & Forsberg, S. (1985a). *Matematikboken 7A*. Stockholm: Esselte Studium.

Undvall, L., Olofsson, K.-G., & Forsberg, S. (1985b). *Matematikboken 7S*. Stockholm: Esselte Studium.

Undvall, L., Olofsson, K.-G., & Forsberg, S. (2001). *Matematikboken X*. Stockholm: Liber.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the Book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht: Kluwer Academic Publishers.

van Dormolen, J. (1986). Textual analysis. In B. Christiansen, G. Howson & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 141-171). Dordrecht: D. Reidel Publishing Company.

Paper III

Textbooks as instruments

Three teachers' way to organize their mathematics lessons

MONICA JOHANSSON

This paper reports a study of three teachers way to organize their lessons and how textbooks are incorporated in their work. Despite the differences between the teachers it is noticeable that in these three classrooms, the textbooks, to a large degree, guide the teaching. The textbooks are present: (a) in the students' individual work, (b) in many of the examples presented on the board, (c) as a source for background and motivational discussions, (d) in how mathematics is presented, and (e) for homework.

1. INTRODUCTION

From previous research, we find that textbooks influence *what* to teach. If a mathematical topic is presented by the textbooks it is very likely that the teacher introduces it in the classroom. On the other hand, if a topic is not in the textbook, it is most likely not presented by the teacher (e.g. Freeman & Porter, 1989; Reys, Reys, Lapan, Holliday, & Wasman, 2003). Textbooks are also a primary information source in deciding *how* to present the content (e.g. Schmidt et al., 2001). The instructional approach of the material can even influence the teacher's pedagogical strategies (e.g. Reys et al., 2003). This study is an attempt to further analyze the influence of textbooks in mathematics classrooms, especially how it can be defined and discussed from a Swedish perspective.

The study focuses on the use of textbooks[i] by teachers. Under the assumption that a teacher uses a textbook because he or she has a more or less conscious idea that the book is important (cf. B. Englund, 1999), this paper discusses the *influence* of textbooks in classrooms. The research questions are:

- How do the teachers organize their teaching in terms of type of classroom interaction, organization of students, and content activity?
- When and how, direct or indirect, are the textbooks used in the different types of organization of teaching in the three classrooms?
- In what respect do the textbooks influence (or not influence) the mathematical work and how do the teachers highlight key ideas?

The three issues will be discussed in separate sections of this paper (3, 4 and 5 respectively).

The study is guided by a theoretical perspectives that is based on what Englund (1997) describes as the third stage of the frame factor theory[ii]. This means that the choice of educational content and contextualization of teaching is emphasized. A fundamental assumption is that students are offered different possibilities to create and construct meaning depending on, for example, what content is chosen and what context the textbook offers. In other words, different choices can be made, more or less consciously, which have crucial implications for teaching and learning.

The main purpose of the study is not to compare the three teachers' teaching methods or to make generalizations about mathematics teaching in Sweden. The study is not supposed to be normative either. It is not a criticism of teachers or textbooks. But rather, it is to describe and analyze some mathematics classrooms with the aim to reveal teachers' practices and relations to textbooks. The main purpose is to, mainly quantitatively, portray mathematics teaching in some Swedish classrooms.

2. METHODOLOGY

A study of Swedish classrooms, the CULT-project[iii], forms the empirical background for this paper. Three mathematics teachers were identified for their locally-defined 'teaching competence' and for their situation in demographically diverse government schools in major urban settings. Observation data is, in short, gathered using a three-camera-approach, with complementing wireless microphones, focusing teacher, class, and a group of students. Video-recorded classroom data were collected for at least ten consecutive mathematics lessons. Further data involves post-lesson video-stimulated interviews with the teachers and the students.

For this study, parts of the following types of the CULT-data are used: video-recorded lessons, teacher interviews, and teacher questionnaires. From the three teachers, Mr. Andersson, Mr. Svensson, and Mr. Larsson[iv] (labeled as SW1, SW2, and SW3 in the CULT-data), a total of thirteen lessons (679 minutes) are analyzed. The selected lessons have two common features, they are consecutive (with one exception), and the sequences start when the teachers introduce a new chapter in the textbooks. Five video-recorded lessons are chosen from Mr. Andersson's classroom: lessons 6, 7, 8, 10, and 11. In lesson 9, the students were occupied with a diagnostic test, which is the reason why this lesson is excluded for this study. Video-recorded lessons 4-7 are chosen from Mr. Svensson's classroom, and 2-5 from Mr. Larsson's classroom. Practical reasons, such as a lack of time, are behind the decision to not include all lessons in this study. Nevertheless, four lessons seem sufficient enough to expose variations and highlight interesting phenomena in each of the three teachers' classroom practices. Table 1 shows the length of each lesson and its label in parenthesis.

Teacher	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Mr. Andersson	60 min (SW1-L06)	42 min (SW1-L07)	38 min (SW1-L08)	60 min (SW1-L10)	64 min (SW1-L11)
Mr. Svensson	40 min (SW2-L04)	53 min (SW2-L05)	55 min (SW2-L06)	50 min (SW2-L07)	
Mr. Larsson	67 min (SW3-L02)	39 min (SW3-L03)	70 min (SW3-L04)	41 min (SW3-L05)	

Table 1: *Length of each lesson*

2.1. THE PARTICIPANTS

Mr. Andersson and his students (SW1)

Mr. Andersson is a bit more than thirty years old and has seven years of teaching experience, which he has gained at the school where the study is conducted. His subjects are Mathematics and Science and he has been teaching grade eight students for four years. The class, which consists of 26 grade eight students, is a mixed-ability group. The students are mainly working on an individual basis but every second or third week he organizes them to work in groups.

The mathematical content, which is treated during the period of video-recording for the CULT-project, concerns mathematical relationships (i.e. coordinate system, proportionality, and linear equations). It is, in most parts, a new subject area for the students. The teacher practice is *individual pace learning* (cf. Löwing, 2004). This means that students are working with tasks in the textbook[v] in their own pace. Mr. Anderson's idea of individualized teaching is indicated in an interview. When one of his students works faster and reaches further than the other, he offers her another textbook and asks her to work with the more demanding tasks in that book.

Mr. Andersson does not assign homework on a regular basis. He thinks it is difficult to go through the homework in a whole class setting, when the students are working at different speeds.

Mr. Svensson and his students (SW2)

Mr. Svensson is about sixty years old. He has been working on the current school almost all the time of his thirty-three years of employment as a teacher. The school practice is tuition in ability groups and the twenty-five ninth-grade students in his class are regarded as high-achievers. According to Mr. Svensson, the students in this class intend to be prepared for the Natural Science program, which they attend after finishing compulsory school.

The mathematical content that is treated during the period of video-recording for the CULT-project is, according to the teacher, partly new and partly repetition. It is about equations. About ten to fifteen lessons are what the teacher usually plans for each subject. Homework is assigned on a regular basis; in order to practice new skills, to establish a topic, and ending work that was started during the lesson, the teacher says.

The teacher has mixed feelings about the textbook, which is a book in one of the most common textbook series in grade seven to nine in the compulsory school in Sweden. Some tasks are quite good, he says, but the word problems are too unrealistic.

> Well, the equations as such, the ready-made, they are okay. They review algebraic knowledge and some other understanding also. But then, the word problems, some of them you can be without. If you are supposed to count the three consecutive even numbers (refers to a task in the textbook) … they are a bit 'non-realistic', I think. (translation to English by the author)

The students do not solve all tasks in the textbook. Some problems, for example the A-tasks[vi], are left out and some new are jointly constructed in the classroom.

Mr. Larsson and his students (SW3)

Mr. Larsson is about sixty years old. He has a long-standing experience of teaching and a long period of employment, more than thirty years of teaching, at the school where the study is conducted. Besides mathematics, he teaches Physics and Technology in grade eight and nine.

The textbook[vii], which is used in this particular class, is a book in the same textbook series as Mr. Svensson's. The teacher seems to adhere very closely to the textbook in the private as well as the public part of the lesson, even if he from time to time brings up examples from outside the book. In one of the interviews, he confirms the strong reliance on the book. He was asked why he uses concrete numbers to show the students how to simplify an expression. The teacher answered:

> It is generally so that I just follow the usual way to do it … this is normally how it is done in all books and I have not wondered about it so much, I think it is a system that works (translation to English by the author).

The class consists of 22 grade eight students. The school practice is tuition in ability groups and the students in this class are identified as high achievers. According to Mr. Larsson, they are quite homogeneous. He thinks about them as a hard working group that concentrates on mathematics. It is a good group, he says, nice and positive, sometimes a bit too chatty, but ambitious. When the teacher decides to give the students homework, which is not on regular basis, it is because he needs it for the grading or to gather the students for the next chapter in the textbook. As regards how the students work in the textbook, he seems to think it is important to keep them together. He also uses the thematic tasks in the textbook to gather them.

The mathematical content that is treated during the period of video-recording for the CULT-project is, according to the teacher, partly new and partly repetition. The chapter in the textbook has the title *Negative numbers, variables and expressions*.

2.2. THE CODING PROCEDURE

A coding procedure is used in order to capture sequences of the lessons that are of special interest for this study. These sequences are analyzed from three different perspectives: (1) the type of classroom interaction, organization of students, and content activity; (2) the use of textbooks, when and how, indirect or direct; and (3) the role of textbooks in different types of teacher activity. Two types of codes are used, coverage codes (see paragraph 2.2.1) and occurrence codes (see paragraph 2.2.2).

The coding procedure is in principle based on the coding manual of the TIMSS Video Study (Jacobs et al., 2003). There are however some differences. Before the description of each code, I will try to explain these differences. First and foremost, the TIMSS study focuses to a large extent on *Problems* while the main focus of this study is the use of textbooks. The concept *problem* is in TIMSS defined in the following way:

> Problems contain an explicit or implicit *Problem Statement* that includes an unknown aspect, something that must be determined by applying a mathematical operation, and they contain a *Target Result*. When a solution is checked, this is considered part of the problem (Jacobs et al., 2003).

For this study, the code *Problems and tasks* corresponds, in principal, to the definition of problems in the TIMSS study. The label is however changed for the reason that, in research of mathematics education, there are different ways to define the concept 'problem'. In order to avoid a restriction of the code or make the reader confused about its meaning, the word *tasks* is added. This means that all kind of mathematical tasks that students are working with during a lesson counts. Moreover, for the TIMSS, the *problems* are categorized in a different way. In this study, it is the *activity of the teacher* that is categorized and not *what kind* of problems the students are dealing with.

For the TIMSS study, the code *Mathematical generalization* is organized into three categories; procedural, definitional, and conceptual. Every mathematical generalization, in verbal or written form, is marked: MG1 for the first appearance, MG2 when it is repeated and so on. For this study, the marking of repetition has been excluded. Moreover, a category *Name/define*, which corresponds to the code *Labels and Symbols* in the TIMSS, is added. This implies that the code *Mathematical generalization or statements* also marks instances when the teacher points out the name or the symbolic representation of a mathematical concept or idea.

The code *Background – motivational* is based on the code *Historical background* in the TIMSS, though expanded to also include the use of a particular mathematical knowledge in the society, in other school subjects, or within (school-) mathematics. This code is especially relevant if one considers the syllabus (kursplanen) of mathematics for compulsory school in Sweden[viii].

In the TIMSS, there is a code, which refers to the direct and visible use of resources such as chalkboard, calculator or real world object. *Textbook or Worksheet* is one category within this code. However, for the perspective of this study, which focuses on the use (and not use) of textbooks, it is not sufficient. The code *Textbook influence* is therefore added. It involves direct and indirect use but also 'absence' of a textbook, for example when the teacher uses other resources such as real world objects.

2.2.1. Coverage codes

The coverage codes are used to code a lesson, or a defined period of a lesson, in its entirety. A coverage code has at least two mutually exclusive and exhaustive options. Thus, only one of these options is applied to each defined period in the lesson. The following graphs show the coverage codes and their categories.

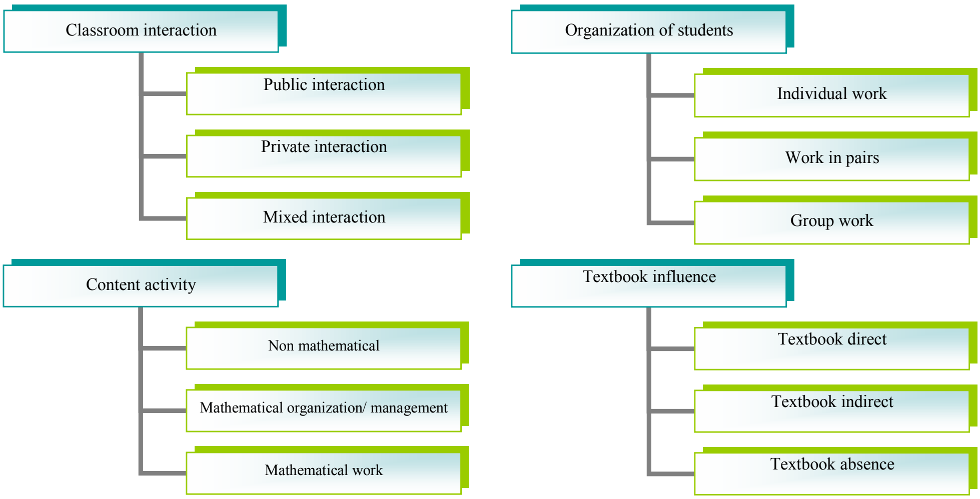


Figure 1: *Coverage codes*

Classroom interaction: *Classroom interaction* is a coverage code for all parts of the lessons. This means that all points in the lessons are coded as one of the three, mutually exclusive categories: *Entirely public interaction, Entirely private interaction,* and *Mixed interaction*. There may however be brief periods, less than one minute, in between two segments that are not coded according to the categorization. This can happen, for example, in a *public interaction* segment if the teacher asks the students to complete a small task (private work) and then return to a public dialogue.

Entirely public interaction: There is a public dialog directed by the teacher or one or more students. All students are supposed to participate or listen, but their contribution may, however, be minimal. Talk may or may not be accompanied by written information.

Entirely private interaction: All students are working in their seats. Students may discuss problems with one another. The teacher may assist individual students or small groups of students, either verbally or both verbally and in writing.

Mixed interaction: The teacher or a student presents information in public, in either verbal or written form. Students can choose to pay attention to it or ignore, but there must be a clear signal from the teacher that student attention is optional.

ORGANIZATION OF STUDENTS: *Organization of students* is a coverage code for all parts of the lesson that are coded as periods of *Entirely private interaction* or *Mixed interaction***.** How the students are working, individually, in pairs or in groups, is categorized in order to describe student organization and cooperation during the 'non-public' part of the lesson.

CONTENT ACTIVITY: *Content activity* is a coverage code for all parts of the lesson. This means that all points in the lessons are coded as one of the three, mutually exclusive categories: *Non mathematical work, Mathematical organization,* and *Mathematical work*. There may however be brief periods, less than 30 seconds, in between two segments that are not coded according to the categorization.

Non mathematical work: Parts of the lesson when there is no mathematical activity going on. For example: announcements about school activities; interruptions by someone outside of the class requesting the teacher's attention; discussions by the teacher of non-mathematical events (e.g., the music concert the night before); disciplinary actions by the teacher in response to students' misbehavior. Note that the teacher is the focus so if the students are doing mathematical work but the teacher is not – it still counts as a non-mathematical activity.

Mathematical organization/management: This part of the lesson includes references to mathematics (e.g., mathematics tools, resources, homework, tests), but does not entail a mathematical content. It involves, for example, general organizational descriptions of a future test or a quiz but not issues specific to a mathematical problem. A frequent activity that is coded as *Mathematics organization/management* is when the teacher tells the students which pages in the textbook to work with.

Mathematical work: The teacher is supposed to engage in some kind of mathematical activity. In order to be counted as a mathematical activity it is not necessary that there is a clear mathematical content. A sufficient condition is that the activity cannot be coded as one of the other two categories (*Non-mathematical* or *Mathematics organization/management*). When the teacher, for example, walks around the classroom, observing students, which are working with tasks in the textbook, or tries to motivate them to work, it counts as mathematical work.

TEXTBOOK INFLUENCE: Whenever a textbook is explicitly or implicitly used in the classroom one can think about it as an influential factor. If the textbook really *influences* the teaching is a matter of believes and values of the teacher and the students. Thus, the actual influence of a textbook is based on a more or less conscious idea that the book is important (B. Englund, 1999). For this study, under the assumption that teachers use textbooks because they think they are important, the code *Textbook influence* marks the role of the textbook in segments of the lessons. *Textbook influence* is a coverage code for all parts of the lessons that are coded as periods of *Mathematics organization/management* and *Mathematical work.* This means that all points in these segments of the lessons are coded as one of the three, mutually exclusive categories: *Textbook direct, Textbook indirect,* and *Textbook absence*. There may, however, be brief periods, less than 30 second, in between two segments that are not coded according to the categorization. Note that all material connected to the textbook (e.g. the teacher's manual) count as the textbook.

Textbook direct: There is an open and explicit use of the textbook: the students are working individually or in groups with exercises from the textbook; the teacher makes, explicitly, comments about a text, a problem, or a picture in the textbook; or the teacher reads directly from the textbook.

Textbook indirect: The teacher makes explicit verbal or written statements that are parallel and comparable with the text in the textbook without referring to it. Examples: the teacher explains a graph without declaring that there is a similar graph in the book; the teacher shows a worked example on the board that is similar or exactly as an example in the textbook; the teacher talks about mathematical aspects (generalization/statements) in the same way as the textbook.

Textbook absence: There is a clear difference between how the teacher introduces, explains, draws, or comments on a mathematical subject and how it is presented in the textbook. Examples: the teacher makes connection to other mathematical areas but the textbook does not show this link; the teacher makes connections to every day life or applications, which are not in the textbook.

2.2.2. Occurrence codes - Teacher activity

Occurrence codes are used in order to highlight *how many times* and *where* a specific event occurs within a particular lesson. Since the focus is on the teacher it is the activity of the teacher that counts. An occurrence code can be applied several times within a lesson but it can also be the case that there is no event to apply to. *Teacher activity* includes eight occurrence codes for all parts of the lessons that are coded as *Mathematical work*: (1) Problems and tasks, (2) Assignment of homework, (3) Assessment, (4) Mathematical generalization or statements, (5) Link to lesson, (6) Goal statement, (7) Background/motivational, and (8) Summary of lesson. These categories are marked each time they occur. The following graph shows the occurrence codes and their categories.

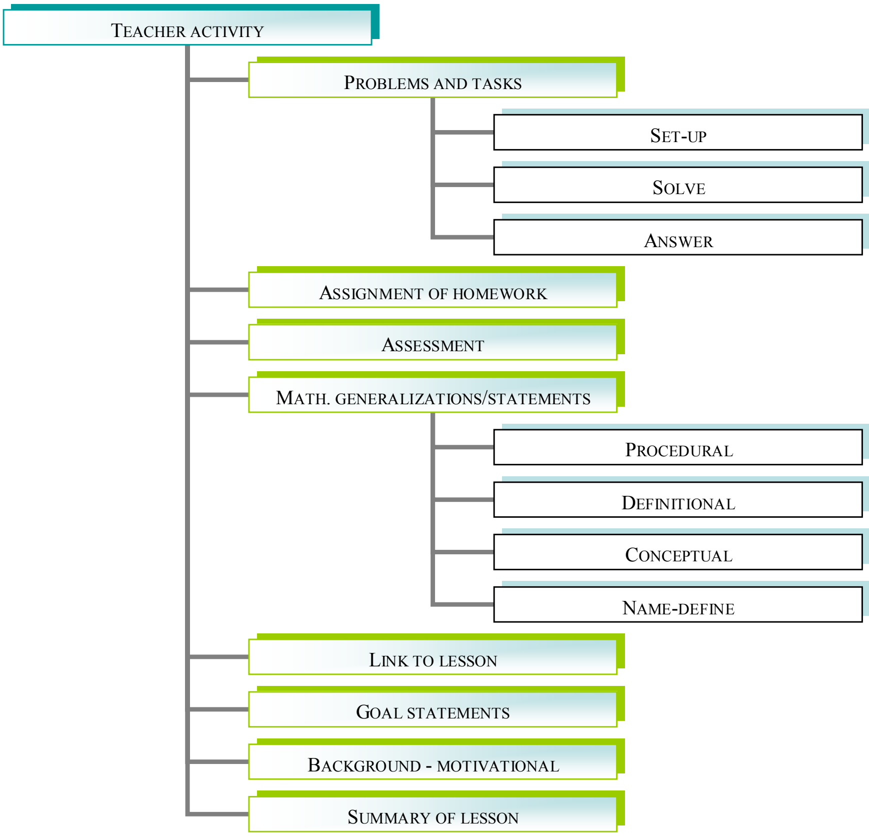


Figure 2: *Occurrence codes*

Problems and tasks: If the code *Problems and tasks* is marked, it means that there is an interaction, public or private, between the teacher and the students. The discussion concerns a mathematical problem, for example a task in the textbook. The activity of the teacher is categorized into three types of interaction: Set-up, Solve, and Answer.

Set-up: The teacher presents the problem in writing or orally, for example by reading the problem text. It could also be that the teacher tries to motivate or push the student to start working with the problem/task.

Solve: The teacher solves the problem/task on the chalkboard or on a paper in public or for an individual student or group of students. The teacher "funnels" or "scaffolds" a student until s/he reaches or comes closer to the solution of a task.

Answer: The teacher gives or checks an answer.

Assignment of homework: The teacher assigns homework for the students to complete after the lesson ends.

Assessment: The teacher checks or marks a test/diagnosis or a worksheet.

Mathematical generalizations or statements: There are two conditions that must be evident for an occurrence to be coded as *Mathematical generalizations*. There must be generalized mathematical information and there must be an explicit attempt to point out the generality. Mathematical generalizations are marked for each explicit statement, verbally or written. An example is when the teacher says that *the angles of a square always add up to 360 degrees*. Exception: if the statement is made during an individual guidance as a support in a problem-solving sequence and the generality is a part of the solution-strategy. There are four categories of mathematical generalizations: *Procedural, Definitional, Conceptual,* and *Name-define.*

> **Procedural** mathematical generalization is marked each time the teacher describes a general solution procedure used for a class of problems. For example: *the first step to solve an equation is to simplify, then you collect the variables on the side where most of them are and the constant terms on the other side of the equal sign, the last step is to isolate x*.
>
> **Definitional** mathematical generalization is marked for each statement that can be regarded as a traditional or an accepted mathematical definition, convention or rule. Examples: *2x is the same as 2·x and x is the same as 1·x; (a,b) is a point in a coordinate system where a and b denote its position in relation to the x-axis and the y-axis, respectively.*
>
> **Conceptual** mathematical generalization is marked each time the teacher describes the conceptual or structural nature of mathematics. Example: *these two lines are parallel because they have the same slope.*
>
> **Name – define** is marked when an explicit attempt is made to use precise mathematical language to point out the "name" given to the particular mathematical idea/concept or its symbolic representation. For example: *the point where the two axes intersect is called the origin.*

Link to lesson: There is an explicit verbal reference by the teacher that connects particular mathematical ideas discussed or worked on within the current or a different lesson. The link should help students organize related information. For example: *now we are working with expressions and formulae and since you learn about sound in Physics this fall we shall look at the formula of the sound wave.*

GOAL STATEMENTS: Explicit verbal or written statements made by the teacher about the mathematical topic, which will be covered in the specific lesson.
BACKGROUND/MOTIVATIONAL: The teacher connects the mathematical content to its historical background (e.g., Pythagoras as the originator of a mathematical theorem). The teacher connects mathematical content to its practical use in or outside the school context.
SUMMARY OF LESSON: The mathematical content of the current lesson is summarized by the teacher. The statements refer to work that has been completed during the lesson, or describe the main point of the lesson.

2.3. METHODOLOGICAL DISCUSSION

To use video-recorded classroom lessons as a basis for educational research can be a solution for some of the problems that a researcher encounters in the complex milieu of a classroom. An advantage is, for example, that it is possible to analyze and re-analyze the data from different perspectives. However, the methodological approach, which of course is guided by the objectives of a certain study, defines the range of possible perspectives. One of the objectives in the TIMSS Video Study was "To develop objective, observational measures of classroom instruction to serve as appropriate quantitative indicators of teaching practices in each country" (Hiebert et al., 2003, p. 1). It included more than a hundred schools from each of the seven participating countries. In contrast, the CULT project, which is based on the research design set out for the Learner's Perspective Study[iii], has a "methodological approach that offers an informative complement to the survey-style approach of the TIMSS video study" (Clarke & Mesiti, 2003, p. 2). In the Learner's Perspective Study, the data involves video-recordings from at least ten consecutive lessons from each teacher in the participating countries. Thus it has potential to address consistency as well as degree of variation in lesson structure (Clarke & Mesiti, 2003).

Since this study is based on the data of the CULT-project, there are some issues that have to be clarified. First of all, I am not a member of the research team and I had no part in the planning of the project or the data gathering. Following the guiding principle for the project, for example protecting the anonymity of the participants, I had permission to use the data for this study. However, the responsibility for the methodological approach (the coding procedure), the results from the analysis, and the conclusions presented in this paper are mine.

Coding reliability is measured by a method, percentage agreement, which was also used in the TIMSS[ix]. A re-coding of one of the thirteen lessons was made one year after it was coded for the first time. The reliability score is calculated by dividing the number of agreements by the number of agreements plus disagreements. All codes included, the score is 95,8 %. Because of technical problem with the software[x], it was difficult to mark in- and out-point, i.e. exact start and end of each segment. In order to control that the distribution of the coverage code is acceptably correct, the time differences are also measured. On average, the coverage codes are identically coded to 96,2 %. For the occurrence codes, the results show two different pictures. Looking at the eight codes, the reliability score is 95,2 %, but if the subcategories are included, the reliability score is only 69,6 %. This implies that the subcategories for the codes *Mathematical generalizations* and *Problems and tasks* are not coded in a satisfactory way. A closer look at these two subcategories reveals the reason for this. The main problem is related to the category *Problems and task* and the difficulties to distinguish between the level *Set-up* and *Solve*. For that reason, the subcategories are not considered in the analysis of the data. Thus, only the reliable parts of the coding procedure are used for the study.

In this paper, most of the transcripts, and all translations from Swedish to English of transcripts, are made by the author. For all possible errors, the author takes full responsibility. One remark, however, is in place: The text should be seen as a report of the conversation in the classroom. Thus, making an allowance for readability, it is not a word for word description of what the teacher and the students say in the classroom.

3. THE ORGANIZATION OF THE LESSONS

In this section of the paper, the teachers' way to organize their teaching in terms of type of classroom interaction, organization of students, and content activity will be discussed. In the analysis of the thirteen video-recorded lessons, one can observe that there are in principle two types of interaction in the classrooms, private and public. A lesson normally starts with a public part. The teacher stands in front of the class. He writes on the board, presents problems, poses questions and verifies or disproves answers.

During the private-work part of the lesson, the students, at least most of them, are engaged in 'practice'. This means that they are working, mainly on an individual basis, with tasks in the textbook. The teacher walks around the classroom, he observes and interacts with the students.

A third type of interaction, mixed interaction, is observed in Mr. Svensson's lessons. These are occasions when a student stands in front of the class, writing a solution on the board, and the other students' participation is optional. Table 2 shows the average distribution of private, public and mixed interaction. The teachers' way to divide their lesson time between private and public interaction is rather similar. In all three teachers' classrooms, private interaction is more common than public interaction[xi].

Classroom interaction	Mr. Andersson (SW1)	Mr. Svensson (SW2)	Mr. Larsson (SW3)
Public interaction	35,86 %	47,77 %	44,97 %
Private interaction	64,14 %	48,56 %	55,03 %
Mixed interaction	0,00 %	3,67 %	0,00 %

Table 2: *Classroom interaction, percentage of total time of lessons*

Figure 3 displays the distribution of private and public interaction for all thirteen lessons. It shows that the teachers' way to interact with the students varies from lesson to lesson. Mr. Andersson's lessons four and five (labeled SW1-L10 and SW1-L11), for example, show notable differences. About eight percent of lesson four is devoted to public interaction in comparison to about fifty-five percent of lesson five.

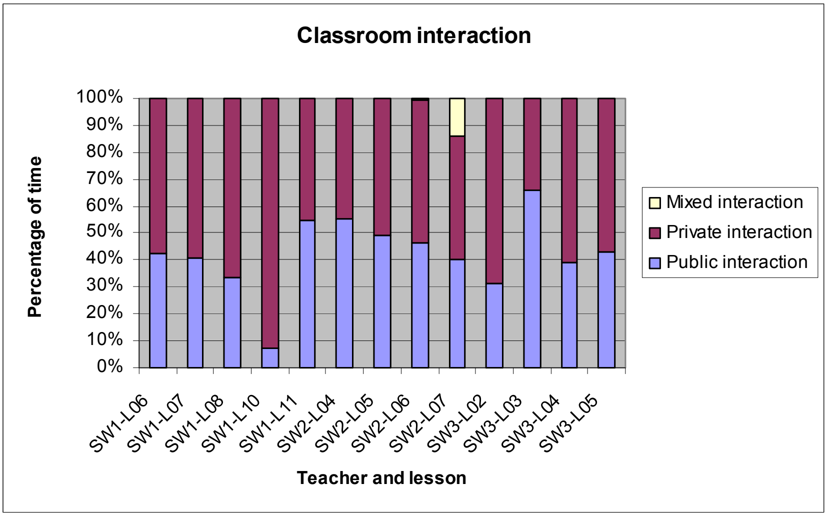


Figure 3: *Classroom interaction in all thirteen lessons***.**

The tables and the diagrams, however, do not display the whole picture of the interaction in the three teachers' classrooms. Two teachers can spend the same amount of lesson time in, for example, a public-work setting. Still, the interaction is of different type if the teacher alters between public work and private work several times in a lesson or distributes it in two separate parts of the lesson. A different way to describe a lesson is through an illustration of how it is subdivided, for example showing how they shift between interaction types.

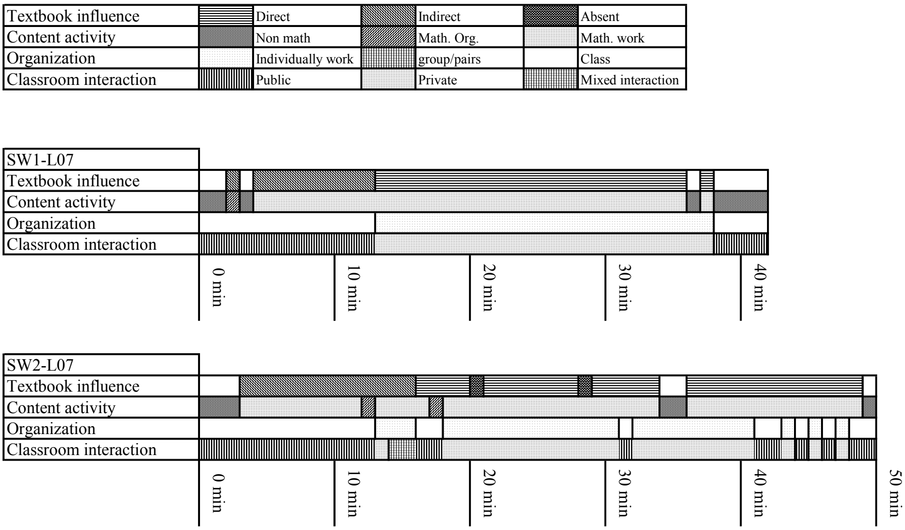


Figure 4: *Sequences of lessons*

Figure 4 makes it possible to compare two lessons with similar proportion of public interaction. It shows that Mr. Andersson (labeled SW1-L07) starts with a public interaction for about 13 minutes and then continues with private interaction the remaining part of the lesson. The last four minutes, which actually is a non-mathematical-work part of the lesson, is also in a whole class setting. For Mr. Andersson, and also Mr. Larsson, this lesson is representative regarding how the interaction progresses during the lessons, first a public part and then private work. The lesson labeled SW2-L07 is representative for Mr. Svensson who, on the other hand, alters between types of classroom interaction several times in his lessons. This means that the students in his class are, in principal, kept together as regards to their work during the whole lessons. Another difference between Mr. Svensson and the other two teachers is that he sometimes chose to let a student write the solution on the board, which in this case is marked as *Mixed interaction* in the lesson marked SW2-L07.

In all three classrooms, most of the students are working by themselves during private work time. There were no occasions of group work throughout the thirteen lessons in this study. Some of the students seem, however, to work in pairs from time to time, but this is optional and not incited by the teacher. Hence, individual work is even more common in this study than was found in the TIMSS Video Study. Among the participating countries of the TIMSS, on average, between 73 and 95 percent of private work time involved students completing tasks individually (Hiebert et al., 2003). The findings of this study show, however, not the whole picture. At least if one takes into account what the teachers, they themselves, state. In one of the interviews, Mr. Andersson (SW1) declares that he organizes group work every second or third week and Mr. Svensson (SW2) states, in a questionnaire, that most of the students work in pairs.

With regards to *Content activity*, Figure 5 displays the percentage of time devoted to mathematical work, mathematical organization, and non-mathematical work for each lesson. From the picture we can read that the three teachers spend most of their lesson time on mathematical work and that there are no big differences between them.

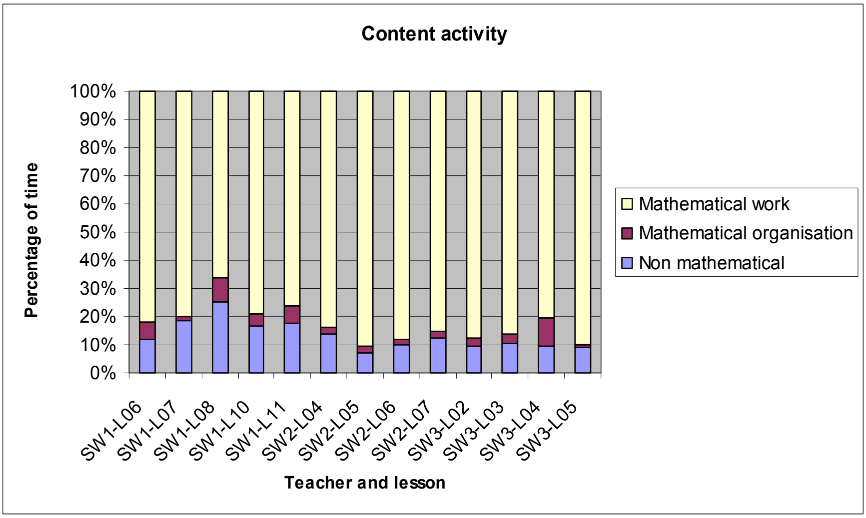


Figure 5: *Content activity in all thirteen lessons.*

Table 3 below, displays the average percentage of time devoted to mathematical work, mathematical organization, and non-mathematical work for each teacher. Still, the diagram and the table does not give an idea about *when*, in the lesson, each type of activity is offered. But a closer look at the sequence of each of the lessons, as in Figure 4 and Appendix C, reveals that segments of non-mathematical work commonly appear at the beginning or at the end of the lessons.

Content activity	Mr. Andersson (SW1)	Mr. Svensson (SW2)	Mr. Larsson (SW3)
Non mathematical	17,93%	10,77%	9,62%
Mathematical organization	5,44%	2,29%	4,39%
Mathematical work	76,62%	86,94%	85,99%

Table 3: *Content activity, percentage of total time of lessons*

Mr. Andersson seems to use more time on non-mathematical work than the other two teachers. From the video-recordings of his lessons, one finds that he more often is involved in disciplinary discussions with students but also conversations that can be regarded as ‘from one friend to another’.

In comparison to the results from the TIMSS Video Study, where at least 95 percent of eighth-graders’ lesson time consists of mathematical work, it seems like the teachers in this study spend less time on mathematics (Hiebert et al., 2003). However, the non-mathematical segments of the teachers, Mr. Svensson and Mr. Larsson, take almost always place in the beginning or end of a lesson. Thus, a slightly different way to define the start and the end of the lesson has consequences for the result. While the TIMSS coders mark the start when the teacher says ‘welcome’ or actually starts the teaching, the start in this study is marked when the door opens and the students take their seats. Another deviation from the TIMSS is that the focus in this study is on the teachers. So, even if the majority of the students are working with mathematics but the teacher is not, the segment will be coded as a non-mathematical activity.

4. THE USE OF TEXTBOOKS

In all lessons, there is an extensive use of textbooks, especially if one looks at the private-work part of the lessons. The textbooks are in direct use about sixty percent of the time. For Mr. Andersson and Mr. Svensson, the textbook is definitely the main resource. There are very few occasions when the textbook influence seems to be 'absent'[xii]. For Mr. Larsson, the picture is slightly different. He uses other sources than the textbook in about one fifth (18,40 %) of the time. Table 4 shows the average distribution of *Textbook influence* for the three teachers, measured during lesson periods of *Mathematical work* and *Mathematical organizations*.

Textbook influence	Mr. Andersson (SW1)	Mr. Svensson (SW2)	Mr. Larsson (SW3)
Textbook direct	57,55%	61,67%	57,54%
Textbook indirect	23,92%	24,41%	14,44%
Textbook absence	0,60%	3,15%	18,40%

Table 4: *Textbook influence, percentage of total time of lessons (non-mathematical work is excluded)*

The use of textbooks differs from lesson to lesson for the same teacher. The following graph (Figure 6) shows, for example, that the textbook has direct influence on the mathematical work almost all of the time in one of Mr. Andersson's lessons (the fifth lesson – labeled as SW1-L10). Mr. Larsson, on the other hand, devotes almost forty percent of one of his lessons to non-textbook work (labeled as SW3-L03).

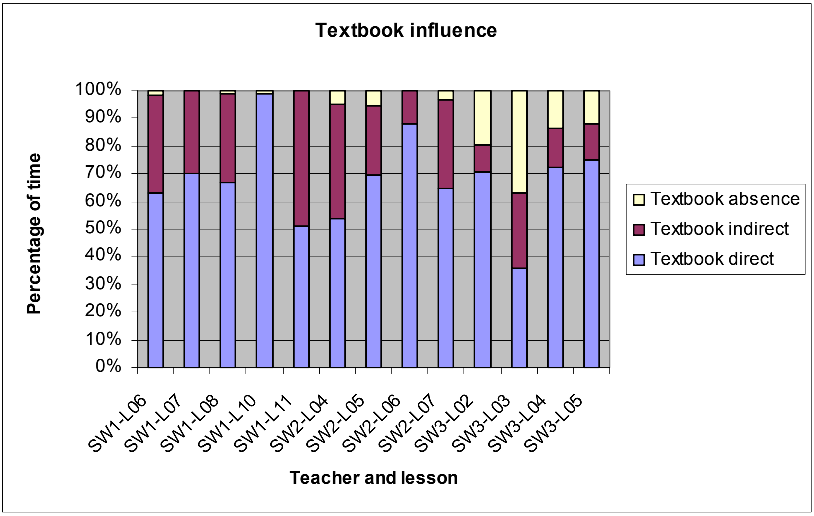


Figure 6: *The influence of the textbook, percentage of time of each lesson (non-mathematical work is excluded).*

In all three classrooms, there were few occasions of non-textbook tasks and the mathematical content is to a large extent introduced and elaborated via the textbooks. But what kind of mathematical activity is offered when the teacher teaches without the textbook? The following example is from a lesson of Mr. Larsson (SW3-L03).

Example of a non-textbook task: The teacher presents an information sheet from the municipalities. It shows the price of water and drainage supply. The formula $A \cdot x + B \cdot \sqrt{x}$ describes the charge. At the request of the teacher, one of the students, Jonas, reads what it says while the teacher writes on the board (see Transcript 2 in Appendix A). The teacher points at the *x* on the board and says that *x* stands for consumption in cubic meter. He explains that *x* is a variable and that *A* and *B* are constants. Mr. Larsson also asks the student if they know how much a cubic meter is and suggests that they should think about it as a big dice, one times one times one meter (see Transcript 3 in Appendix A).

As regards the information sheet from the municipality, the teacher explains the reason for writing a formula like this. It is because the municipality can change the price next year, if more money is needed, just by changing the value of *A* and *B*, he says. In the information sheet, the numbers corresponding to *A* and *B* are quite uneven (A = 8,80 and B = 47,60) so the teacher rounds up and writes $A=10$ and $B=50$ on the board in order to make it easier to calculate. After that, he asks about different costs for different kind of households.

> Teacher: Let us look into some examples of the effect of this. We start with a very low consumption then (writes *a)* $25m^2$ on the board) … no, not square meter … cube meter of course (change to m^3 on the board). We start, like this, by saying that low consumption … one lonely person that seldom takes a shower and doesn't have a washing machine. What do we get here then?

One of the students, Jonas, solves the equation: "ten times twenty-five plus fifty times twenty-five … eh … square root of twenty-five … is equal five-hundred". The teacher writes the solution on the board and presents a second example, a person that is a bit more wasteful with water and spends a hundred cubic meters. For this person's water and drainage supply, the cost is calculated to 1,500 kr (see Transcript 4 in Appendix A). The last example concerns the costs for a household that uses 400 m^3. The teacher, together with the students, calculates the costs and comes up with the result that the fee will be 5,000 kr.

On the board, there are now three examples of costs for water and drainage supply for three different types of consumers, a lonely person that consumes little amount of water, a lonely person that consume a larger amount of water, and a family. The teacher suggests that they could compare the costs. First the teacher asks how many times bigger the consumption is for the person that consumes a hundred cubic meters of water in comparison to the person that uses twenty-five. One of the students says it will be four times bigger. Thereafter, the teacher asks about how many times bigger the fee will be. Three times, another student says. “What should it be, if it would be really fair?”, the teacher asks. One of the students suggests that it should have been 2,000 kr instead of 1,500 kr. The teacher goes on, comparing a person that uses four hundred cubic meters waters, just as a household he says, with a person that uses a hundred. One of the students concludes that it should have been 6,000 kr instead of 5,000 kr. The teacher asks the class if they know why the municipality makes the price like this. “Because they want people to use as much water as possible”, is one suggestion from the students. The teacher replies:

> Teacher: Well, one can say that it is some kind of quantity discount here, it is, and one can understand that there is an idea behind it. Because there are costs for the municipality to keep the water sewage pipes in good shape. They sometimes break and then you have to dig and replace and so on. And if you only get a small amount of money, then it is not good. And at the same time there are also costs for reading off the water meter and send out bills and so on.

In this part of the lesson, which is coded as *Textbook absence,* the teacher uses a resource that is regarded as a real world object. It is a task derived from an information sheet, which the teacher solves together with the students. The episode entails a repetition of some known concepts, square root and cubic meter, but also a concrete example of how constants and variables work in an equation and affect the results. There is a task in the textbook (see Figure 7), which in some sense concerns the same topic. The differences between these two tasks are left to the reader to think about. I could offer one inference, and that is that the textbook task is less related to the use of mathematics in the ‘real world’[viii].

5078
The cost for water supply in a private house per year can be calculated with the formula
$K = 295 + 16x + 6{,}5y$.

K = the cost in kronor
x = the amount of consumed water in cubic meter (m^3)
y = the amount of consumed cold water in cubic meter

a) How much is the cost for a family in one year when the family consumes 60 m^3 warm water and 200 m^3 cold water?
b) What do the numbers 16 and 6,5 stand for in the formula?

Figure 7: *A textbook task* (Undvall et al., 2002, p. 219, translation by the author).

5. THE ROLE OF THE TEXTBOOKS

The activity of the teachers is indicated by means of occurrence codes. Thus, the data from the coding procedure describes number of occasions when a certain activity is noted (see Appendix B). The length of each lesson is therefore relevant for the interpretation of the data. Table 1 (on page 3) shows the length of each lesson.. The role of the textbooks, in the activity of the teachers, will be discussed in this part of the paper. First, how the teachers arrange for mathematical work (section 5.1), and thereafter how they highlight key-ideas (section 5.2).

5.1. TO ARRANGE FOR MATHEMATICAL WORK

In this study, as well as in the TIMSS Video Study (Hiebert et al., 2003), teachers and students spend a considerable part of each mathematics lesson solving problems or tasks. But mathematical work can also involve assessment and homework. This part of the paper reports how the teachers arrange for mathematical work and the way textbooks are incorporated.

Problems and tasks are offered in two different settings, in the public part of the lesson when the teacher (or sometimes a student) stands at the board in front of the class, and in the private part of the lesson when the teacher walks around in the classroom and gives individual assistance. In the latter type of interaction, during private-work, the problems and the tasks origin from one exclusive source: the textbook. In the public part of the lesson there are a few occasions, less than one percent, when the task seems to be derived from a different source.

Considering the number of occasions that involve problems and tasks, Mr. Andersson (SW1) seems to be most busy among the three teachers in this study. Figure 8 displays, however, that this varies between lessons. The first column shows, for example, that the teacher was involved in forty-seven *Problems and tasks* situations during the lesson. In three of his other lessons (L07, L08, and L11), the number of occurrences is between fifteen and eighteen. Mr. Svensson (SW2) is, on average, involved in nineteen *Problems and tasks* situations. Mr. Larsson (SW3) shows a slightly different pattern. In two of his lessons (L02 and L04) he is engaged in thirty-two and thirty occasions, but in the other two, the number of occurrences is nine and nineteen. A plausible explanation is the length of the lessons, every second of Mr. Larsson's lessons are about seventy minutes and every second are about forty minutes (see Table 1 on page 2).

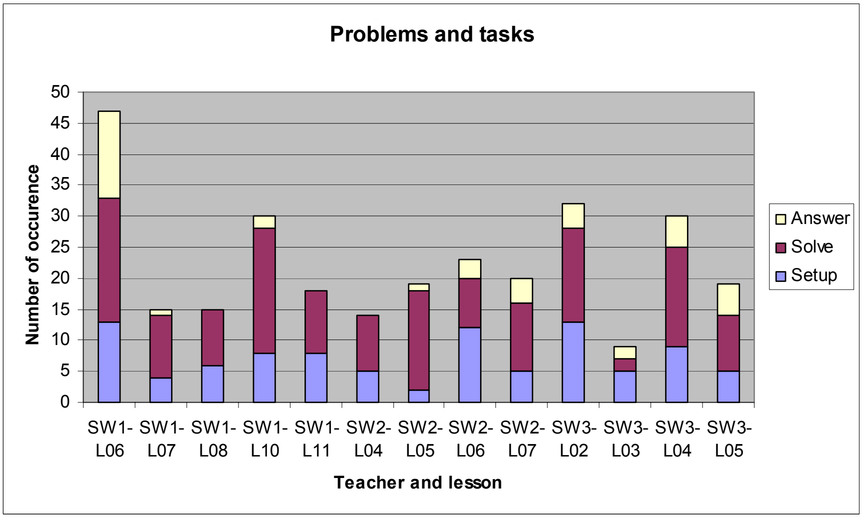


Figure 8: *Problems and tasks*

In order to illustrate how busy a teacher can be during a private-work part of the lesson, I will give you two concrete examples. The first concerns how many *Problems and tasks* situations the teacher is involved in. In the first lesson (SW1-L06), Mr. Andersson assists students' problem solving processes thirty-three times in a period of thirty-four minutes of private work. On average, this would be about one minute per occasion. The second example is also from Mr. Andersson's lessons. He is teaching a mixed-ability group of students, which means that the students are working at a different pace. The whole class begins a new chapter in the textbook together (L06) but in the fourth lesson (L10) they are spread out. As a consequence, the teacher assists students that are working on problems from task 7 to task 40 in the textbook, which makes it difficult to give instructions to the whole class in a meaningful way (cf. Löwing, 2004). In this respect, one can wonder why he, despite of the range, which the students work within, chooses to teach in a whole class setting, i.e. *Public interaction*, in the fifth lesson (L11).

In the thirteen lessons included in this study, no time is spent on assessments or diagnostic tests. This does not mean that there are no occasions of assessment in these classrooms. First of all, one has to consider the kind of indefinable assessment that could be a part of the teachers' performance in the teacher-student interaction. Secondly, Mr. Andersson uses one whole lesson, lesson 9 in the CULT-study, for a test. For this study, however, this lesson was excluded, mainly because there was not much of teacher activity.

Only one of the teachers, Mr. Svensson (SW2), claims that he assigns homework on regular basis. However, during the four lessons included in this study he does not tell the students to do homework. During all thirteen lessons, there are two occasions when the students get homework. One occasion occurs in a lesson of Mr. Andersson and one occasion occurs in a lesson of Mr. Larsson. Both times, the assignments concern tasks in the textbooks. Mr. Andersson tells the students that if they have not completed the tasks till page number 177, they should work with it at home. Mr. Larsson writes *Homework: 5090-5096* on the board[xiii].

5.2. TO HIGHLIGHT KEY IDEAS

How can a teacher help students to identify key mathematical points in a lesson? To verbally, or in writings, offer mathematical generalizations or statements is of course one way to do it. It can also be helpful to explain the goals for the current lesson or connect to a different lesson, in mathematics or another school subject. An additional approach is to talk about the historical background of a mathematical concept or the use of a specific mathematical knowledge in everyday life, the society or within school. In this subsection of the paper, I will try to illustrate the three teachers' ways to highlight key ideas and how the textbook is involved, or not involved, in this.

In all thirteen lessons, 119 occasions of *Mathematical generalizations or statements* have been identified. Most of them, eighty-one, are found in the public part of the lessons. Only nineteen of them are recognized as different from the textbook. This implies that the textbooks, to a large extent, influence *how* mathematics is presented in terms of mathematical procedures and concepts.

Figure 9, shows how often the code is marked in each lesson. Mr. Andersson (SW1) seems to offer mathematical generalizations more often than the other two teachers, especially if one considers the first lesson. One plausible reason could be that the specific chapter, which is about 'relationship' (i.e. coordinate system, diagram, and linear relationships), involves many, for the students, new concepts and terms. The subjects in the other teachers' lessons are partly new and partly repetition for the students.

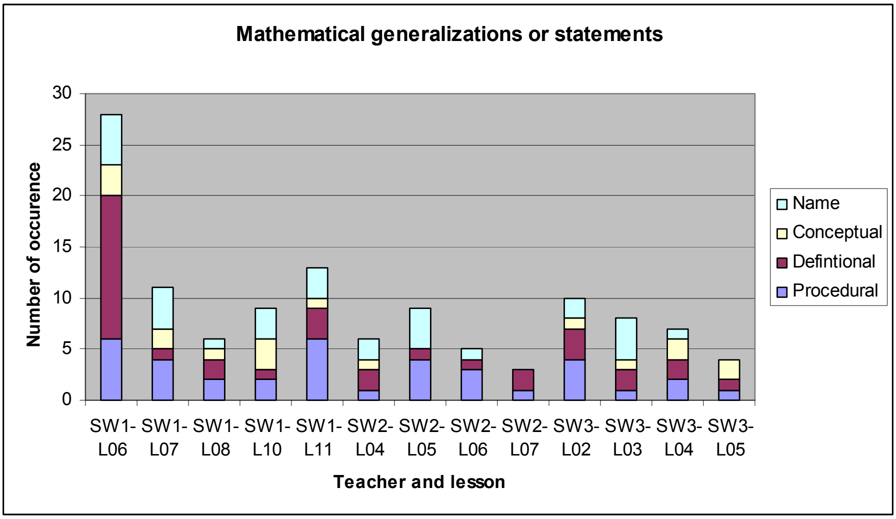


Figure 9: *Mathematical generalizations or statements, number of occasions.*

Mathematics is a subject that can be related to other school subjects such as Physics, Chemistry, and Social science. Links can also be made between different topics within mathematics. *Link to lesson* is marked each time the teachers make connections to other lessons, in mathematics or other school subjects. Fourteen occasions are recognized in all thirteen lessons. Some of them are rather vague links to previous or future lessons and some of them refer to Physics lessons.

The following example, which includes a task that deviates from the textbook, illustrates a rather clear link to Physics. It is an episode in one of Mr. Larsson's lessons (labeled SW3-L04) and it is about sound waves.

Example of a link to a lesson in Physics: The teacher starts by explaining that sound is a wave movement and that there is a formula connected to this. He writes $\lambda = \frac{v}{f}$ on the board. After a discussion about the different parts of the formula, he holds up a tuning-fork and hits it with a pen. This particular tuning-fork has the frequency 1700 Hertz, he explains. In cooperation with the students, the teacher calculates the wavelength to twenty centimeters (see Transcript 5 in Appendix A).

A difference between the teachers in this study concerning non-textbook work (see Figure 6), which is discussed in section 4 in this paper, could be explained by the fact that Mr. Larsson can rely on his knowledge as a teacher in Physics when he presents examples. As regards to the current topic in his lessons, which is about formulae and equations, this is especially suitable. However, Mr. Larsson makes a choice that could be discussed. In this study, he always presents these examples in the public part of the lesson and not as tasks for the students to work with, individually or in groups. It would certainly be a different experience if the students, by themselves, work with this type of activity. I am not saying that it would be better, but it would be different.

Concerning goal statements, there is also a difference between the teachers in this study. Mr. Svensson (SW2) presents the goal each lesson[xiv]. Mr. Andersson (SW1) and Mr. Larsson (SW3) presents the goal in two (of five) and one (of four) lessons, respectively. There are three types of goal statements, which emphasizes: (a) the subject: "today we are going to work with … which is about …", (b) the target: "this work will lead to …", or (c) what is most important.

Goal statements can also be found in the textbooks, often in the beginning of a chapter. They are of different kind and more or less explicit. In the textbook that Mr. Andersson uses, the text, which describes the goals, is placed in a framed textbox on the first opening of the chapter:

Goal
After you have studied this chapter, you should be able to:

- draw and identify points in a coordinate system
- use proportional relationships, i.e. cost-per-unit prices
- use relationships that consists of a fixed and a flexible part
- interpret different types of linear relationship

Figure 10: *Goal statement in the textbook* (Carlsson et al., 2002, p. 171, translation by the author).

The example above shows the kind of learning goals that the authors had in mind when they made this particular chapter. There are goal statements in the other two textbooks as well, not framed in textboxes though. For example:

> In this introductory part, we repeat how to solve equations. In the next part, you practice your skills on different problems (Undvall, Olofsson, & Forsberg, 2003, p. 96, translation by the author).

Concerning the aims for teaching mathematics, the following citation is from the mathematics syllabus for compulsory school:

> Mathematics is an important part of our culture and the education should give pupils an insight into the subject's historical development, its importance and role in our society (Skolverket, 2001, p. 23).

What is this good for? How can we make use of this? Mathematics is a school subject that we sometimes take for granted. However, some students, for example Beata in Mr. Andersson's class, seems to need more motivation and justifications than others. In the first lesson, when Mr. Andersson introduces a new topic (the chapter in the textbook starts with an introduction to coordinate systems), Beata asks: "why do we have to know this?". "Well, first of all, coordinate system is very useful if you are going to New York", the teacher says. He explains that the streets are arranged like a coordinate system. Beata, and some other students too, seem to be a bit doubtful. “You don’t need to learn a whole system for that?”, she says. It seems like a reasonable comment, it really surprised me that the teacher gives such an example. However, a look into the textbook, which shows a map of New York on the first page of the chapter, reveals the likely source of inspiration.

In order to convince his students, Mr. Andersson offers more examples of usefulness. He says that they are going to draw curves and do estimates of costs. Later in the lesson, Beata calls for the attention from the teacher:

Transcript 1

Beata	Well, why do I have to do this when there is probably a computer somewhere and a program that can calculate this rather easy
Teacher:	Yes, but why should you do anything at all, Beata?
Beata	But …
Teacher:	But this, I mean …
Beata	Yes, I want to do funny stuff (laughing)
Teacher:	Yes, but perhaps a computer can do that too for you … we don’t have to do so much
Beata	No, because these … it feels so meaningless if I don’t know how I can use it

Three lessons later (SW1-L10), Beata is still not convinced about the usefulness of the current topic. She calls for the attention of the teacher again and in the three minutes which follow, Mr. Andersson tries to persuade her and makes her keep on working with the textbook tasks (see Transcript 6 in Appendix A).

The other two teachers seem not to have the same type of motivational discussions with their students, most likely because they are teaching students that are regarded as high achievers and, perhaps, therefore more willing to learn mathematics. When Mr. Svensson justifies a subject or suggests aims for learning a certain topic, he often refers to future work in the textbook. “This group of students accepts the somehow poor explanation that it will be useful for them later on”, says Mr. Svensson.

Motives and background are often offered implicitly by Mr. Larsson when he presents examples on the board. For instance when he, at the beginning of one of his lessons (labeled SW2-L02), writes $E=mc^2$ on the board and asks the students if they recognize the formula. The episode is coded as *Textbook absence* and *Background motivational*. However, the textbook might have inspired the teacher. On page 217 one can read: *Formulae are often used within Science. One example is Einstein's famous formula* $E=mc^2$. But, since Mr. Larsson elaborates and discusses this further, this episode is not regarded as influenced by the textbook (see Transcript 7 in Appendix A).

Another kind of support for students' recognition of key ideas in a lesson is a summary statement. This is when the teacher, near the end of the lesson, highlights points that the students have been study. In this study, there were no occasions at all that could be regarded as lesson summaries[xv].

6. Main results

The main purpose of the study is not to compare the three teachers' teaching methods or to make generalizations about mathematics teaching in Sweden. For this particular study, it would not even be fair to make such comparisons between the three teachers since one of them, Mr. Andersson, works under different condition than the other two. He is less experienced as a teacher and teaches a mixed ability group of students. Furthermore, it is not a matter of proposing criticism towards teachers or textbooks. The intention is rather to analyze some mathematics classroom in order to reveal teachers' practices and relations to textbooks, which hopefully will stimulate discussions about choices.

An underlying assumption in the TIMSS Video Study is that there exists a culturally-based 'lesson script' (cf. Clarke & Mesiti, 2003). In this study of three teachers' way to organize their lessons, a definite 'script' is not recognized. However, with regards to the use of textbooks, there are some observable patterns. What is noticeable is that the textbooks to a large degree, guide teaching in these three classrooms:

a) Students are exclusively working with tasks in the textbook during the private work part of the lesson, which on average is more than half the time of a lesson.
b) In the public part of the lesson, the examples and the tasks that the teachers present are mainly from the textbook. An exception is the teacher Mr. Larsson who uses his experiences as a Physics teacher in some of the examples on the board.
c) The way that mathematics, as a scientific discipline, is presented is comparable with the approach in the textbook. A hundred of totally

119 occasions of *Mathematical generalizations or statements* are coded as comparable or the same as in the textbook. In principal, this means that hardly any other definitions, conventions, or rules than the textbook offers are presented to the students. It also means that the mathematical procedures, for example how to solve an equation, and how the structural features of mathematics are portrayed, are mainly the same as in the textbook.

d) Two of the teachers, Mr. Andersson and Mr. Svensson, use their textbooks as the main sources for background and motivational discussions.
e) Homework is not assigned on a regular basis. However, when the teachers do give assignments, students are supposed to work with tasks from the textbooks.

Besides these results of quantitative nature, there are some aspects of the three teachers' teaching that I would like to highlight as well. It is 'uniqueness' of each teacher's teaching style.

Mr. Andersson is teaching a mixed-ability group of students, which means that the students are working at a different pace. As a consequence, after just a few lessons of a new chapter, the class is spread out in terms of the tasks in the textbook. He seems to be the busiest teacher among the three teachers in this study, at least during the private-work part of the lessons. In one of his lessons, he assists students' problem solving processes thirty-three times in a period of thirty-four minutes of private work.

Mr. Svensson sequences his lesson differently than the other two teachers; he alters between types of classroom interaction several times in his lessons. In this case, it means that each student in his class is probably working with the same task as all the other students throughout the lesson. Other differences between Mr. Svensson and the other two teachers are that he sometimes chose to let a student write the solution on the board and that he presents the goal each lesson.

Mr. Larsson uses supplementary sources more often than the other two teachers when he presents examples on the board. Instead of taking the examples from the textbook, he often relies on his knowledge as a teacher in Physics. In this study, he always presents these examples in the public part of the lesson and not as tasks for the students to work with, individually or in groups.

7. Discussion

A fair question to ask in the discussion about the results from the study is: *Does it matter if mathematics textbooks guide the teaching?* First of all, we need to keep in mind that the textbook facilitates the daily work of the teacher. It is not rational or even realistic to just expect that the teachers' dependence on textbooks will be reduced without good reasons. Further advantageous is that, within the school system, the textbooks serve as some kind of agreement and support for the uniformity. A textbook is often organized in such a way that it covers the topics that students should encounter during a particular school year. Thus, teachers can defend their decision to follow the textbook closely by arguing that it prevents them from skipping important topics or teaching topics out of an appropriate sequence (Freeman & Porter, 1989). Moreover, in many textbooks, at least in Sweden, the tasks are graded by level of difficulty. This means that the students can work individually and hopefully be challenged at their own level (see Brändström, 2005, p. 71). If these arguments are convincing enough, we should not be worried. Thus, the answer to the question would be: *No*, *it does not matter that textbooks guide mathematics teaching*.

If we, on the other hand, find it problematic and believe that *it does matter that textbooks guide mathematics teaching*, we need to discuss the options. Considering, for example, the discussion between the student Beata and her teacher about monotonous work with tasks in the textbook, one could argue that the textbook fails to encourage the student's joy to learn (cf. Lindqvist, Emanuelsson, Lindström, & Rönnberg, 2003). One could also contrast the textbook task about the water consumption in a household (Figure 7) with the example that Mr. Larsson brought up on the same topic using an information sheet from the municipality. This could be a starting point in a discussion of textbook tasks from the perspective of their richness and relatedness to out-of-school 'reality' (for a detailed study on this see Palm, 2002).

Another issue concerns individualized teaching, which is emphasized in the national curriculum in Sweden. This means that teaching should be adjusted to each student's ability and needs. In a mixed-ability group of students, such as Mr. Andersson's class, this could be more or less complicated. But since textbooks facilitate private work, one could think about them as proper tools to accomplish individualized teaching, especially if the tasks are graded by level of difficulty. Then the students can work on their own level and pace. However, if the students are working individually and with different tasks, just as Mr. Andersson's students do, it could be difficult to give instructions to the whole class in a meaningful way. Furthermore, it is also important to question if a certain textbook offers something for *each* student's ability[xvi] and needs and if it is appropriate for all student to learn mathematics through a textbook and by themselves (and with some help of the teacher of course).

A final comment may raise the question if there are any differences between the school subject Mathematics and other school subjects, i.e. *Is a study of the influence of textbooks more relevant in research in mathematics education than in educational research in general?* I believe it is, but to prove that the assumption is correct is out of the scope of this study. Nevertheless, Sosniak and Stodolsky (1993) noticed that none of the teachers in their study used textbooks in the same way when teaching different subjects. One of the teacher reports that the textbook for the reading program serve as "food for thoughts" but the mathematics textbook frees her from "having to do much thinking at all about her mathematics program" (Sosniak & Stodolsky, 1993, p. 260).

Notes

[i] Textbooks are in this study defined as "fairly large and printed objects, which intend to guide students' work throughout the year" (Johansson, 2003, p. 20)

[ii] The frame factor theory, or model, origins from work of the Swedish educationalist U. Dahllöf and his colleagues in the 1960s. In its early stage, it focus on how political decisions regarding teaching and education (e.g. time schedules, grouping, etc.) influenced the pedagogical work (Lundgren, 1998).

[iii] For a comprehensive description of the methodological and technological design in the CULT-study, see Häggblom (2005). Information about the study can also be found on: http://www.ped.uu.se/kult/default.asp. The fieldwork and the data collection in the CULT-project is based on the research design set out for the Learner's Perspective Study (http://extranet.edfac.unimelb.edu.au/DSME/lps/).

[iv] All names in this paper are fictitious.

[v] The teacher, Mr. Andersson, uses the textbook Matte Direkt, år 8 (Carlsson, Hake, & Öberg, 2002).

[vi] The textbook, which is Matematikboken Z Röd (Undvall, Olofsson, & Forsberg, 1997), differentiate the tasks. A-tasks require the lowest demands.

[vii] The textbook is Matematikboken Y Röd (Undvall, Olofsson, & Forsberg, 2002). According to the authors, it is intended to be used by students who are interested and have good skills in mathematics.

[viii] One of the aims of teaching mathematics in the compulsory school in Sweden is, according to the syllabus, that "Mathematics is an important part of our culture and the

education should give pupils an insight into the subject's historical development, its importance and role in our society. The subject aims at developing the pupil's interest in mathematics, as well as creating opportunities for communicating in mathematical language and expressions. It should also give pupils the opportunity to discover aesthetic values in mathematical patterns, forms and relationships, as well as experience satisfaction and joy in understanding and solving problems" (Skolverket, 2001, p. 23).

[ix] For the TIMSS Video Study, the minimum acceptable reliability score for an individual coder was 85 %.

[x] According to the manual of the software Videograph, you should be able to watch a movie and code at the same time. An unknown technical problem forced me to play the movie via Quick Time in a separate window. Since these two programs were not time-synchronized, it makes the reliability testing a bit problematic.

[xi] In the TIMSS Video Study, the teachers from different countries divided their time between public or private interaction differently. However, apart from two countries, a greater percentage of lesson time was spent in public interaction. In Australia, there was no detectable difference between time on public and private interaction. In the Netherlands, fifty-five percent of lesson time was spent in private interaction (Hiebert et al., 2003).

[xii] Since 'absent' in this case means that one cannot notice an influence of the current textbook, influence of another textbook cannot be excluded.

[xiii] In the TIMSS Video Study, the teachers in all seven countries, except Japan, assign homework in at least 57 percent of the lessons (Hiebert et al., 2003).

[xiv] How frequently the teachers state the goal of a lesson differs between the countries in the TIMSS Video Study. The teachers from the Czech Republic, for example, do it in almost every lesson (91 % of the lessons) but the teachers from the Netherlands state the goals in only 21 % of the lessons (Hiebert et al., 2003).

[xv] In the TIMSS Video Study, lesson summaries, which are less common than goal statements, were found in at least twenty-one percent of the lessons in Japan, the Czech Republic, and Hong Kong SAR, and in ten percent of lessons in Australia (Hiebert et al., 2003).

[xvi] Brändström (2005) analyzed tasks in mathematics textbooks in order to reveal their level of difficulty and how they are differentiated. She found that though textbooks tasks are offered on different levels, "the processes and required demands are too low" (p. 75) on all levels.

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8. References

Brändström, A. (2005). *Differentiated tasks in mathematics textbooks: an analysis of the levels of difficulty.* Luleå university of technology, Luleå.

Carlsson, S., Hake, K.-B., & Öberg, B. (2002). *Matte Direkt, år 8*. Stockholm: Bonnier.

Clarke, D., & Mesiti, C. (2003). Addressing the challenge of legitimate international comparisons: lesson structure in Australia and the USA. In L. Bragg, C. Campbell, G. Herbert & J. Mousley (Eds.), *Mathematics Education Research: Innovation, Networking, Opportunity, Proceedings of the 26th Annual Conference of the Mathematics Education Research Group of Australasia, Vol. 1* (pp. 230-237).

Englund, B. (1999). Lärobokskunskap, styrning och elevinflytande. *Pedagogisk forskning i Sverige, 4*(4), 327-348.

Englund, T. (1997). Towards a dynamic analysis of the content of schooling: narrow and broad didactics in Sweden. *Journal of Curriculum Studies, 29*(3), 267-287.

Freeman, D. J., & Porter, A. C. (1989). Do textbooks dictate the content of mathematics instruction in elementary schools? *American Educational Research Journal, 26*(3), 403-421.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., et al. (2003). *Teaching mathematics in seven countries*. Washington, D.C: US Govt. Printing Office.

Häggblom, J. (2005). *From analogue tapes to digital drives: On the development of methods and techniques for classroom interaction research.* (D-uppsats). Uppsala University, Uppsala.

Jacobs, J., Garnier, H., Gallimore, R., Hollingsworth, H., Givvin, K. B., Rust, K., et al. (2003). *TIMSS 1999 Video Study. Technical report:*

Mathematics. Washington, D.C: National Center for Education Statistics.

Johansson, M. (2003). *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum* (Licentiate thesis). Luleå: Department of Mathematics, Luleå University of Technology.

Lindqvist, U., Emanuelsson, L., Lindström, J.-O., & Rönnberg, I. (2003). *Lusten att lära - med fokus på matematik* (No. 221). Stockholm: Statens skolverk.

Lundgren, U. P. (1998). The making of curriculum making: reflection on educational research and the use of educational research. In B. G. Gundem & S. Hopmann (Eds.), *Didaktik and/or curriculum: an international dialogue* (Vol. 41, pp. 149-162). New York: Peter Lang.

Löwing, M. (2004). *Matematikundervisningens konkreta gestaltning. En studie av kommunikationen lärare - elev och matematiklektionens didaktiska ramar.* Acta Universitatis Gothoburgensis, Göteborg.

Palm, T. (2002). *The realism of mathematical school tasks. Features and consequences.* Umeå University, Umeå.

Reys, R., Reys, B., Lapan, R., Holliday, G., & Wasman, D. (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education, 34*(1), 74-95.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H., Wiley, D. E., Cogan, L. S., et al. (2001). *Why schools matter : a cross-national comparison of curriculum and learning*. San Francisco: Jossey-Bass.

Skolverket. (2001). *Syllabuses for the compulsory school*. Stockholm: Fritzes.

Sosniak, L. A., & Stodolsky, S. S. (1993). Teachers and textbooks: Materials use in four fourth-grade classrooms. *The Elementary School Journal, 93*(3), 249-275.

Undvall, L., Olofsson, K.-G., & Forsberg, S. (1997). *Matematikboken Z röd*. Stockholm: Liber AB.

Undvall, L., Olofsson, K.-G., & Forsberg, S. (2002). *Matematikboken Y Röd*. Stockholm: Liber.

Undvall, L., Olofsson, K.-G., & Forsberg, S. (2003). *Matematikboken Z, röd*. Stockholm: Liber.

Appendix A

Transcript 2

Teacher: Now I write, *A* times *x* plus *B* …

Jonas: … times square root of *x*

Teacher: Exactly, and what is a *square root*? This I think at least some of you know a bit about, but maybe not all of you. What is a *square root*? We take an example, which makes it easier to understand. I say like this … let's say that *x* is equal to four, then it is easy (the teacher writes the radical sign and 4 under it on the board). What does it mean, the square root of four, Ralf?

Ralf: It is four times four

Teacher: No, that is not correct. That was wrong direction … Olof?

Olof: It is two times two

Teacher: Exactly, the number … yes, do you want to say something? (to a student)

Student: It is the number that will be … that times itself will be the number under

Teacher: Yes, exactly, the number that times itself becomes what is under the radical sign … four in this case … and that is two, isn't it?

Transcript 3

Teacher: By the way, how much is a cubic meter … John

John: A thousand liters

Teacher: Exactly, and that is pretty much isn't it … one cubic meter. Imagine a cube then, like a big dice, one times one times one meter. That is a cubic meter … a thousand liter. Nevertheless, *A* and *B* they are constants. So this, we say, is a variable (points at x on the board). This is how much water the households consume. But *A* and *B*, they are fixed numbers

Transcript 4

Teacher: Then we take *b* instead … another person here … a bit more wasteful with water. He spends a hundred cubic meter (writes *b) 100* m^3 on the board). Well, now I choose this figure to make it easy for you to calculate. Is there someone who can say what I shall write here now, somebody else than Jonas … Maria?

Maria: Ten times a hundred plus fifty times square root of a hundred.

Teacher: Ten times a hundred plus fifty times square root of a hundred. (writes $10\cdot 100 + 50\cdot\sqrt{100} =$ on the board). We don't need to write, we calculate directly … mental arithmetic. How much can this be? Ten times a hundred plus … yes this is easy to handle … square root of a hundred is … what, Johan?

Johan: Ten

Teacher: Yes, one thousand plus …, put it together. If you count fifty times it will be (points at $50 \cdot \sqrt{100}$ on the board)

Student: Five-hundred

Teacher: Yes, and together

Student: One-thousand and five-hundred (the teacher writes *1500* after the equal sign on the board)

Teacher: One thousand and five hundred, yes. Let us take a *c* here as well. Let us say four hundred cubic meters (writes *c) 400* m^3 on the board).

Transcript 5

Teacher: This is a nice formula, don't you think, Erik

Student: Yes

Teacher: Then, what is this? (points at the Greek letter λ on the board)

Student: A Greek letter

Teacher: A Greek letter?

Student: Epsilon

Teacher: No, perhaps my drawing is unclear but it should not be epsilon. Anyone, who knows its name? I can say that it corresponds to our letter L, lambda it is. […] Thus, this is the Greek letter lambda. We can write the explanation here then (the teacher writes λ *= wavelength in m*; *v = the speed of sound in m/s; f = frequency* on the board). You have been talking about frequency, I think. What is that? One can speak of frequency in connection to sound and other oscillations. What is that?

Student: (...)

Teacher: Exactly, and what unit do you use to measure frequency? This you may also recognize from Physics. I don't have the pleasure to teach you in physics but I guess you have read about it … Kristina.

Student: Hertz

Teacher: Mm (the teacher writes *in Hertz* after *frequency* on the board)

Student: (...)

Teacher: Yes, we can write the abbreviation (writes *Hz* after *Hertz*) … mm […] Now we shall elaborate this. I think you have played with … what is this? (shows a tuning-fork) … in Physics … some of you … Kristoffer.

Student: A mortise chisel (in Swedish, the first part of the names of these two tools are alike, tuning-fork is *stämgaffel* and mortise chisel is *stämjärn*)

Teacher: No, not a mortise chisel. That I think you use in Crafts. What is the name?

Student: Tuning-fork

Teacher: A tuning fork (the teacher hits the tuning-fork with a pen and holds it up). This is how it sounds and this particular tuning fork has the frequency of one thousand seven hundred hertz. Seventeen hundred hertz, a rather high tone one can say. I think we can calculate the wavelength. When this sounds, it compresses the air so that condensations (förtätningar) and rarefies (förtunningar) wander in the air in a certain direction. The wavelength then, it is how far it is between two condensations or two rarefies. Let us calculate how far then … this is nothing one can see but it is in the air. This makes it possible to hear, at all. Then we insert the frequency, in this case, one thousand seven hundred hertz (writes *1700Hz* under *frequency*). But we need to know something more to find the wavelength under these circumstances. What is it we need to know more … Patrik?

Student: (...)

Teacher: Certainly, and if we think that this is air … you normally use a standard value on speed of sound in air … Ralf?

Student: (...)

Teacher: Exactly (the teacher writes *340 m/s* after *speed of sound*). I think it is about … just as in this room. Maybe we can measure a bit more than three hundred and forty meter in here … around twenty-two degrees. Because it depends on the temperature. If it is cold, then it will be slower. If it is warm, then it will be faster. But okay, the formula then. Let us insert and calculate our lambda then (writes λ = on the board). Which values shall I insert here then …Erika

Student: (...) (the teacher writes *340/1700* after λ =)

Teacher: And what is the unit on this then? The units are important in such formulae, since it is often given what type of unit to use, and if you put the wrong unit then it will be totally crazy. What is the unit in this case … Patrik?

Student: (...)

Teacher: What? What is the unit … on lambda … on the wavelength? … it says (points at λ = *wavelength in m* on the board)

Student: meter

Teacher: Yes, it is meter. And what can this be if we calculate it then? (writes λ =). You, who are good in mental calculations, you can see that it is chosen so it will be easy to calculate … John?

Student: Zero point five

Teacher: No, it is not zero point five … zero point …

Student: Zero point two

Teacher: Yes, zero point two it is (writes *0,2* after λ =) […] Zero point two meter, twenty centimeters then. So, when I hit this (hit the tuning-fork with the pen), we have condensations on a twenty centimeters distance here (writes *The wavelength is 20 cm* on the board). Is this hard to understand?

Student: Yes

Teacher: You think that? Well, it can be like that. We could derive this but this is more about Physics so I think we leave it for now.

Transcript 6

Student: Where is it all? (turning pages in the textbook). What is the name?

Teacher: Yes, what do you mean?

Beata Relationships

Teacher: Yes, you draw lines … yes you do

Beata Yes, but it's so … I don't know how to explain it. It's just that … will I ever use the ability to draw lines? I mean … I do understand a thing like this but why should I be able to show where this point is in comparison to that?

Teacher: Yes, but this is very good. It's good, very good.

Beata But I don't want to do this … on about twenty pages or so … and continue … and than more

Teacher: Yes, but do you know *how* then?

Beata Yes, I think so

Teacher: Well, then I think you keep on. Because in the read course (another part of the textbook) … then there definitely are things that you don't know. It will be more … and different

Beata How can we use this?

Teacher: How we can use this? Yes, like what it's good for?

Beata Mm

Teacher: Well, it's not always this easy. If you can read and understand the difference between pears and apples there (task 14 on page 176 in the textbook)… then it's good

Beata Mm

Teacher: But that's a rather … that's a very simple graph. But I think it's good if you practice on this because I think it's important that all of you can. If you have a graph in a newspaper, then you should be able to understand what the curve means. And then we will talk a little about …

Beata Yes, but I do know that … it's about …

Teacher: Well that's good … Then I think you should continue with this a little while

Beata But there are so many pages

Teacher: No, there are not so many pages

Beata Yes

Teacher: Yes

Beata And then it continues … it's so many pages. I mean, how can one make up so many lines (laughing). I don't understand that.

Teacher: Mm, how can one make up so many lines … mm

Mari Look, these are different (laughing) (to Beata)

Beata It's so strange, you know, which bag is most expensive … this is, you know … it's so silly

Transcript 7

Teacher: Is there someone who knows what this is about, actually?

Student: The theory of relativity.

Teacher: Yes, it has connection to that. Who do they say is the originator of the formula, Patrik?

Student: Einstein.

Teacher: Exactly, but what is this particular formula about then?

Student: The energy is equal to mass times square … or cube … of the speed of light.

Teacher: Yes, two it stands here (points at the formula on the board) … thus it is square. Good, that's correct. A rather difficult formula to understand, one can say. I cannot understand how fast the speed of light … how fast is the speed of light, by the way, Patrik?

Student: The highest (some students laugh quietly).

Teacher: The highest yes (laugh quietly). Yes, it is the highest … the speed of light in vacuum. Do we have a number for the speed of light? We talk about the speed of light in air and vacuum, what can it be? I guess you have heard it some time. You use to say that … three hundred thousand kilometer per second. If I write it using the scientific notation … you get … meter per second … you get three times ten to the power of eight (writes $3 \cdot 10^8$ *m/s* on the board). What does it mean that we have cube here? We shall, by the way, not practice on this formula, that comes later, but it can be fun to have seen this anyway. If I take the cube of the speed of light then. Thus, three times ten to the power of eight, three times ten to the power of eight? We talked a little about 'ten to the power' last fall. Any suggestions, someone?

Student: Nine times ten to the power of sixteen.

Teacher: Yes, that is correct. That is quite much, isn't it? Now, this is how much energy can be extracted here, yes, and it is an enormous quantity of energy that can be extracted if the mass transforms into energy.

Appendix B

Teacher SW1 lessons	L06		L07		L08		L10		L11	
	Public	Private	Public	Private	Public	Private	Public	Private	Public	Private
Mathematical generalization	19	9	5	6	6	0	0	9	11	2
Link to lesson	2	2	1	0	1	0	0	0	1	0
Problems and tasks	14	33	1	14	2	13	0	30	2	16
Assignment of homework	0	0	0	0	0	0	1	0	0	0
Goal statement	2	1	0	0	0	0	0	1	0	0
Background motivational	4	0	1	0	2	0	0	3	1	0
Summary of lesson	0	0	0	0	0	0	0	0	0	0
Assessment	0	0	0	0	0	0	0	0	0	0

Table 1: *Number of occasions in public respective private interaction*

Teacher SW2 lessons	L04		L05		L06		L07	
	Public	Private	Public	Private	Public	Private	Public	Private
Mathematical generalization	6	0	8	1	4	1	0	4
Link to lesson	0	0	2	0	1	0	0	0
Problems and tasks	8	6	4	15	9	14	7	13
Assignment of homework	1	0	0	0	0	0	0	0
Goal statement	2	0	4	0	2	0	2	0
Background motivational	1	0	2	0	1	0	0	0
Summary of lesson	0	0	0	0	0	0	0	0
Assessment	0	0	0	0	0	0	0	0

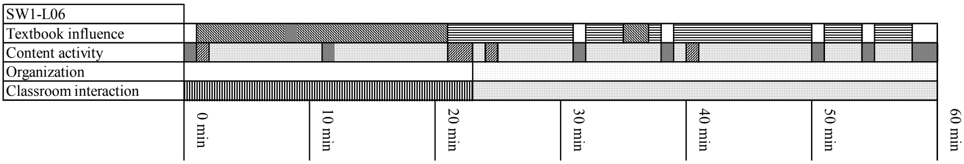
Table 2: *Number of occasions in public respective private interaction*

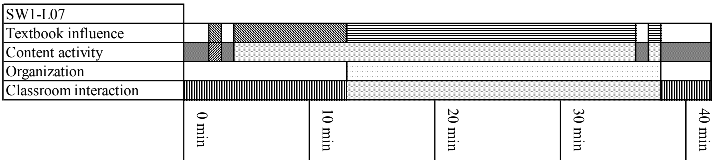
Teacher SW3 lessons	L02		L03		L04		L05	
	public	private	public	private	public	private	public	private
Mathematical generalization	7	3	7	1	6	1	3	1
Link to lesson	2	0	0	0	1	0	1	0
Problems and tasks	9	23	4	5	10	20	5	14
Assignment of homework	0	0	0	0	1	0	0	0
Goal statement	1	0	0	0	0	0	0	0
Background motivational	2	0	2	0	1	0	0	0
Summary of lesson	0	0	0	0	0	0	0	0
Assessment	0	0	0	0	0	0	0	0

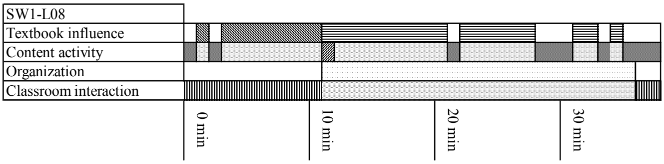
Table 3: *Number of occasions in public respective private interaction*

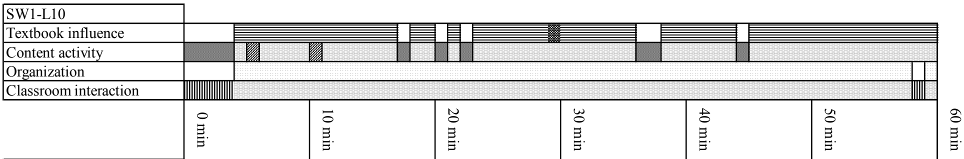
Appendix C

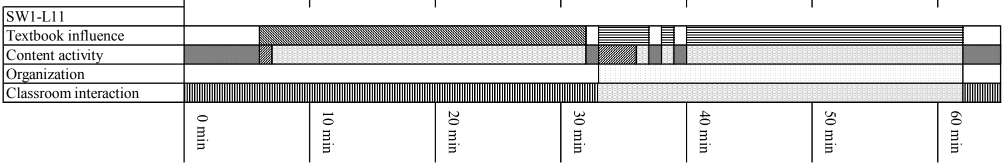
Textbook influence	Direct	Indirect	Absent
Content activity	Non math	Math. Org.	Math. work
Organization	Individually work	group/pairs	Class
Classroom interaction	Public	Private	Mixed interaction

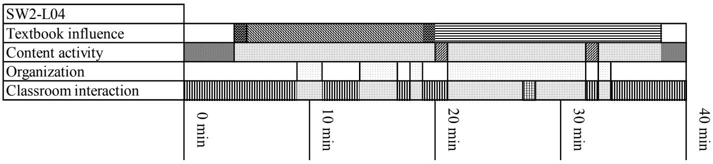


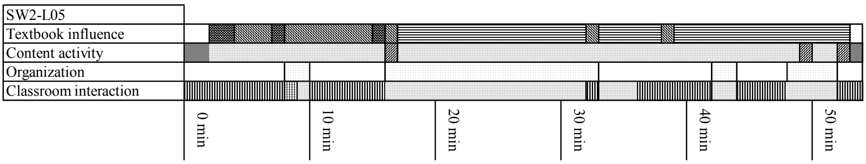


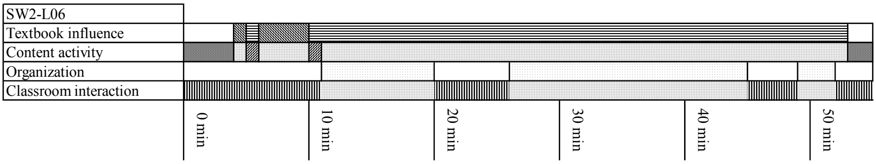


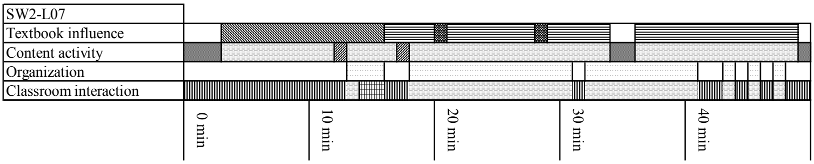


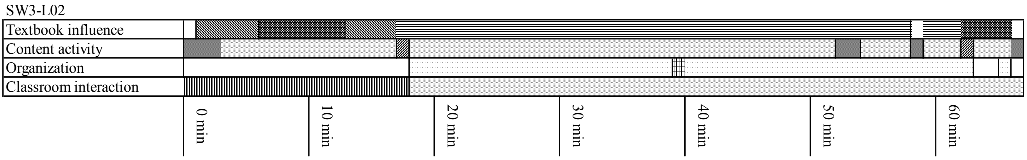


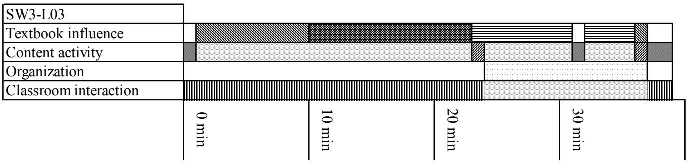


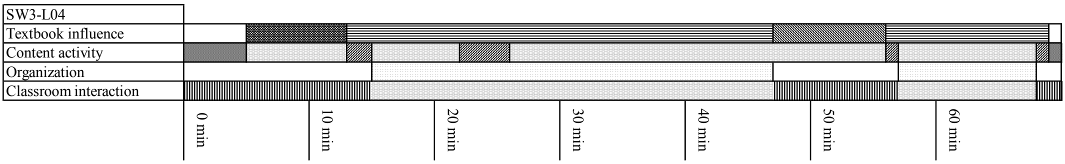


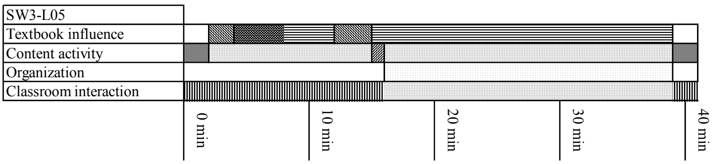

SW2-L06
Textbook influence
Content activity
Organization
Classroom interaction
0 min
10 min
20 min
30 min
40 min
50 min


SW2-L07
Textbook influence
Content activity
Organization
Classroom interaction
0 min
10 min
20 min
30 min
40 min
50 min


SW3-L02
Textbook influence
Content activity
Organization
Classroom interaction
0 min
10 min
20 min
30 min
40 min
50 min
60 min


SW3-L03
Textbook influence
Content activity
Organization
Classroom interaction
0 min
10 min
20 min
30 min
40 min


SW3-L04
Textbook influence
Content activity
Organization
Classroom interaction
0 min
10 min
20 min
30 min
40 min
50 min
60 min


SW3-L05
Textbook influence
Content activity
Organization
Classroom interaction
0 min
10 min
20 min
30 min
40 min

Paper IV

Mathematical meaning making and textbook tasks

MONICA JOHANSSON

This paper reports from a study of mathematics classrooms. It is about the interaction that emerges when students are solving tasks in the textbook and the teacher gives individual assistance. The interest lies in the task and the way in which the teacher-student interaction is influenced by the textbook. An analysis of classroom episodes reveals control features of the textbook and shows that a textbook task, in an interrelationship with a teacher, can cause ambiguity as well as generate mathematical discussions.

1. INTRODUCTION

In the late 1970s, Bauersfeld (1979) notices a trend shift in the research field of mathematics education. He states that the difficulty to generalize results from studies made in a laboratory context has contributed to an increased interest of doing research studies of real classroom situations. Looking at the current focus of much research in mathematics education it seems like Bauersfeld's conclusion is correct, there has been a trend shift. In a survey of the research community, Sfard (2005) finds that most researchers use carefully recorded classroom interactions as their empirical data. She also notices that "we may now be living in the era of the teacher" (Sfard, 2005, p. 409). This is a change from the previous foci that were on the learner and, before that, the curriculum. Sfard thinks that we have come a long way since the 1960s and 1970s and "the era of the curriculum […] when the main players in the educational game were the developer and the textbook" (ibid.). There have been changes with regards to research methods as well. At present, most researchers use qualitative methods and from a perspective of participation. Instead of trying to find out ‘what works in the classroom’ the focus is on *how* things work and *what* the alternative possibilities are (Sfard, 2005).

Looking at how things work in the classroom, one can see that teaching is an activity that can take place from at least two different locations in the classroom. It can be an entirely public interaction with a teacher (or a student) standing in front of the class. It can also be a more private interaction where the teacher walks around the classroom tutoring individual students or groups of students. In Japan, the term Kikan-Shido means ‘between desks instruction’. It is a term that describes the time in the lesson when the students, on an individual basis or in groups, are engaged in ‘practice’ and the teacher walks around the classroom, observing and sometimes interacting with the students (Clarke, 2004). This is a familiar activity, well recognized by researchers and teachers in Sweden as well as in many other countries. Kikan-Shido refers to an activity of a teacher, so what are the students occupied with during that time? Solving tasks in the textbook perhaps? This is probably the most common activity in Sweden where the textbook seems to define ‘school mathematics’ as well as ‘learning paths’ for the majority of students (Lindqvist, Emanuelsson, Lindström, & Rönnberg, 2003).

So, besides the teacher and the students there is a third player in the game of teaching mathematics – the textbook. The book is of course a dead object and could not play an active role in the interaction. But it is a pedagogical text, or “a book designed to provide an authoritative pedagogic version of an area of knowledge” (Stray, 1994, p. 2). As such, it shall mediate a school subject from someone who knows about it to someone who does not know but is supposed to get hold of the subject (Selander & Skjelbred, 2004). The textbook has an authoritarian position, partly because a teacher has authorized it, or whoever decides which textbook to use. It also reveals underlying beliefs of what mathematics is and how it can be learned (cf. Johansson, 2005; Luke, de Castell, Fraser, & Luke, 1989).

This paper reports from a study of a Swedish classroom. It is about the interaction that emerges when students are solving tasks and the teacher gives individual assistance. The research question is: In what way does the textbook influence, or not influence, the teacher-student interaction in the Kikan-Shido part of the lesson? The influence of textbooks and how textbooks are used in the mathematics classroom are recognized issues in previous research. Some of the studies are mostly quantitative and measure, for example, time allocation (cf. Fan, 2000). Other studies are more qualitative and focus on how the teachers interact with a new reform-oriented textbook (cf. Remillard, 2000). In the last ten to fifteen years, there has been a flood of the latter type of studies, stemming from the reform movement in the United States[i]. The purpose of these studies is often to expose changes or difficulties connected to changes (cf. Wilson & Goldenberg, 1998). The change of teaching and learning in mathematics in Sweden is not associated with a reform of textbooks similar to the one in the United States. Consequently, the textbook in this study could not be regarded as 'new' even if it is a new edition[ii]. Nevertheless, it is still interesting to study the interrelationships between the textbook and the teacher. The intention is to explore the role of the textbook, in a Swedish classroom, under the current state of affairs, rather than examine the effects of a reform.

The purpose of this paper is twofold; it is to increase the awareness of *how* textbooks influence the teaching and learning of mathematics but also to stress the content issue. When doing research in mathematics education, we should not forget about the curriculum and the textbook. Looking at how things work in the classroom we sometimes notice phenomena that are hard to explain in terms of, for example, the teachers mathematical knowledge or beliefs about mathematics and teaching of mathematics.

1.1. THEORETICAL PERSPECTIVES

The study is guided by three theoretical perspectives. The first is based on what Englund (1997) describes as the third stage of the frame factor theory[iii]. It emphasizes the choice of educational content and contextualization of teaching. The fundamental assumption is that different choices can be made, more or less consciously, which have crucial implications for teaching and learning. The student is offered different possibilities to create and construct meaning depending on, for example, what content is chosen and what context the textbook offers.

The second perspective concerns interaction. Following Voigt's (1994) arguments, mathematical meaning is a matter of negotiation, a product of social interactions. Given that a person's beliefs and background knowledge offer definite clear-cut meanings of tasks, questions, symbols, etc., it is in this perspective helpful to consider the ambiguity of objects in the mathematics classroom. Interaction, from the view of symbolic interactionism, is about how the participants monitor their actions in accordance with what is assumed to be the other person's background understandings, expectations and intentions. Thus, mathematical meaning is negotiated although the participants in the interaction do not explicitly argue from different points of view.

The third perspective relates to the textbook. To begin with, the textbook is designed in a certain way. It is specially made for the purpose to be appropriate to the receiver, often a student, and the educational context. It is also colored by a view of learning, even if it is not explicitly stated. For example, a textbook that focuses on getting the right answers on well-defined questions corresponds to the ideas of behaviorism. From a constructivist and socio-cultural perspective, it would be more important to start from the students own experiences and create problems that nurture discussions and cooperation (Selander & Skjelbred, 2004). Can one say that textbooks *influence* the teaching and learning? It is of course questionable whether a dead object like a book or a text really can lead people in a certain direction in a pedagogical process and this would probably lead into a fruitless discussion. Instead one should think about the influence of textbooks as something that is related to peoples' beliefs and values. The influence of textbooks is based on a more or less conscious idea that the book is important (B. Englund, 1999).

Combining these three perspectives, I will analyze the choices that a teacher makes in the classroom by looking at the teacher-student interaction. At the same time, the textbook and the role of the textbook in the specific classroom will be taken into account.

1.2. METHODOLOGY

A study of Swedish classrooms forms the empirical background for this paper. The classrooms are video-recorded as a part of the data collection that is made within the CULT-project[iv] in Sweden. Three teachers from schools in a large community of Sweden participated in the CULT-study. The data consist of split-screen video recordings, video stimulated interviews, lesson material, an International Benchmark Test, and teacher questionnaires. More than fifteen consecutive lessons are observed and video-recorded in each classroom.

For this paper, one of the three teachers and video-recordings from four consecutive lessons are chosen. A coding procedure and a computer software[v] is used in order to categorize activity and settings in the classroom. This is the quantitative part of the study. The lessons are also transcribed. The results from the coding procedure together with the associated transcripts make it possible to identify specific sequences and analyze them in a qualitative manner. The focus in this paper is on the private interaction between the teacher and the students when the students are solving tasks in the textbook, i.e. the Kikan-Shido, as it is defined in the specific classroom. Sections 2 to 4 describe forms of interaction between the teacher and the students. Section 2 illustrates typical activities and includes representative examples of interactions. Sections 3 and 4 portray, by some means, unexpected and non-prototypic teacher-student interactions. The episodes are chosen for the purpose to highlight the control features of the textbook.

1.2.1. *The coding procedure*

The coding schedule is, in principle, guided by the video coding manual of the TIMSS Video Study[vi] (Jacobs et al., 2003). The TIMSS Video Study emphasizes problem and problem-solving and has an advanced coding system for that purpose. The coding procedure in this study is however simplified and adjusted in order to capture what is most important from the perspective of this study.

Two types of codes are used, coverage codes and occurrence codes. The *Organization of students* and if *Classroom interaction* is public or private are examples of coverage codes. When the teacher for example writes a problem on the board it is coded as an occurrence code: *Problems, exercises or tasks,* category 0 (Set-up). The same category is used when the teacher clarifies a textbook task for an individual student, during the private-work part of the lesson, in order to make him or her able to start working on the task. The following graph is not showing a complete picture of the coding procedure. It illustrates however the main focus.

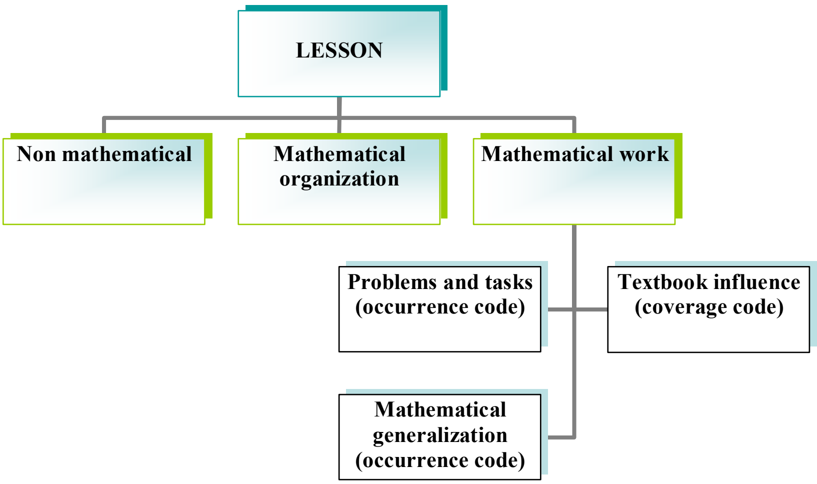


Figure 1 *The coding procedure*

1.2.2. *The participants*

The teacher, Mr. Larsson[vii], is 62 years old. He has a long-standing experience of teaching and a long period of employment at the school where the study is conducted. Thirty-one of totally thirty-six years of teaching he has been working in that school. Besides mathematics, he teaches physics and technology in grade eight and nine. The results from the quantitative part of the study, i.e. the coding procedure, shows that, on average, almost 60 % of the lessons consist of private work (median value 58.8 %) where the students are solving tasks in the textbook and the teacher gives individual guidance. The rest of the time, in the public part of the lesson, the teacher stands in front of the class. He writes on the board, presents problems, poses questions and verifies or disproves answers. The desks are organized in pairs but group work was not found in this study.

The textbook[viii], which is used in this particular class, is a book in one of the most common textbook series in grade seven to nine in the compulsory school in Sweden. The teacher seems to adhere very closely to the textbook in the private as well as the public part of the lesson, even if he from time to time brings up examples from outside the book. In one of the interviews[ix] he confirms the strong reliance on the book. He was asked why he uses concrete numbers to show the students how to simplify an expression. The teacher answered:

> It is generally so that I just follow the usual way to do it … this is normally how it is done in all books and I have not wondered about it so much, I think it is a system that works.

The class consists of 22 grade eight students. The school practice is tuition in ability groups and the students in this class are identified as high achievers. According to Mr. Larsson, they are quite homogeneous. He thinks about them as a hard working group that concentrates on mathematics. This is confirmed by the quantitative analysis, which shows that non-mathematical activities are rare in Mr. Larsson's classroom. In this sense, one can say that the classroom environment is a comfortable one.

2. THE 'STANDARD' PATTERN OF TEACHER-STUDENT INTERACTION

In a mathematics classroom, one can notice certain patterns of interaction where the participants follow hidden regulations, which they actually seem to be unaware of. One example is the 'funnel' pattern. Bauersfeld (1988) uses the metaphor 'funnel' to describe an activity where the teacher provides individual guidance through a step-by-step reduction of the demands.

Sometimes this process culminates so that one expected word from the student makes the teacher complete the solution by himself. The ‘funnel pattern of interaction’ is a well-known activity in many mathematics classrooms (cf. Alrø & Skovsmose, 2002; Kilborn, 1979; Lundgren, 1983) and is also observed in this particular classroom. The following transcript serves as an example of a common type of interaction where the teacher, Mr. Larsson, funnels the student until he almost reaches the solution[x]. The task, which the student is trying to solve, is as follows:

> **5033**
> x is an odd number, any one. Write an expression for the two consecutive odd numbers.
>
> *The solution is presented in the answer key: $x+2$ and $x+4$*

Transcript 1:

Student: (...) odd numbers

Teacher: Odd numbers, yes. Any one, yes … two consecutive. If you think of an odd number, for example eleven, what is the next odd number then?

Student: Thirteen

Teacher: Yes. How do you get eleven then … you add …?

Student: Plus two

Teacher: Yes, and then the next number … how much should you add then do you think?

Student: Two more

Teacher: Yes of course

Student: Yes, okay

Discussions between Mr. Larsson and his students are not exclusively about specific tasks. Now and then, the comments involve general aspects, conventions or rules of mathematics. These kind of clarifications can be an answer to a question of a student. For example[xi]:

Transcript 2:

Student: When should you use brackets and when should you not use brackets?

Teacher: Bracket are used in order to … so you don’t have to write the unit two times. If you put brackets, then you can write *kronor* behind

Student: Aha

Teacher: Precisely. I wrote *10 x kronor* minus *5 y kronor*, which works as well. But now you are putting brackets and then you write *kronor* after the whole expression there, yes.

The overall picture of the teacher activity in the private-work part of the lessons can be described as follows: The teacher interacts with some of the students, mostly individually, in order to help them solve the tasks in the textbook. Sometimes he just controls their answers or gives hints, which could help them start working on the task, and sometimes he helps them all the way through the solution of the task. In principal, the interaction starts from the task in the textbook that the students are working with. Questions about the task, sometimes initiated by the teacher and sometimes by a student, lead to an interaction between the student and the teacher. The kind of guidance that each individual student gets from the teacher differs depending on the task and how far the student has come in his or her efforts to solve it and other factors. The teacher's objectives seems however to be the same, to arrive at a correct solution to the problem and/or to clarify mathematical properties, rules or conventions.

3. CRITICAL INCIDENT 1

Even if the interaction in the classroom is colored by routines and regularities one can observe phenomena that are interesting to analyze more deeply. This can for example be an instance of teacher decision making in which the inherent learning potential significantly depends on the outcome of that decision[xii]. For this study, two critical incidents are chosen. The first example is chosen because it illustrates discrepancy from the smooth 'standard' pattern of teacher-student-task interaction and highlights the role of the textbook. It starts in the private-work part of the lesson when the students are working by themselves solving tasks in the textbook[xiii]. One of the tasks in the textbook is especially 'challenging' in the sense that many students call for help from the teacher. The task is the following:

5099

Svante Gruvberg works as a miner. Every second week he works in daytime and every second week he works in the night. Svante earns 118 SKr per hour on daytime and 152 SKr per hour on nighttime.

a) How much does Svante earn in one year with 44 working weeks and 40 hours per week?
b) Write an expression of how much Svante earns in one year with *x* working weeks if he works *y* hours per week

The solution to this task is, according to the key at the end of the textbook, the following:

a) 237 600 kr, and b) $(\frac{x}{2} \cdot y \cdot 118 + \frac{x}{2} \cdot y \cdot 152)$ *kr.*

During the period of private work, the teacher discusses this particular task individually with ten of the students. The first time, he just reads the a-problem and checks if the student has arrived at a correct answer. The following discussion comes up the second time he sees the task:

Transcript 3:

Teacher: It was good that it was forty-four weeks. If it had been forty-three, could one possibly give an exact answer then?

Student: It depends on what week

Teacher: Exactly

Student: (…) then it can be two different answers

Teacher: It can be that, yes … surely … depending if it is twenty-two weeks of nightshift or twenty-one weeks of nightshift. So, one cannot really answer precisely

The teacher recognizes that the task would be quite different if the number of weeks would be odd and the student seems to follow his reasoning stating that it can be two different answers.

Later in the same lesson, the teacher discusses the b-problem with another student. The teacher seems a bit puzzled about how the student solves the problem.

Transcript 4:

Teacher: Now, look here. I don't think I recognize this you know [turn pages in the textbook] … *y* hours per week … aha

Student: (…) working weeks

Teacher: Mm … but you … did you put *z* here … where did that come from?

Student: This will be *z*

Teacher: Aha … well okay … that is *z* you mean … yes. But you should rather not write it together then. Eh … a bit doubtful if that formula will be totally correct there. If you take it like in two steps instead. On the one hand you take what he earns on the dayshift weeks and what he earns on the nightshift weeks … then you get … you certainly get a more accurate formula.

It is not clear if the teacher looks at the solution at the end of the textbook when he turns the pages. However, in the interaction with the student he does not mention that an odd number of weeks gives two different solutions[xiv]. He leaves the student before he finishes the task. Shortly after this, he approaches another student that also works with the b-problem.

Transcript 5:

Teacher: Of course … then one can write these together … but you can have it as in two parts … you put these together wouldn't you? But a rather questionable formula actually. It depends on how … if it is even or odd and with what he starts with, don't you think?

Student: Mm

Teacher: Thus, it is not certain if the formula is going to work. It depends a little on how he starts working … if he work more day weeks.

Student: (…)

Teacher: What did you say?

Student: He works half time day and half time night you know

Teacher: Yes, but if it is … no, he doesn't … he works whole weeks you know. Then if it is three weeks … it could be two weeks that he works night and one week that he works day

Student: (…) but if he works night every second week and day every second week

Teacher: Yes, but let us say that he works three … if you count on a three-weeks period

Student: (…) one year … that is fifty-two weeks

Teacher: Well, okay … mm … I suppose it evens itself out … sure

Student: (…) forty-four weeks

Teacher: Forty-four working weeks … then it should work. But if you only work temporarily for three weeks then it cannot … you cannot know for sure

Once again, the teacher tries to convey the message that there is a problem with the formula. This particular student seems however quite confident with his solution. He does not care about the case of an odd number of weeks and argues "one year … that is fifty-two weeks". The teacher appears to be unsatisfied and ends the discussion by uttering, "you cannot know for sure".

After that discussion, it seems like the teacher abandons his mission to show that an odd number of weeks gives two possible solutions. He helps three more students with the same task without mentioning the numbers of weeks. Then he approaches a student that shows a quite different solution of the problem.

Transcript 6:

Teacher: Yes, no problems?

Student: No, but now, look on this

Teacher: To write that expression yes … the last one you mean [the b-problem]

Student: See … isn't it easier if one takes, like this, the mean value of a hundred … or … between a hundred and eighteen and a hundred and fifty-two?

Teacher: Yes, one could probably do so too

Student: and like that [points at a spot in the textbook]

Teacher: That you could do … sure … it works yes … took the mean value of a hundred and eighteen and a hundred and fifty-two and then you multiply with the number of weeks and how many hours per week. Yes … should work … it evens itself out on a very long … the longer time he works it is working very well to do like that yes

In this sequence, one of the students presents a solution that the teacher seems to regard as just as good as the one in the textbook. Taking the mean value of 118 and 152 you get the solution that Svante earns $135 \cdot y \cdot x$ kr. The teacher looks at the solution and approves it, adding that it works because it evens itself out in the long run.

Less than one minute after that, the teacher approaches a pair of students that also are working with the same task. He starts by saying that "yes it is quite difficult", then he funnels the students through the task to the same answer as the textbook, without any sign of hesitation.

Transcript 7:

Teacher	How many weeks are there … *x* it is yes … how many of these will he work days and how many will he work nights?
Student A	Fifty percent
Teacher	Yes, half the time you can say yes. How do you write half of *x*? How do you write that? Wait … *x* divided by two you can write then … *x* divided by two or zero point five *x* you can also write. That's the same yes … a half *x*.
Student B	One hundred and eighty *y* times zero point five *x*
Teacher	Yes, you can write that … and then the other thing … then you have to reckon the number of hours to yes … so it also is included there. How many hours he works per week … that is … this
Student B	(…)
Teacher	Yes, it is *y* yes … sure … yes. If you put *y* times *x* half and *x* half again yes … then you get a complete formula that works.

At the beginning, the teacher is obviously not aware of the difficulty that is embedded in the particular textbook tasks. When he realizes that there is a problem in the textbook he has to decide whether to stick to what he thinks is a correct solution or follow the textbook. One can see that the teacher-student interaction changes during the lesson and different students encounter different types of signals about the solution.

4. Critical incident 2

A second example of a critical incident is chosen because it highlights the role of the teacher in a teacher-student-task interaction. The example is taken from a session of public interaction, i.e. the part of the lesson where the teacher stands in front of the class organizing discussions and writing on the board, but it origins from the lesson before, in the Kikan-Shido part of the lesson.

Early in the lesson, the teacher talks about division by negative numbers. The topic seems to be a deviation from what the class is working with in the current chapter of the textbook, which is about formulae. The teacher explains that he wants to discuss an interesting question that was raised by one of the students in the previous lesson. The student, John[xv], was working with the following task in the textbook:

> **5080**
> In USA and many other countries the temperature is measured in Fahrenheit (°F). The relationship between the scale of Fahrenheit and our scale is the following,
> $C = \frac{F-32}{1{,}8}$, where F = temperature in degree of Fahrenheit and C = temperature in degree of Celsius. Use the formula and calculate how many degree in Celsius corresponds to
> a) 212°F b) 32°F c) 14°F
>
> *The solutions, which one can find at the end of the textbook, are the following: a) 100°C; b) 0°C; and c) -10°C.*

In the preceding lesson, John attracted the teacher's attention and asked him if the numbers always get bigger if one divides a negative number.

Transcript 8:

John	When you divide a negative number, they are getting bigger? If you divide minus eighteen you get minus ten.
Teacher	How do you mean?
John	If one divides a negative number it is getting bigger.
Teacher	No, it doesn't have to be that way … it depends … the important thing is whether the number … so to speak … is bigger or smaller than one. If you divide a number that is smaller … in between zero and one …
John	[interrupts the teacher] But if it is less than zero … if it is above one … look … look here [shows the result on the calculator].
Teacher	Well, okay.
John	(…)
Teacher	If you put in a value here that … what was it … what did the formula look like [searching in the textbook]
John	It is minus eighteen.
Teacher	That's right.
John	It is divided.
Teacher	Then you get what … you get minus.

John — That will be minus eighteen … or? Yes, it will be minus eighteen up there. Then divide it by one point eight. It will be minus ten, wouldn't it? That's what the calculator says anyhow.

Teacher — Hmm, it will be minus ten … for sure

John — But that was what I was asking about.

Teacher — Well, okay … yes, yes. No, I didn't quite understand what you meant.

John — Obviously!

What John discovers when he is working with the task is that $\frac{14-32}{1,8}$ = $\frac{-18}{1,8}$ = -10, i.e. that the quotient is bigger than the numerator. He seems however to be rather annoyed and unsatisfied with the response from the teacher. The teacher, on the other hand, appears to be a bit puzzled when he walks away from John's desk. About five minutes later[xvi], another student calls on the teacher. He is working with the same task (problem c) and his reaction is that "it is raising". Again, the teacher seems puzzled but the student repeats, "raising". "Yes, but you are dividing a negative number, so …", the teacher says. He does not finish the sentence; he just walks away from the student's desk. However, at the beginning of the next lesson, the teacher picks up the question again. The teacher starts by writing $\frac{-18}{1,8}$ on the blackboard, which was a part of the solution of problem c, and asks for the solution. Then he writes $\frac{18}{2}$ = 9 on the board and asks the students what happens to the quotient.

Transcript 9:

Teacher — I take eighteen and divide it by two, which is, as you all know, nine. This number is bigger than one, isn't it [he makes an arrow from 2 and writes >1]. Two is bigger than one and you write it like this. What is happening with the quotient here [points at the 9]? When I divide with a number that is bigger than one, what happens to the solution, the quotient here? It is of course …

Student — (…)

Teacher — It's getting smaller yes [writes <]. Nine is smaller than the number that we divide. Nine is smaller than eighteen, isn't it?

The teacher continues writing on the board and asks about the solution of $\frac{18}{0.5}$. One of the students says that it is thirty-six "because it is the same as multiplying with two if it is divided by zero point five". That's right, the teacher says.

Transcript 10:

Teacher	We divide by a number that is less than one [writes an arrow from 0.5 and writes < 1] … then the quotient is bigger. Thirty-six is bigger than eighteen

The teacher continues in the same manner with $\frac{-18}{2}$ and $\frac{-18}{0.5}$, comparing quotient and numerator. At the end of this sequence, which lasts a little bit more than four minutes, the teacher summarizes and generalizes the results by saying that one gets a reversed result when dividing negative numbers compared to the positive numbers.

What does this story tell us? Well, the episode starts in the previous lesson (transcript 8) when the students are working by themselves solving tasks in the textbooks. It is in the interaction between the teacher and two of the students when they are working on an individual basis solving tasks in the textbook. Some of the students raise questions that the teacher, at first, did not understand. But in the lesson that follows, the teacher seems to become conscious about the learning opportunity that the questions could create. He takes the empirical phenomenon that the student discovered (when you divide a negative number it gets bigger?) as a starting-point when he bring up the questions in front of the whole class.

5. Discussion and conclusion

In the 'standard' pattern of interaction, the teacher interacts with the students in a confident way in order to help them solve tasks in the textbook. Sometimes he just controls their answers or gives hints, which could help them start working on the task and sometimes he helps them to arrive at a solution.

In the first example of critical incidents (transcripts 3 to 7), the teacher has to decide whether to keep to his judgment that the formula does not work for an odd number of weeks or accept the solution in the textbook. As an observer, it is difficult to understand why the teacher, in this case, alters between these two standpoints. One reason can be that he believes that some of the students will have problems to understand the logic, at least one of them did not follow his line of argument (see transcript 5). However, besides that the students are regarded as high-achievers the teacher is very experienced and, in general, shows no hesitation when it comes to correct mistakes made by the students. So, why this time? The decision is perhaps made more or less unconsciously and one can only speculate about the reasons behind it. One thing is however clear, it would make a difference if the textbook did not present any answers. The result, this time, is that the teacher does not argue against the solution in the textbook, which in this case becomes the authority.

In the second example of critical incidents, one of the students makes a statement (or question) that can be interpreted as an attempt to generalize: *When you divide a negative number, they are getting bigger*. It is a task, a quite ordinary task in the textbook that triggers him to call for the teacher and discuss this issue. At first, the teacher does not understand what the student means. But at some point, the teacher decides to leave the book and make a whole class discussion about this subject. What is the role of the task in this case? Obviously, without a response from a teacher, the task will be as any other task in the textbook. It will probably not lead to a general discussion about mathematical properties. Nevertheless, the discussion origins from a question that is raised by a student working on a specific task, which makes him think in terms of mathematical generalizations.

The analysis above shows how the textbook influences the teacher-student interaction in the Kikan-Shido part of the lesson. First and foremost, the tasks in the textbook guide the activity of the students. Their work is to find a solution to each problem. The teacher walks around, looks over the shoulders, asks and answers questions that are related to the tasks. One can say that the activity is 'framed' by the textbook, which, like a painting, offers a static picture of mathematics. The picture is colored by both pedagogical ideas and traditions. In the 'standard' pattern of interaction, the teacher becomes the guide who explains and clarifies. Both textbook and teacher can be regarded as the authority since it is the textbook that offers the text and the tasks but it is the teacher who selects the tasks and sometimes guides the students (or funnels them) to the correct solution.

The situation changes however when there is a discrepancy between the answer in the textbook and what the teacher thinks is a correct solution (transcripts 3 to 7). The teacher becomes ambiguous. Instead of arguing against the textbook, and perhaps address the question to the whole class, he interacts with the students one by one. The teacher's mathematical knowledge seems to be less important in this case and the result is an inconsistent mathematical meaning making. The frame, which is constituted by the textbook, is kept intact and the book becomes the authority.

But in the second example of critical incidents (transcript 8 and onwards), Mr. Larsson shows that he can go outside the frame and deviate from the textbook. So even if the textbook is influential, it is not in charge of everything. This I think is very important to highlight. First of all, teachers need to be aware of how they use their books and secondly, they should act as the textbooks' superior.

Furthermore, the analysis presented in this paper can serve as an illustration of the complexity of a mathematics classroom. In order to understand the decision made by the teacher in the first example of critical incidents (transcript 3 to 7), it is not enough to consider the teacher's mathematical knowledge or beliefs about teaching and learning mathematics, something else has to be taken into account. I argue that this 'something' can be the textbook, or rather the role of the textbook in the classroom.

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6. References

Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education. Intention, reflection, critique* (Vol. 29). New York: Kluwer.

Bauersfeld, H. (1979). Research related to the mathematical learning process. In *ICMI: New trends in mathematics education, Vol IV* (pp. 199-213). Paris: Unesco.

Bauersfeld, H. (1988). Interaction, construction, and knowledge: alternative perspectives for mathematics education. In D. A. Grouws, T. J. Cooney & D. Jones (Eds.), *Perspective on research on effective mathematics teaching* (Vol. 1, pp. 27-46). Reston, VA: National Council of Teachers of Mathematics.

Clarke, D. (2004). *Kikan-Shido - between desks instruction.* Paper presented at the Annual meeting of AERA, San Diego, USA.

Englund, B. (1999). Lärobokskunskap, styrning och elevinflytande. *Pedagogisk forskning i Sverige, 4*(4), 327-348.

Englund, T. (1997). Towards a dynamic analysis of the content of schooling: narrow and broad didactics in Sweden. *Journal of Curriculum Studies, 29*(3), 267-287.

Fan, L. (2000). The influence of textbooks on teaching strategies: an empirical study. *Mid-Western Educational Researcher, 13*(4), 2-9.

Jacobs, J., Garnier, H., Gallimore, R., Hollingsworth, H., Givvin, K. B., Rust, K., et al. (2003). *Third International Mathematics and Science Study 1999, Video study technical report, Volume 1: Mathematics*: NCES.

Johansson, M. (2003). *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum* (Licentiate thesis). Luleå: Department of Mathematics, Luleå University of Technology.

Johansson, M. (2005). The mathematics textbook: from artefact to instrument. *Nordic Studies in Mathematics Education, NOMAD, 10*(3-4), 43-64.

Kilborn, W. (1979). *Pumprojektet: bakgrund och erfarenheter*. Stockholm: Liber Utbildningsförlaget.

Lindqvist, U., Emanuelsson, L., Lindström, J.-O., & Rönnberg, I. (2003). *Lusten att lära - med fokus på matematik* (No. 221). Stockholm: Statens skolverk.

Luke, C., de Castell, S., Fraser, S., & Luke, A. (1989). Beyond criticism: The authority of the school textbook. In S. de Castell, A. Luke & C. Luke (Eds.), *Language, authority and criticism: Readings on the school textbook*. London: The Falmer Press.

Lundgren, U. P. (1983). *Att organisera omvärlden. En introduktion till läroplansteori.* Stockholm: Liber.

Remillard, J. T. (2000). Can curriculum materials support teachers' learning: Two fourth-grade teachers' use of a new mathematics text. *The Elementary School Journal, 100*(4), 331-350.

Selander, S., & Skjelbred, D. (2004). *Pedagogiske tekster: för kommunikasjon og læring*. Oslo: Universitetsforlaget.

Sfard, A. (2005). What could be more practical than good research? On mutual relations between research and practice of mathematics education. *Educational Studies in Mathematics, 58*, 393-413.

Skott, J. (2001). The emerging practices of a novice teacher: the roles of his school mathematics images. *Journal of Mathematics Teacher Education, 4*, 3-28.

Stray, C. (1994). Paradigms regained: towards a historical sociology of the textbook. *Journal of Curriculum Studies, 26*(1), 1-29.

Undvall, L., Olofsson, K.-G., & Forsberg, S. (2002). *Matematikboken Y Röd*. Stockholm: Liber.

Wilson, M. S., & Goldenberg, M. P. (1998). Some conceptions are difficult to change: One middle school mathematics teachers' struggle. *Journal of Mathematics Teacher Education, 1*, 269-293.

Voigt, J. (1994). Negotiation of mathematical meaning and learning mathematics. *Educational Studies in Mathematics, 26*, 275-298.

Notes

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[i] In 1989, the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards.* The *Standards* call for increased emphasis on mathematical reasoning, understanding, and problem solving. In many school districts, the first step to respond to the call for changes was to adopt a new textbook. Research on teaching raises however questions about the effectiveness of this way to implement the reforms (Remillard, 2000).

[ii] Johansson (2003) examines three editions of this particular textbook series (published 1979, 1985, and 2001) and states that the books are in many respects comparable.

[iii] The frame factor theory, or model, origins from work of the Swedish educationalist U. Dahllöf and his colleagues in the 1960s. In its early stage, its focus is on how political decisions regarding teaching and education (e.g. time schedules, grouping, etc.) influence the pedagogical work.

[iv] Information about the study can be found on: http://www.ped.uu.se/kult/default.asp. The fieldwork and the data collection in the CULT-project is based on the research de-

sign set out for the Learner's Perspective Study (http://extranet.edfac.unimelb.edu.au/DSME/lps/).

[v] The software that is used for the coding and transcribing procedure is Videograph.

[vi] Appendix I in TIMSS 1999 Video Study, Mathematical video coding manual, http://nces.ed.gov/pubs2003/2003012_C.pdf (retrieved 2006-02-03)

[vii] The name Mr. Larsson is fictitious.

[viii] The textbook is *Matematikboken Y Röd* (Undvall, Olofsson, & Forsberg, 2002). According to the authors, the book is intended to be used by students that are interested and have good skills in mathematics.

[ix] The interview is made after the seventh lesson by means of a stimulated recall technique (SW3-L07-IT1, time 00:29:00)

[x] The transcript is from the second video-recorded lesson in the CULT data (SW3-L02, time 00:24:02)

[xi] The transcript is from the second video-recorded lesson in the CULT data (SW3-L02, time 00:40:50)

[xii] Skott (2001) uses the term *critical incident of practice* as an analytical focal point to high-light lesson episodes in which a teacher decision making is critical; to his *School Mathematics Images,* to the further development of the classroom interaction, and for the students' learning opportunities.

[xiii] The transcript is from the fifth video-recorded lesson in the CULT data (SW3-L05, time 00:19:10)

[xiv] The formula works fine if one separates the number of working weeks, for example let x_1 be the number of weeks he works days and x_2 the number of weeks he works nights and write: $(x_1 \cdot y \cdot 118 + x_2 \cdot y \cdot 152)$ kr. The textbook suggests the following answer: $(\frac{x}{2} \cdot y \cdot 118 + \frac{x}{2} \cdot y \cdot 152)$ *kr.*

[xv] The name John is fictitious.

[xvi] The transcript is from the fourth video-recorded lesson in the CULT data (SW3-L04, time 00:40:03)